

Design of PI Sliding Mode Controller For Magnetically Suspended Balance Beam System

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Abstract— This paper presents the design of an Proportional Integral sliding mode controller(PISMC) for a highly unstable non-linear system , magnetically suspended balanced beam(MSBB) and analysis of its effects. The modelling of Magnetically Suspended Balance Beam (MSBB) and design of Integral sliding mode controller are carried out using MATLAB/SIMULINK. The robustness of the controller is investigated and the performance of the Integral sliding mode controller is compared with that obtained by a conventional state feedback controller with integral action when a disturbance force applied.

Index Terms— non-linear system , MSBB, State feedback controller, gap displacement, Proportional Integral sliding mode controller (PISMC).

I. INTRODUCTION

For the control of nonlinear systems, Sliding mode control is a robust technique. The complete compensation of disturbances acting on the control input channel is the most positive feature of PI sliding mode control is it does not have reaching phase; hence it ensures insensitivity of the desired trajectory with respect to matched uncertainties starting from the initial moment. These properties make attractive the study of ISM, when the system is in the *sliding phase* and a *sliding mode* is enforced. The compensated dynamics become insensitive to matched disturbances and uncertainties under sliding mode control. This insensitivity causes the chattering in control when the system dynamics are vulnerable to disturbances/uncertainties

The proportional integral sliding mode (PISMC) technique was first proposed by V.I.Utkin et.al as a solution to the reaching phase problem for systems with matched disturbances only. The ISM control can also be regarded as a way to combine the use of the sliding mode controller with that of Integral action controller. The latter aims at stabilizing the nominal system. Systems compensated with this type of controllers are of full order. When the system is subjected to

external bounded perturbations, it is natural to try to compensate such perturbations by means of an auxiliary control that retains the effect of the controller designed for the unperturbed system. The sliding mode based auxiliary controller that compensates the perturbation from the very beginning of the control action, while retaining the order of the uncompensated system, is the PISMC controller.

II. MATHEMATICAL MODEL OF MAGNETICALLY SUSPENDED BALANCE BEAM SYSTEM

The Magnetically Suspended Balance Beam (MSBB) is a balancing system that used two magnetic coils to balance the beam as shown in Figure 1. These two magnetic coils are placed at each end of the beam, one at the right hand side and one in the left hand side. It can be easily described as a small see-saw.

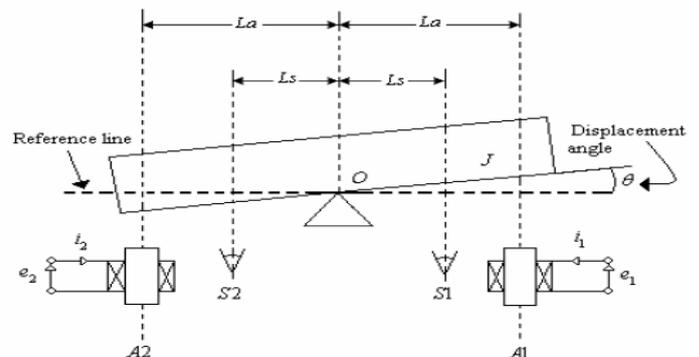


Figure 1: Symmetric Balance Beam on two Magnetic Bearings

The main objective is to control the gap displacement angle of the beam. If the gap displacement angle is equal to the set point, it can be concluded that the designed controller is successful in controlling the angle and make the beam become stable.

The MSBB consists of two magnetic bearings, two sensors and the balanced beam. The geometry of the symmetric

balance beam with two horseshoe shaped magnetic bearings A_1 and A_2 . are shown in Figure 1. These magnetic bearings will produce a force to make displacement angle, θ of the balanced beam equal to the set point. The magnitude of the force can be controlled by controlling the input voltage, e_1 and e_2 . The sensors S_1 and S_2 are used to detect the displacement angle of the beam from the reference line. Table 1 shows each parameter of the balance beam system.

The torque that can be produced at the balanced beam if there is a ‘pulling action’ from either one of the magnetic bearings can be described by

$$J\ddot{\theta} = L_a(f_1 - f_2) + f_d \quad (1)$$

The force that had been produced at A_1 in the balanced beam can be represented mathematically as

$$f_1 = \mu_0 A_g N^2 \frac{(i_0 + i'_1)^2}{2(g_0 + L_a\theta)^2} \quad (2)$$

Similarly, the force that had been produced at A_2 in the balanced beam is

$$f_2 = \mu_0 A_g N^2 \frac{(i_0 + i'_2)^2}{2(g_0 - L_a\theta)^2} \quad (3)$$

By using the kirchoff voltage law, the control voltage, e_1 at magnetic bearing, A_1 is

$$e'_1 = Ri'_1 + L \frac{di'_1}{dt} \quad (4)$$

Similarly, the control voltage, e_2 at magnetic bearing, A_2 is

$$e'_2 = Ri'_2 + L \frac{di'_2}{dt} \quad (5)$$

Table 1: Parameter of the Balance Beam System

Parameter	Symbol	Value	Units
Angular Position	θ	-	rad
Half Bearing Position	L_a	0.1412	m
MassMI about pivot position	J	0.0948	Kgm ²
Coil current in bearing 1	i_1	-	A
Coil current in bearing 2	i_2	-	A
Control voltage in bearing 1	e_1	-	V
Control voltage in bearing 2	e_2	-	V
Coil resistance	R	0.7	Ω
Coil inductance	L	0.728	mH
MB open loop stiffness	K_x	2826.32	N/m
Actuator current gain	K_i	1.074	N/A
Steady current	i_0	1	A
Steady gap	g_0	380	μm

where,

- J Mass moment of inertia about the pivot point
- L_a Half bearing span
- L_s Distance from pivot point to sensor
- f_1, f_2 Electromagnetic attractive force on bearings 1&2
- f_d External disturbance force
- μ_0 Permeability of free space

- N Number of turns in coil
- A_g Pole face area
- R Coil resistance
- L Coil inductance
- e'_1, e'_2 Control voltages in bearing 1&2
- i'_1, i'_2 Instantaneous coil currents in bearing 1&2

Equations (1)-(5) can be linearized with the assumption that the $(i'_1)^2, (i'_2)^2$ and $(\theta)^2 \approx 0$.

$$f_1 = \frac{\mu_0 A_g N^2 (i_0^2 + 2i_0 i'_1 + (i'_1)^2)}{2(g_0)^2 + 4g_0 L_a \theta + 2L_a^2 \theta^2} \quad \text{and}$$

$$f_2 = \frac{\mu_0 A_g N^2 (i_0^2 + 2i_0 i'_2 + (i'_2)^2)}{2(g_0)^2 - 4g_0 L_a \theta + 2L_a^2 \theta^2} \quad (6)$$

By assuming $(i'_1)^2, (i'_2)^2$ and $(\theta)^2 \approx 0$, Thus:

$$f_1 = \frac{\mu_0 A_g N^2 (i_0^2 + 2i_0 i'_1)}{2(g_0)^2 + 4g_0 L_a \theta} \quad \text{and}$$

$$f_2 = \frac{\mu_0 A_g N^2 (i_0^2 + 2i_0 i'_2)}{2(g_0)^2 - 4g_0 L_a \theta} \quad (7)$$

The total current at the right hand side coil A_1, i_1 is the total between the steady current, i_0 and the instantaneous current, (i'_1) . so $i_1 = i_0 + i'_1$ and the total current at the left hand side coil A_2 , is the total between the steady current, i_0 and the instantaneous current, (i'_2) . i.e. $i_2 = i_0 + i'_2$. If $i_1 + i_2$, the answer will be zero because $i'_2 = -i'_1$. But if the overall instantaneous, i' will exist can be written as:

$$i' = i'_1 - i'_2 \quad (8)$$

The overall forces, F that exist in MSBB system can be calculated as follows:

$$F = f_1 - f_2 = \frac{\mu_0 A_g N^2 (i_0^2 + 2i_0 i'_1)}{2(g_0)^2 + 4g_0 L_a \theta} - \frac{\mu_0 A_g N^2 (i_0^2 + 2i_0 i'_2)}{2(g_0)^2 - 4g_0 L_a \theta}$$

Considering $(\theta)^2 \approx 0$, Thus;

$$F = f_1 - f_2 = \frac{(\mu_0 A_g N^2 i_0^2 + 2i_0 i'_1 \mu_0 A_g N^2)(2(g_0)^2 - 4g_0 L_a \theta)}{4(g_0)^4} - \frac{[(\mu_0 A_g N^2 i_0^2 + 2i_0 i'_2 \mu_0 A_g N^2)][(2(g_0)^2 + 4g_0 L_a \theta)]}{4(g_0)^4} \quad (9)$$

Let us assume,

$$\begin{aligned}
 A &= \mu_0 A_g N^2 i_0^2 & B &= 2i_0 i_1' \mu_0 A_g N^2 & C &= 2(g_0)^2 \\
 D &= 4g_0 L_a \theta & E &= 2i_0 i_2' \mu_0 A_g N^2 \\
 F &= \frac{(A+B)(C-D)}{4(g_0)^2} - \frac{[(A+E)(C+D)]}{4(g_0)^2} \\
 F &= -\frac{2(\mu_0 A_g N^2 i_0^2)(4g_0 L_a \theta)}{4g_0^4} & (10) \\
 &+ \frac{2(g_0)^2 [2i_0 \mu_0 A_g N^2 (i_1' - i_2')]}{4g_0^4} \\
 &+ \frac{4g_0 L_a \theta [2i_0 \mu_0 A_g N^2 (i_1' - i_2')]}{4g_0^4}
 \end{aligned}$$

But, $\frac{4g_0 L_a \theta [2i_0 \mu_0 A_g N^2 (i_1' + i_2')]}{4g_0^4} = 0$, because

$$i_1' + i_2' = 0$$

Thus,

$$\begin{aligned}
 F &= f_1 - f_2 = -\frac{2\mu_0 A_g N^2 i_0^2 L_a \theta}{g_0^3} + \frac{\mu_0 A_g N^2 i_0 (i')}{g_0^2} & (11) \\
 &= -2K_x L_a \theta + K_i i'
 \end{aligned}$$

Where $K_x = \frac{\mu_0 A_g N^2 i_0^2}{g_0^3}$ $K_i = \frac{\mu_0 A_g N^2 i_0}{g_0^2}$

From equation (1)

$$J \ddot{\theta} = L_a (f_1 - f_2) + f_d$$

$$\ddot{\theta} = \frac{(2K_x L_a^2 \theta)}{J} + \frac{K_i L_a i'}{J} + \frac{f_d}{J}$$

Define an auxiliary equation:

$$\begin{aligned}
 e' &= e_1' + e_m - (e_2' - e_m) = 2e_m + e_1' - e_2' & (12) \\
 &= 2K_i \dot{\theta} + e_1' - e_2'
 \end{aligned}$$

Where e_m is back emf and $e_m = K_i \dot{\theta}$

Substituting $e_1' = R i_1' + L \frac{di_1'}{dt}$ and $e_2' = R i_2' + L \frac{di_2'}{dt}$

into equation (10),

$$\begin{aligned}
 e' &= 2K_i \dot{\theta} + R i_1' + L \frac{di_1'}{dt} - R i_2' - L \frac{di_2'}{dt} \\
 \frac{e'}{L} - \frac{2K_i \dot{\theta}}{L} - \frac{R(i_1' - i_2')}{L} &= \frac{d(i_1' - i_2')}{dt} \\
 \frac{e'}{L} - \frac{2K_i \dot{\theta}}{L} - \frac{R(i')}{L} &= \frac{d(i')}{dt} & (13)
 \end{aligned}$$

From (10) and (11), the state space equation of the MSBB system shown in figure 1 can be represented as:

$$\dot{X} = AX + Bu + E f_d \quad (14)$$

$$Y = CX$$

Where,

$$X = [x_1 \quad x_2 \quad x_3]^T = [\theta \quad \dot{\theta} \quad i']^T$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 \frac{K_x L_a^2}{J} & 0 & \frac{K_i L_a}{J} \\ 0 & -2 \frac{K_i}{L} & \frac{-R}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}; \quad E = \begin{bmatrix} 0 \\ \frac{1}{J} \\ 0 \end{bmatrix} \quad \text{and} \quad C = [1 \quad 0 \quad 0]$$

$$u = e' \quad (15)$$

E is an external disturbance matrix.

Rearrange equation (14) gives

$$\begin{aligned}
 \dot{x}_1 &= \dot{\theta} = x_2 \\
 \dot{x}_2 &= \ddot{\theta} = -2 \frac{K_x L_a^2}{J} \theta + \frac{K_i L_a}{J} i' + \frac{1}{J} f_d & (16) \\
 \dot{x}_3 &= i' = -2 \frac{K_i}{L} \dot{\theta} - \frac{R}{L} i' + \frac{1}{L} u
 \end{aligned}$$

Equation (16) can be written in a vector-matrix form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -2 \frac{K_x L_a^2}{J} & 0 & \frac{K_i L_a}{J} \\ 0 & -2 \frac{K_i}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{1}{J} \\ 0 \end{bmatrix} f_d$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (17)$$

By using the parameters tabulated in Table 1, the linearized model of the MSBB system can be computed as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1188.8 & 0 & 1.6 \\ 0 & -2951 & -962 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1374 \end{bmatrix} u + \begin{bmatrix} 0 \\ 10.55 \\ 0 \end{bmatrix} f_d$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (18)$$

III. STATE FEEDBACK CONTROLLER DESIGN

This design technique begins with a determination of the desired closed-loop poles based on the transient-response and/or frequency-response requirements, such as speed, damping ratio, or bandwidth, as well as steady-state requirements for displacement angle control of MSBB system the desired specifications are 10% of overshoot, and Settling time of 0.1 secs.

The obtained values of damping ratio and the natural frequency are $\xi = 0.6$ and $\omega_n = 66.67$ rad/sec (19)

The desired characteristic equation and poles are

$$s^2 + 80s + 4445 = 0$$

$$s_1, s_2 = -40 \pm j53.33 \quad (20)$$

A. *Design of State Feedback Controller (Without Integral Control)*

The poles are $s = -40 + j53.33$ and $s = -40 - j53.33$. To design state feedback controller using pole placement technique, one more pole is required to be added to make the characteristic equation as a 3rd order equation. One pole that had been added is $s = -240$. This pole is chosen because they are 6 times bigger than those two dominant poles. So by adding $s = -240$ to the s-plane, the system still behaves like a second order characteristic equation. So the new characteristic equation is:

$$(s + 40 + j53.33)(s + 40 - j53.33)(s + 240) = 0$$

$$s^3 + 240s^2 + 23465s + 1066800 = 0 \quad (21)$$

The feedback vector K can be calculated. Using equation (18),

$$|sI - (A - BK)| = 0$$

$$s[(s^2 + 962s + 1374k_3s) + (4721.6 + 2198.4k_2)]$$

$$- (-1)[(1188.8s + 1143625.6 + 1633411.2k_3) + 2198.4k_1] = 0$$

$$s^3 + (962 + 1374k_3)s^2 + (5910.4 + 2198k_2)s$$

$$+ 1143625.6 + 1633411.2k_3 + 2198.4k_1 = 0 \quad (22)$$

Equation (21) and equation (22) are compared to each other the feedback vector K is:

$$K = [k_1 \quad k_2 \quad k_3] = [312.20 \quad 8.067 \quad -0.5] \quad (23)$$

By substituting equation (23) into u , where u input control voltage

$$u = -KX = -\begin{bmatrix} 312.20\theta & 8.067\dot{\theta} & -0.5i \end{bmatrix}$$

$$u = -312.20\theta - 8.067\dot{\theta} + 0.5i \quad (24)$$

B. *Design of State Feedback Controller with Integral Control*

To design the state feedback controller using the pole Placement technique, two more poles are required to be

added to make the characteristic equation as a 4th order equation. Two more poles that had been added are $s = -240$ and $s = -250$. These two poles are chosen because they are 6 times bigger than those two dominant poles. So by adding $s = -240$ and $s = -250$ to the s-plane, the system still behaves like a second order characteristic equation. So the new characteristic equation is

$$(s + 40 + j53.33)(s + 40 - j53.33)(s + 240)(s + 250) = 0$$

$$s^4 + 570s^3 + 10364s^2 + 6976036s + 266453400 = 0 \quad (26)$$

The feedback vector K can be calculated as

$$|sI - (A' - B'K)|$$

$$= s^4 + (962 + 1374k_3)s^3 + (4721.6 + 2198.4k_2)s^2 + 1188.8s^2$$

$$+ (1,142,436.8 + 1,633,411.2k_3 + 2198.4k_1)s + 2198.4k_{n+1} \quad (27)$$

Equation (26) and equation (27) are compared to each other to gives the required gain vector, K :

Hence, the feedback vector, $K' = [k_1 \quad k_2 \quad k_3 \quad k_{n+1}]$

$$K' = [2865.32 \quad 44.46 \quad -0.285 \quad 121,203.33] \quad (28)$$

The control input is

$$u = -[2865.32 \quad 44.46 \quad -0.285] \begin{bmatrix} \theta \\ \dot{\theta} \\ i' \end{bmatrix} + [121,203.33]x_{n+1} \quad (29)$$

$$= -2865.32\theta - 44.46\dot{\theta} + 0.285i' + 121,203.33x_{n+1}$$

IV. PROPORTIONAL INTEGRAL SLIDING MODE CONTROLLER DESIGN

The design problem of proportional integral sliding-mode controller consists of two items. The first item is concerned with the design of a sliding surface σ upon which the desired dynamic behavior can be guaranteed for the nominal system. The second item is concerned with the selection of a proper nonlinear control law to handle strong nonlinearities of the initial states of the magnetically suspended system.

To enhance the control system rejection ability, an integrator as a state variable is introduced into equation . The integrator output z is expressed as the difference between the integrated reference angular position r and integrated angular position θ written as

$$z = \int (r - \theta) \quad (4.2.13) \quad (30)$$

But $r = 0$ for nominal design. So equation (30) become

$$\dot{z} = -\theta \quad (31)$$

An additional state variable, z has been added hence the order of the system increases by one. The state variables is written as $x = [z \ \theta \ \dot{\theta} \ i']^T$, and the modified state-space model of MSBB i

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{-2K_x L_a^2}{J} & 0 & \frac{K_i L_a}{J} \\ 0 & 0 & -2\frac{K_i}{L} & -\frac{R}{L} \end{bmatrix}, \quad (32)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}, \quad C = [0 \ 1 \ 0 \ 0]$$

The equation (32) is decomposed as:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \quad (33)$$

Using the desired specifications 10% of overshoot, and Settling time of 0.1 secs.

After substituting parameters from Table 1

$$A_{11} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & -1188.8 & 0 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 \\ 0 \\ 1.6 \end{bmatrix}$$

Let the switching surface be defined as $\sigma = SX$ (34)

$$\sigma = [S_1 \ S_2] \begin{bmatrix} X_1 \\ x_2 \end{bmatrix} \text{ where } S = [S_1 \ S_2] \text{ and } S_1 = s_2 K$$

By selecting K as follows and by taking $s_2 = 1$

$$S_1 = [-666270 \ .55 \ 14035.44 \ 200] \quad (35)$$

$$S = [-666270 \ .55 \ 14035.44 \ 200 \ 1]$$

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1188.8 & 0 & 1.6 \\ 0 & 0 & -2951 & -962 \end{bmatrix} \quad (36)$$

$$SA = [0 \ -23167.65 \ -3547.57 \ -929.54] \quad (37)$$

Since (A_{11}, A_{12}) controllable then the control input is

$$u = -(SB)^{-1} [SAX + \rho \text{sgn}(\sigma)] \quad (38)$$

$$SB = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1374 \end{bmatrix}$$

$$(SB)^{-1} = -0.000728 \quad (39)$$

Let $\rho = 2120$

$$SAX = [0 \ -23167.65 \ -3547.57 \ -929.54] \begin{bmatrix} z \\ \theta \\ \dot{\theta} \\ i' \end{bmatrix} \quad (40)$$

$$u = -0.000728 [-23167.65\theta - 3547.57\dot{\theta} - 929.54i' + 2120 \text{sgn}(\sigma)]$$

V. SIMULATION AND RESULTS

The simulated model of highly unstable and non-linear MSBB system in open loop is shown in fig 2

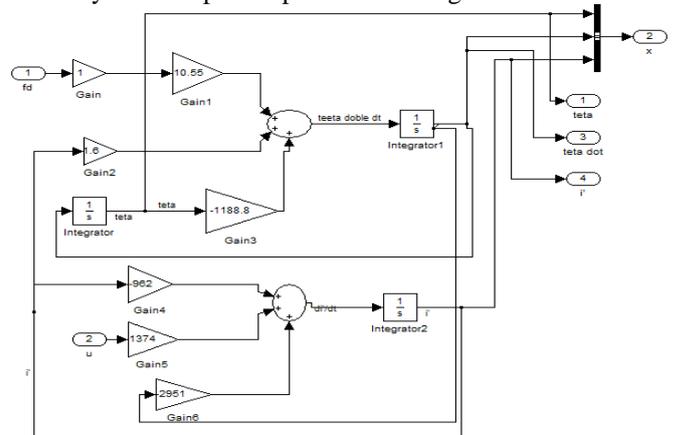


Fig 2: MSBB Matlab/Simulink model

The simulation of MSBB with State Feedback Controller without Integral action and with Integral action using MATLAB/SIMULINK is shown in fig 3 and 4.

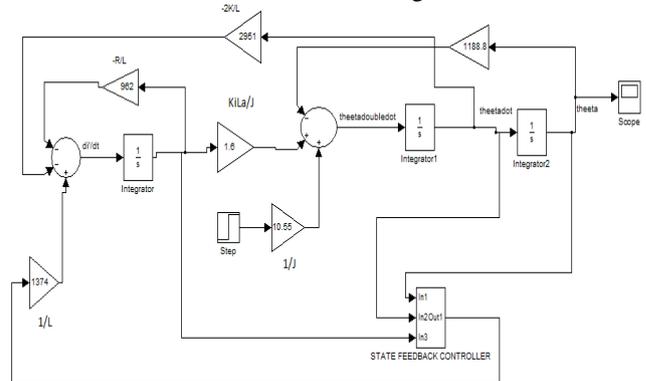


Fig 3: MSBB With State Feedback Controller (without integral control)

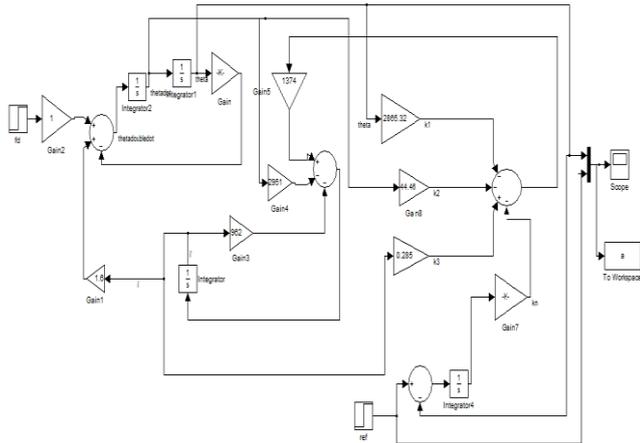


Fig 4: State Feedback Controlled MSBB with Integral action

The comparison of responses (gap displacement) of MSBB with State Feedback Controller without and with integral action using MATLAB/SIMULINK is shown in fig 5

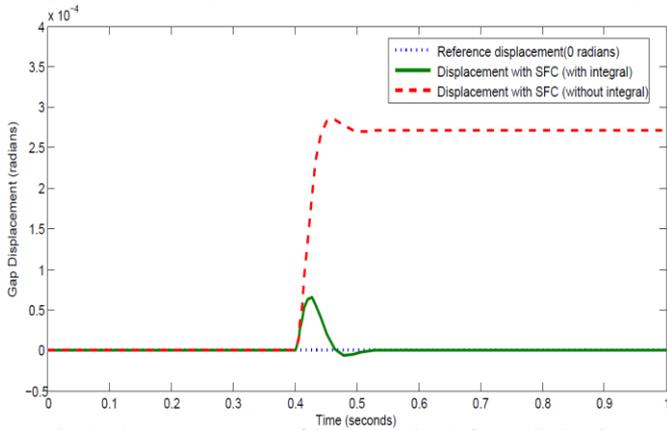


Fig 5: The step response of State Feedback Controlled MSBB

From the figure 5, it can be observed that the designed State Feedback Controller with Integral control stabilizes the balance beam within less time and eliminates steady state error than State Feedback Controller without Integral control after applying a disturbance force 1N-M at 0.4 seconds .

The simulated diagram of MSBB with Proportional integral Sliding Mode Controller using ATLAB/SIMULINK is shown in Fig 6

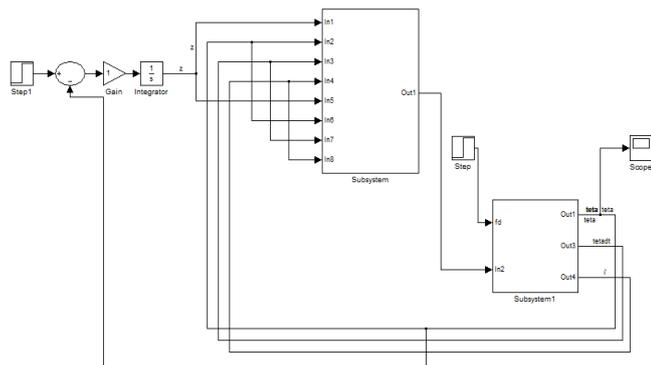


Fig 6: Proportional Integral Sliding Mode Controlled MSBB Matlab/Simulink.

The response (gap displacement) of MSBB with PI SMC using MATLAB/SIMULINK is shown in fig 7

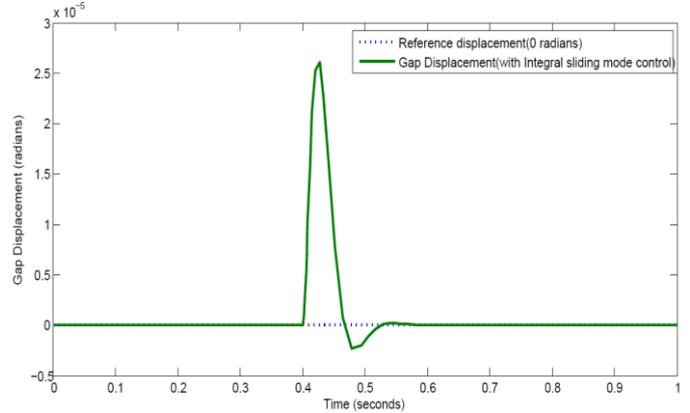


Fig 7: The step response of PISMC Controlled MSBB

From the figure 7, it can be observed that the PISMC stabilizes the balance beam (Zero steady state error) within 0.15 seconds after applying a disturbance force 1N-M at 0.4 seconds.

The comparison of responses (gap displacement) of MSBB with Integral State Feedback Controller and PI Sliding Mode Control using MATLAB/SIMULINK is shown in fig 8.

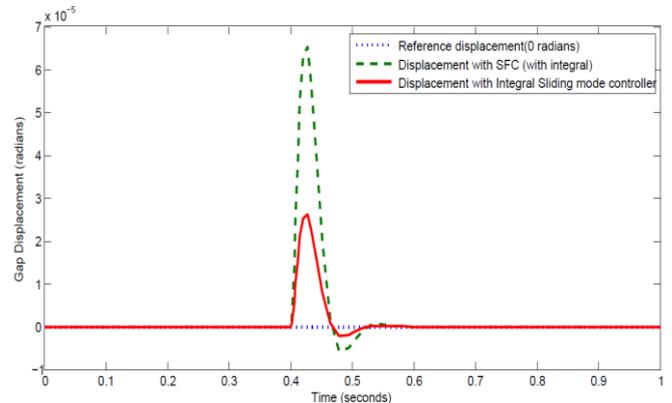


Fig 8:The step response of SFC and PISMC controlled MSBB

From the figure 8, it can be observed that the designed PISMC reduces the peak over shoot in the response(gap displacement) value from 7×10^{-5} radians to 2.5×10^{-5} radians and eliminates steady state error than Integral State Feedback Controller after applying a disturbance force 1N-M at 0.4 seconds

VI. CONCLUSIONS

An approach for controlling the highly openloop unstable, nonlinear system, Magnetically Suspended Balance Beam system(MSBB) has been presented. The simulation of Magnetically Suspended Balance Beam system, State Feedback Controller (with and without integral action) and Integral Sliding Mode Controller are carried out using MATLAB/SIMULINK .

From the simulation results it is observed that in both sliding mode controller and state feedback controller with integral action completely eliminate the steady state error.

It is also observed that the Proportional Integral sliding mode controller drastically reduces the peak overshoot in the response (gap displacement) than the state feedback controller, when the system is subjected to a sudden disturbance.

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