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Interpretation of the Principia's Third Law of Motion

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ABSTRACT
Newton's masterpiece the Principia was originally published in the Latin in three editions i.e. 1687, 1713 and 1726. It was translated for first time from third edition in English in 1729 by Andrew Mott, this version is used for critical analysis. But now this book is available on the internet free of cost. The theme of mentioning this is that it is now easily available and accessible for global critical analysis. Newton's third law is also implies Law of conservation of linear momentum when statement is mathematically interpreted. This law establishes precise equality between action and reaction in all three examples or applications given by Newton. The main apparent limitation of the law is that it neglects the properties of projectiles (shape, mass, inherent composition, nature, flexibility, elasticity, plasticity, rigidity, size, mode of interactions etc.). Further the law puts no constraints on the target i.e. floor, table, stretched sheets of steel or sheet of newspaper may act as target. It has resemblance with elastic collisions in one dimension. It is concluded that characteristics of projectile and target may play significant role which are neglected in Newton's third law of motion. Thus law is theoretically generalized i.e. Action =C Reaction, where C is coefficient of proportionality. These aspects can be confirmed experimentally. Thus re-analysis of various aspects of the Principia is meaningful.
Keywords: Principia, Third law, generalized law, characteristics of projectile

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I. THE RESEMBLANCE OF THIRD LAW OF MOTION WITH ELASTIC COLLISIONS

Newton [1] stated Third Law of Motion in the Principia that action and reaction are always equal and opposite (Action=-Reaction or Force exerted by one body = -Force exerted by other body) for all bodies in all conditions i.e. universally.

Action=-Reaction
or Force exerted by one body = -Force exerted by other body (1)

Newton's law does not put any constraint on precise equality of action and reaction force in exactly opposite direction. Thus it universal in nature like law of gravitation. Any law or mathematical formulation is justified by consistent theoretical deductions followed by repeatable and consistent experimental confirmations. Newton justified the third law or axiom of motion without any mathematical equation i.e. qualitatively only. Practically, in first two examples given by Newton [1] to elaborate static part (stone remains at rest when pushed by finger or pulled by horse). Apparently Newton expressed action and reaction in terms of force. The third example explains dynamic part (when one body strikes other body) i.e. change in momentum of projectile is equal and opposite in case of target. In this case Newton expressed action and reaction in terms of momentum, mv (or velocity). Both parts are separately interpreted.

Mathematical equations are formulated for the dynamic part of law and third law implies conservation of momentum [2].

change in momentum of target as caused by projectile = - change in momentum of projectile (2)

$$MV_{final} - MV_{initial} = -(mU_{rebound} - mU_{initial}) \quad (2)$$

If target remains at rest , $V_{initial} = 0$

$$V_{final} = (U_{initial} - U_{final})\, m/M \quad (3)$$

where m, $U_{initial}$, U_{final} are mass, initial and final velocities of projectile; M, $V_{initial}$, V_{final} are mass , initial and final velocities of target. These equations clearly neglect the properties (shape, mass, inherent composition, nature, flexibility, elasticity, plasticity, rigidity, size, mode of interactions etc.) of projectile and target [3,4] . Apparently these are significant factors and their role can be checked in specific experiments. Theoretically these factors can be taken in account if the law is generalized (action is regarded as proportional to reaction) i.e.

Action = C Reaction (4)

and co-efficient C accounts for all the elusive factors. This aspect requires quantitative confirmations.

It must be noted that in third application of third law of motion and elastic collision may have similar nature in some respects. If an event is described by two methods (elastic collisions and third law, say) then both methods must give same results. In elastic collisions kinetic energy and momentum both are simultaneously conserved. The third law (third application) implies conservation of momentum. In some cases the predictions of elastic collisions and third law of motion are exactly similar. There are numerous predictions of velocities of projectile and target about elastic collisions theoretically available but are not experimentally confirmed. Also in theoretical predictions are available for third part of third application of the law for confirmation.

The factors cited above about third law of motion are equally significant for elastic collisions (as same events are described) and can be experimentally checked in specific experimental set up. One of the predictions in elastic collision is if projectile is very-2 heavy, then target must move with twice velocity of projectile ($V_{final} = 2U_{initial}$) in all cases [5]. Whereas in this cases prediction from third law of motion is different. The confirmation of predictions of third law of motion and elastic collisions means confirmation of basic conservation laws. Thus proposed experiments (about third law of motion and elastic collisions) would lead to noble results in Thus third law of motion and equations based on elastic collisions must be experimentally confirmed in view of elusive factors (shape, mass, inherent composition, nature, flexibility, elasticity, plasticity, rigidity, size, mode of interactions etc.) cited above. Newtonian Mechanics as effect of many significant factors are not experimentally tested yet.

1.2 The significance of characteristics of target on body

Newton stated Third Law of Motion in the Principia that action and reaction are always equal and opposite (Action=-Reaction or Force exerted by one body = -Force exerted by other body) for all bodies in all conditions i.e. universally. Newton's law does not put any constraint on precise equality of action and reaction force in exactly opposite direction. Newton explained law qualitatively, philosophically and geometrically in three examples or applications (body remains static or moves) and did not give any mathematical equation**.** Thus the law is equally applicable for all projectiles and targets without any constraint in vertical or horizontal motion. A ball (projectile) is placed on the target, say floor (attracted by the earth, action =weight =mg), as reaction the ball rebounds upwards, obeying the law. It can be expressed in terms of distance ($0.5gt^2$). In most of the experiments the earth, floor or table are used as target to explain the law. The definition implies that law is applicable for all targets and not for handpicked targets only.

Like bodies of various properties (shape, mass, inherent composition, nature, flexibility, elasticity, plasticity, rigidity, size, mode of interactions etc.), law is applicable for all possible targets of diverse characteristics. Neither in first two examples (body remains static) nor in third example (change in momentum of projectile is equal and opposite to that of target) Newton specifically mentioned any constraint about characteristics of target. Thus law is universal in nature and hence applicable for all projectiles and targets of all characteristics.

Normally the perception about target in third law is that target is very-2 heavy like floor, table or wall compared to projectile (lighter body, ball say). But it may be regarded as the standard or the most suitable case, as law is applicable for possible targets without any constraint. It is not limited to cases described above, as equality between action and reaction is universal. Thus the law does not restrict floor, table, wall, stretched sheet of metal, stretched sheet of newspaper etc. to act as target. For example, it is observed that a stretched sheet of newspaper in air may act as target (provide equal and opposite reaction force) for lighter bodies only but not for heavier bodies. This aspect needs to be studied for various possible targets in relation to various projectiles.

If the weight of body is 3kg (weight = Action =29.4N) and small insect 10g (0.098N) are placed on floor or table . Then action is balanced by reaction force of floor. The same observation is justified if bodies are placed on stretched sheets of steel (400 megapascal) and carbon steel (841MPa). If the body of mass 3kg is placed on the stretched newspaper then it falls down tearing it, no reaction is observed as in previous cases. However the insect gently moves or stands on it. Consequently the reaction depends upon characteristics of projectile and target. This aspect needs to be taken in account. Thus reaction depends on the characteristics of target (it is not taken in account by the law). It can be done in generalized form the law i.e. Action =C Reaction, the coefficient C accounts for characteristics of target and projectile. Thus coefficient C puts constraints on equality of action and reaction which Newton's original law does not do.

1.3 Discussion of force of friction

Descartes had written book in 1644 Principles of Philosophy quoting three laws of motion [6] before Newton's masterpiece the Principia [1,2]. Descartes first two laws resemble with Galileo's law of inertia and third law is independent of existing laws. Descartes third law of motion describes colliding bodies (practically involve action and reaction) in qualitative and philosophical way without any mathematical equation. Due to conceptual limitations Descartes did not use words action and reaction directly but used words like 'power of continuing' and 'resistance offered by body'. Whereas Newton wrote book The Mathematical Principles of Natural Philosophy in 1686 stating his three laws of motion. Earlier Aristotle (383-322BC) had stated that force is required for movement of body, which is like action. Thus Aristotle may be regarded to have indirectly initiated study similar to third law of motion as force exerted on body is just like action. Hooke's law

$F = -kx$, (5)

where F force , k spring constant, x deformation) was discovered in 1660 but Newton did not at all mention it in the Principia. It is useful when bodies are deformed.

Initially laws of friction were given by Leonardo da Vinci in 1493; further laws were rediscovered and elaborated as three laws of friction by [7,8] in 1699, in Newton's time. The laws of friction are equally significant in study of the third law of motion. However, Newton did not mention the effect of friction while explaining third law of motion in second and final editions of the Principia i.e. 1713 and 1726 or in other works. Without mentioning and interpreting laws of friction the study of Newton's third law may not be regarded as complete. We can write,

Applied force (F_{ap}) = $(f_r)_{max}$ = μ_s N = μ_s mg = R (6)

where F_{ap} is applied force on body of mass m , $(f_r)_{max}$ is limiting friction , μ_s , is coefficient of friction , N is normal reaction, μ_s is maximum limiting friction, and R is reaction.

In 1609, Galileo perceived experiments [3] in hypothetical system when all frictional forces are eliminated from the system. Then in such system (hypothetical) body once set in motion (action) always remains in state of uniform or constant velocity. Thus reaction (the force which opposes action) is regarded as zero or tends to zero in this case. In terms of friction the coefficient of friction is regarded as zero ($\mu_s = 0$) or tending to zero ($\mu_s \rightarrow 0$). Consequently according to eq.(6) the reaction becomes zero or tends to zero. Due to this reason body (set in motion), maintains uniform or constant velocity as there is no reaction or resistive force to counter the applied force. Newton has not mentioned any cause of reaction. Thus reaction is caused by friction, if there is no friction, there would be no reaction. It follows from above experiments or observations.

It is evident that mathematically force of friction is related with weight ,mg as in eq.(6). Newton explained phenomena qualitatively and geometrically without mathematical equations in form of propositions, theorems, lemmas, problems without mathematical equations. The inverse square law is quoted number of times in the Principia. But in the Principia modern form of law of gravitation, its mathematical equation and fact that force of attraction between bodies is proportional to product of their masses (Mm), were never quoted [9]. The various aspects of law of gravitation existed before Newton in one way or other; and Newton put together all existing concepts comprehensively. Newton further had stated at page 388 in Book III of the Principia [2] that all bodies (light feather and heavy gold) fall with constant velocity in vacuum. Newton compared the constant velocity of falling bodies with constant orbital velocity of planets, comets and other bodies around the Sun, which is true. As the body comes nearer to earth due to attraction its velocity must increase [9]. In law of inertia Galileo maintained that bodies move with uniform velocity and first part of Newton's first law of motion implies the uniform of velocity of bodies horizontally. So there have been similarities in Newton's various perceptions of uniform velocity. But bodies fall with constant acceleration due to gravity. Some experiments (as suggested above) would confirm the same beyond any doubt ; the noble results are expected as science is not static body.

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