Computation of Robust PI Controller for Systems with Parametric Uncertainty

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Abstract—This paper describes a new technique of PI controller for systems with parametric uncertainty. A PI controller is designed using necessary and sufficient condition for robust Hurwitz polynomial. The method is illustrated through a typical numerical example available in the literature.

Index Terms — Kharitonov’s theorem, Hurwitz polynomial, parametric uncertainty, PI controller.

I. INTRODUCTION

There has been a great amount of research work on the tuning of PI, PID and lag/lead controllers since these types of controllers have been widely used in industries for several decades [1-5]. However, many important results have been recently reported on computation of all stabilizing P, PI and PID controllers after the publication of work by Ho et al. [6-9]. Robust stability analysis with uncertain parameters has been very important research topic. Since control systems operate under large uncertainty present in the control system causes degradation of system performance and destabilization. An important approach to this subject via expressing the characteristic polynomial by an interval polynomial i.e., a polynomial by whose coefficient each varies independently in a prescribed interval. The stability analysis of polynomials subjected to parameter uncertainty have received considerable attention after the celebrated theorem of Kharitonov [11], which assess robust stability under the condition that four specially constructed extreme polynomials, called Kharitonov [11], polynomials are Hurwitz.

II. PROBLEM STATEMENT

Consider the set of real polynomials of degree n of the form

\[ a(s) = a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n \quad \cdots (1) \]

Where the coefficients lie within given region

\[ a_0 \in [x_0, y_0], a_1 \in [x_1, y_1], \cdots, a_n \in [x_n, y_n] \quad \cdots (2) \]

We assume that the degree remains invariant over the family, so that a \( \not\in [x_n, y_n] \) such a set of polynomial called a real interval family and is referred as an
interval polynomials. The set of polynomials given by is stable if and only if each and every element of the set is a Hurwitz polynomial. A necessary and sufficient condition for robust stability of interval polynomial is proposed using the algebraic stability criterion for fixed polynomial due to Nie which is stated in the following Lemmas

**Lemma1**: The interval polynomial $A(s)$ defined in (1) is Hurwitz for all $a_i \in [x_i, y_i]$ where $i=0,1,2,\ldots,n$. If the following necessary conditions are satisfied

$$y_i \geq x_i > 0, \quad i = 0,1,2,\ldots,n$$
$$x_i x_{i+1} > y_{i-1} y_{i+2}, \quad i = 0,1,2,\ldots,n-2$$

**Lemma2**: The interval polynomial $A(s)$ defined in (1) is Hurwitz for all $a_i \in [x_i, y_i]$ where $i=0,1,2,\ldots,n$. If the following sufficient conditions are satisfied

$$y_i \geq x_i > 0, \quad i = 0,1,2,\ldots,n$$
$$0.4655 x_i x_{i+1} > y_{i-1} y_{i+2}, \quad i = 0,1,2,\ldots,n-2$$

Consider a system whose transfer function with parametric uncertainty is given by

$$G(s, b, a) = \frac{N(s, b)}{D(s, a)}$$

Where the numerator and denominator polynomials are of the form

$$N(s, b) = b_0 + b_1 s + \cdots + b_m s^m$$
$$D(s, a) = a_0 + a_1 s + \cdots + a_n s^n$$

Where vectors $b$ and $a$ lie in given rectangles $B$ and $A$ respectively.

$$a \in A : \{a : a_i^- \leq a_i \leq a_i^+\} \quad \text{for } i = 0,1,\ldots,n$$
$$b \in B : \{b : b_i^- \leq b_i \leq b_i^+\} \quad \text{for } i = 0,1,\ldots,n$$

Where $a_i \in [1,1]$ and the bound on $a_i, a_i^-, b_i, b_i^+$ are specified a priori

Let PI controller transfer function in parametric uncertainty form is given by

$$C(s) = \frac{N_c(s)}{D_c(s)} = K_p + \frac{K_i}{s}$$

where $K_p \in [K_{p_{\text{min}}}, K_{p_{\text{max}}}]$

$$K_i \in [K_{i_{\text{min}}}, K_{i_{\text{max}}}]$$

The characteristic equation of closed loop system of reduced model with PI controller is given as

$$N_c(s)N(s) + D_c(s)D(s) = 0$$

The values of $K_p$ and $K_i$ in parametric uncertain form are obtained by solving the characteristic equation with Routh’s criterion. Closed loop control of the system with PI controller is shown in Fig.1.

**III. NUMERICAL EXAMPLE**

Consider a higher order system whose transfer function with uncertainty is given by [24].

$$G(s) = \frac{[28.5,30.5]s^2 + [6.935,8.935]s}{[1.1]s^6 + [17.47,19.47]s^5 + [46.78,48.78]s^4 + [67.52,69.52]s^3 + [64.86,66.86]s^2 + [43.3,45.3]s + [14.16,16.16]}$$

Let the PI Controller transfer function in parametric uncertainty is given by

$$C(s) = \left[k_{p_{\text{min}}}, k_{p_{\text{max}}} \right] + \frac{[k_{i_{\text{min}}}, k_{i_{\text{max}}}]}{s}$$

By applying necessary and sufficient conditions the values of $K_p$ and $K_i$ in parametric uncertainty are obtained by using the equations (3), (4) and (7).
The PI controller transfer function in parametric uncertainty is given by
\[
C(s) = [0.28384,0.4070] + \frac{[0.00164,0.07969]}{s}
\]

The closed loop step response of the system with PI controller is shown in Fig 2 and Fig 3.

It is observed from the Fig 2 and Fig 3 that the designed PI controller obtained from the proposed method stabilizes the higher order uncertain system.

V. CONCLUSION

A PI controller is designed for higher order uncertain systems from robust Hurwitz polynomial. The proposed PI controller procedure is illustrated through a typical numerical example available in the literature.

It is observed from the simulation results of Fig 2 and Fig 3, that the designed PI controller obtained from the proposed method stabilizes the higher order uncertain system.

VI. REFERENCES

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VII.BIOGRAPHIES

Mangipudi Siva Kumar was born in Amalapuram, E. G. Dist, Andhra Pradesh, India, in 1971. He received bachelor’s degree in Electrical & Electronics Engineering from JNTUCollege of Engineering, Kakinada and M.E and PhD degree in control systems from Andhra University College of Engineering, Visakhapatnam, in 2002 and 2010 respectively. His research interests include model order reduction, interval system analysis, design of PI/PID controllers for Interval systems, sliding mode control, Power system protection and control. Presently he is working as Professor & H.O.D of Electrical Engineering department, Gudlavalluru Engineering College, Gudlavalluru, A.P, India. He received best paper awards in several national conferences held in India.

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