RESEARCH ARTICLE OPEN ACCESS

Natural Frequency, Ride Frequency and their Influence inSuspension System Design

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ABSTRACT
One of the most crucial yet overlooked parameter in the suspension design stage is the ride frequency of the system. The ride frequency plays a major role in determining the vehicle dynamics and it is one of the most important handling characteristics involved in the spring design process.
This paper aims to give an insight on the topic of Natural Frequency, Ride Frequency and how they are valuable in the design process of the springs used in automotive suspension systems.
***Keywords*–**Natural Frequency, Ride Frequency, Suspension, Spring Rate

Date of Submission: 28-02-2019 Date of acceptance: 25-03-2019

I. INTRODUCTION

When talking about a suspension system, spring rates and damping levels are the parameters that are often used. However, the process of arriving at these values is often neglected. Much like spouting peak horsepower without talking about peak torque or better yet the torque curve, spring rates and damping are rather meaningless without taking your suspension frequency into account.

The quality referred to as “ride comfort” is affected by a variety of factors, including high frequency vibrations, body booming, body roll and pitch, as well as the vertical spring action normally associated with a smooth ride. If the vehicle is noisy, if it rolls excessively in turns, or lurches and pitches during acceleration and braking, or if the body produces a booming resonance, occupants will experience an “uncomfortable ride.” The ride frequency primarily determines the level of comfort the occupants experience and the handling of the car. [1]

The concept of ride frequency and its influence will be discussed in detail.

II. NATURAL FREQUENCY

Before we can examine suspension frequency, we need to introduce the concept of natural frequencies. Every [elastic] object, material, etc. has a certain speed of oscillation that will occur naturally when there are zero outside forces or damping applied. This natural vibration occurs only at a certain frequency, known as the natural frequency.

Natural frequency, also known as eigen frequency, is the frequency at which a system tends to oscillate in the absence of any driving or damping force. [2]

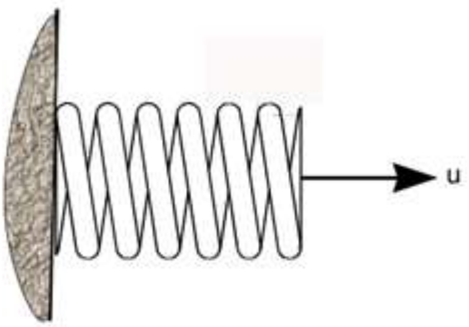
The motion pattern of a system oscillating at its natural frequency is called the normal mode (if all parts of the system move sinusoidally with that same frequency). If the oscillating system is driven by an external force at the frequency at which the amplitude of its motion is greatest (close to a natural frequency of the system), this frequency is called resonant frequency.If a spring which is subject to a vibratory motion which is close to its natural frequency the spring can start to surge. This situation is very undesirable because the life of the spring can be reduced as excessive internal stresses can result.The operating characteristics of the spring are also seriously affected.For most springs subject to low frequency vibrations surging is not a problem. However, for high frequency vibrating applications it is necessary to ensure, in the design stage, that the spring natural frequency is 15 to 20 or more times the maximum operating vibration frequency of the spring.

Nomenclature

C = Spring Index D/d d = wire diameter (m) D = Spring diameter = $(D_i+D_o)/2$ (m) D_i = Spring inside diameter (m) D_o = Spring outside diameter (m) D_{il} = Spring	k = Spring rate = M / δ (N/m.) M = mass supported by spring (kg) n_t = Total number of coils n_a = Number of active coils p = pitch (m) u = motion of spring element(m) y = distance from neutral axis to outer fibre of wire (m)

inside diameter (loaded) (m)
E = Young's Modulus (N/m^2)
F = Axial Force (N)
F_i = Initial Axial Force (N) (close coiled tension spring)
G = Modulus of Rigidity (N/m^2)
K_d = Traverse Shear Factor = (C + 0,5)/C
K_W = Wahl Factor = [(4C-1)/(4C+5)}]+ (0,615/C)
L = length (m)
L_0 = Free Length (m)
L_s = Solid Length (m)
σ = Tensile/com*pressive* stress (N/m^2)
ω_n = Natural angular frequency of spring (rads/s)
f_n = Natural frequency of spring (cycles/s)
τ = shear stress (N/m^2)
τ_i = initial spring stress (N/m^2)
τ_{max} = Max shear stress (N/m^2)
θ = Deflection (radians)
δ = linear deflection (mm)

Consider a spring below subject to a vibration amplitude u at a circular frequency ω

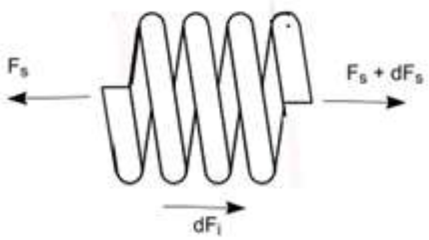


Consider a small element of the spring comprising dn coils. The density of the spring material is ρ and the length of the wire in the element is π.D.dn. If the wire diameter is d then the element has a mass

$$dm = \frac{\pi^2.d^2.\rho.D.dn}{4}$$

For a vibration amplitude u at a circular frequency ω and at any axial location from coil n = 0 to coil n = n_a the inertial force amplitude is as follows.

$$dF_i = dm.\omega^2.u = \frac{\pi^2.d^2.\rho.\omega^2.u.D.dn}{4}$$



The spring rate k for a helical spring is

$$k = \frac{F}{\delta} = \frac{Gd}{8\,C^3 n}\left(\frac{C^2}{C^2+0.5}\right)$$

In practice the term (1 + 0.5/C^2) which approximates to 1 can be ignored.
For the purpose of this analysis this is transformed to

$$k = \frac{Gd^4}{8.n_a.D^3}$$

This equation represents the force for n_a active coils.
The spring force amplitude per active coil resulting from a spring deflection per coil du/dn is

$$dF_s = \frac{Gd^4}{8.n_a.D^3}.\frac{d^2u}{dn^2}.dn$$

Equating the forces on the spring element to zero the equation below is derived.

$$\frac{d^2u}{dn^2} + c^2.u = 0$$

Where, $c = \sqrt{\frac{2.\pi^2.\rho.D^4.\omega^2}{Gd^2}}$

This equation is solved using the equation.

u = A sin (c.n) + B cos (c.n)

A and B are arbitrary constants.
For a spring fixed at one end and free at the other the boundary condition occurs at u = 0 occurs at coil 0 and the condition du/dn = 0 is applied at n = n_a
Resulting in the solution B = 0 and cos(c.n_a) = 0 from which

c.n_a = $\frac{a.\pi^2}{2}$(a= 1,3,5 ......)

This results in a set of natural(surge)frequencies as follows

$$\omega_n = \frac{a.d}{2.n_a.D^2}.\sqrt{\frac{G}{2.\rho}} \quad \text{rad/s}$$

$$f_n = \frac{a.d}{4.\pi.n_a.D^2}.\sqrt{\frac{G}{2.\rho}} \quad \text{cycles/s}$$

Where, a = 1,3,5,….
For the important fundamental natural frequency, this can be simplified to:

$$f_n = \frac{1}{4}.\sqrt{\frac{k}{m_s}} \quad \text{cycles/s}$$

Where,$m_s = mass\ of\ active\ part\ of\ spring$

$$= \frac{\pi^2 d^2.\rho.D.n_a}{4}$$

$$K = spring\ rate = \frac{G.d^4}{8.n_a.D^3}$$

Note: The above analysis relates to a spring with one end against a flat surface and the other end free. The fixed... fixed case i.e. a spring located between and in full contact with two plates results in similar solutions where a = 2, 4, 6......This is also applicable to a spring with one end fixed and against a plate and the other end driven with a sin-wave motion. The fundamental surge frequency resulting from these scenarios are

$$\omega_n = \frac{d}{n_a.D^2}.\sqrt{\frac{G}{2.\rho}}\ rad/s$$

$$f_n = \frac{d}{2.\pi.n_a.D^2} \cdot \sqrt{\frac{G}{2.\rho}}\ cycles/s \qquad [3]$$

III. RIDE FREQUENCY

Ride frequency, derived straight from Natural Frequency, is the suspension frequency found in cars. This is how fast the suspension travels up and then back down to the same point when you drive over a bump. If cars did not have shocks/dampers, the springs would continue to bounce up and down at this rate for quite some time. [4]

By examining the suspension frequency, we are able to fairly accurately predict the handling characteristics of the suspension and how it will 'react' to driver inputs, road surface feedback, brake/acceleration loading, and downforce loading.

The first step in choosing spring stiffness is to choose your desired ride frequencies, front and rear. A ride frequency is the undamped natural frequency of the body in ride. The higher the frequency, the stiffer the ride. So, this parameter can be viewed as normalized ride stiffness.

Based on the application, there are ballpark numbers to consider. [5]

0.5-1.0Hz - Passenger cars, typical OEM
1.0-1.5Hz - Typical lowering springs
1.5-2.0Hz - Rally Cars
1.5-2.5Hz - Non-Aero racecars, moderate downforce Formula cars
2.5-3.5Hz - Moderate downforce racecars with up to 50% total weight in max downforce capability
3.5-5.0+Hz - High downforce racecars with more than 50% of their weight in max downforce

Lower frequencies produce a softer suspension with more mechanical grip, however the response will be slower in transient (what drivers report as “lack of support”). Higher frequencies create less suspension travel for a given track, allowing lower ride heights, and in turn, lowering the center of gravity.

Ride frequencies front are rear are generally not the same, there are several theories to provide a baseline. [6]

Two examples below show exaggerated plots of what happens with unequal ride frequencies front and rear as the car hits a bump. In Figure 1, we can see the undamped vertical motion of the chassis with the front ride frequency higher than the rear. The first period is the most dominant on the car when looking at frequency phase, due to effects of damping.

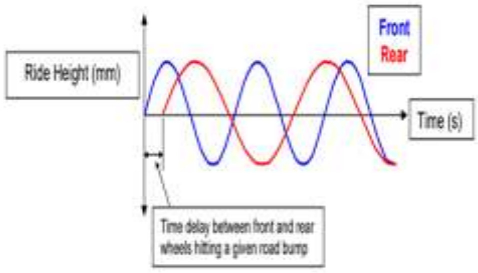


Fig 1

The out of phase motion between front and rear vertical motion, caused by the time delay between when the front wheel and rear wheel hit the bump, is accentuated by the frequency difference. A result of the phase difference is pitching of the body. To reduce the pitch induced by hitting a bump, the rear needs to have a higher natural frequency to “catch up” with the front, as shown in Figure 2. This notion is called producing a “flat ride”, meaning that the induced body pitch from road bumps is minimized.

For a given wheelbase and speed, a frequency split front to rear can be calculated to minimize pitching of the body due to road bumps. A common split is 10 – 20% front to rear. The above theory was originally developed for passenger cars, where comfort takes priority over performance, which leads to low damping ratios, and minimum pitching over bumps. Racecars in general run higher damping ratios, and have a much smaller concern for comfort, leading to some racecars using higher front ride frequencies. The higher damping ratios will reduce the amount of oscillation resultant from road bumps, in return reducing the need for a flat ride. Damping ratios will be explained in the next tech tip in detail. A higher front ride frequency in a racecar allows faster transient response at corner entry, less ride height variation on the front (the aerodynamics are usually more pitch sensitive on the front of the car) and allows for better rear wheel traction (for rear wheel drive cars) on corner exit. The ride frequency split should be chosen based on which is more important on the car you are racing, the track surface, the speed, pitch sensitivity, etc.

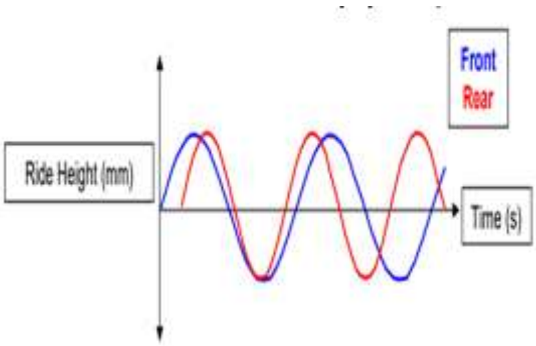


Fig. 2

As an example of ride frequency split front to rear, Figures 3 and 4 shows a simple example of a single degree of freedom vehicle model over an impulse disturbance. The ride frequency difference is 10 percent, 70% critical damping, 100 km/h speed, and 1.75 m wheelbase.

Once the ride frequencies are chosen, the spring rate needed can be determined from the motion ratio of the suspension, sprung mass supported by each wheel, and the desired ride frequency. Starting with the basic equation from physics, relating natural frequency, spring rate, and mass:

Spring rate:

$$f = \frac{1}{2\pi\sqrt{\frac{K}{M}}} \qquad [7]$$

Where, F = NATURAL FREQUENCY (HZ)
K = SPRING RATE (N/M)
M = MASS (KG)

When using these formulas, it is important to take Mass as the total sprung mass for the corner being calculated. That is, the axle weight divided by two, minus an estimated or measured unsprung mass for that corner (things like wheels, tires, brakes, control arms, suspension components etc. all contribute to unsprung mass. Anything that is not supported by the springs).

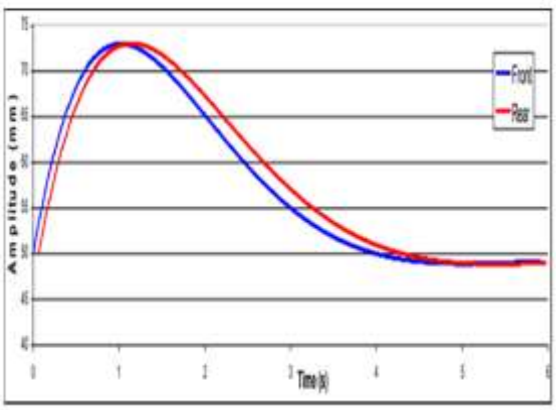


Fig 3. Front ride frequency 10% higher than rear

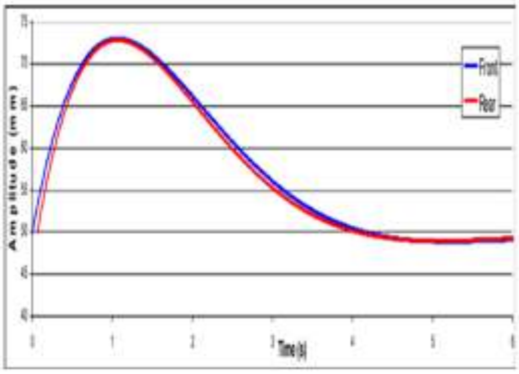


Fig 4. Rear ride frequency 10% higher than front

Solving for spring rate, and applying to a suspension to calculate spring rate from a chosen ride frequency, measured motion ratio, and mass:

$$K_s = 4\pi^2 {f_r}^2 m_{sm} MR^2 \qquad [8]$$

Where, K_s = Spring rate (N/m)
m_{sm} = Sprung mass (kg)
f_r = Ride frequency (Hz)
MR = Motion ratio

Shown below in Table 1 is an example using the above calculation. Table 1 shows spring rates needed for a traditional suspension with one ride spring per wheel, and a motion ratio of one. As the suspended mass per corner and ride frequency are changed, the corresponding spring requirement is shown.

Table - 1

	Sprung Mass per corner							
	100	300	500	700	900	1100	lbs.	Motion Ratio
Ride Frequency (Hz)	45	136	227	318	409	500	kg	1
1.0	10.3	30.8	51.4	72.0	92.5	113.1	lb/in	Spring Rates
	1.8	5.4	9.0	12.6	16.2	19.7	N/mm	
1.5	23.1	69.4	115.7	161.9	208.2	254.5	lb/in	
	4.0	12.1	20.2	28.3	36.3	44.4	N/mm	
2.0	41.1	123.4	205.6	287.9	370.1	452.4	lb/in	
	7.2	21.5	35.9	50.2	64.6	79.0	N/mm	
2.5	64.3	192.8	321.3	449.8	578.3	706.8	lb/in	
	11.2	33.6	56.1	78.5	100.9	123.4	N/mm	
3.0	92.5	277.6	462.6	647.7	832.7	1017.8	lb/in	
	16.2	48.5	80.8	113.1	145.4	177.7	N/mm	
3.5	125.9	377.8	629.7	881.6	1133.5	1385.3	lb/in	
	22.0	65.9	109.9	153.9	197.8	241.8	N/mm	
4.0	164.5	493.5	822.5	1151.5	1480.4	1809.4	lb/in	
	28.7	86.1	143.6	201.0	258.4	315.8	N/mm	
4.5	208.2	624.6	1040.9	1457.3	1873.7	2290.1	lb/in	
	36.3	109.0	181.7	254.4	327.0	399.7	N/mm	
5.0	257.0	771.1	1285.1	1799.1	2313.2	2827.2	lb/in	
	44.9	134.6	224.3	314.0	403.8	493.5	N/mm	

Ride frequencies of different cars are depicted below in Table – 2. The difference in ride frequencies can be observed for cars made for different driving styles and conditions.

	Front Suspension					Rear Suspension					
Vehicle	Ride Rate w/o tire (lb/in)	Corner Weight (lb)	Unsprung Weight (lb)	Sprung Weight (lb)	Frequency (hertz)	Ride Rate w/o tire (lb/in)	Corner Weight (lb)	Unsprung Weight (lb)	Sprung Weight (lb)	Frequency (hertz)	Ride Ratio Rr/Fr
99 Volvo V70 XC	119	1052	100	952	1.12	131	832	100	732	1.32	1.18
2001 MB E320 4-Matic	117	991	100	891	1.13	148	964	100	864	1.29	1.14
Jeep KJ Liberty	126	1086	95	991	1.14	181	914	85	829	1.46	1.28
97 NS Chrysler T&C	148	1173	95	1088	1.15	145	880	85	795	1.34	1.16
Pacifica	160	1256	85	1166	1.16	155	1074	85	989	1.23	1.06
99 MB E320 4-Matic	121	985	100	885	1.16	150	960	100	860	1.31	1.13
97 Peugeot 306 GTI	118	850	85	765	1.19	113	466	85	383	1.7	1.43
99 Audi A6 Quattro	152	1070	100	970	1.24	172	864	100	764	1.48	1.2
2001 MB E320 2WD	131	907	85	822	1.25	144	969	85	884	1.26	NA
	99	907	85	822	1.09						
95 BMW M3	113	763	85	698	1.26	159	780	85	705	1.48	1.18
2001 VW Passat	165	1060	100	960	1.29	136	678	100	578	1.53	1.19
2000 Neon	134	836	75	761	1.31	127	530	65	465	1.67	1.27
2001 PT	161	1009	85	924	1.31	136	607	85	522	1.6	1.22
99 LH Dodge Intrepid	185	1125	85	1040	1.32	152	651	85	566	1.62	1.23
02 Jeep WG Grand Cherokee	197	1170	85	1085	1.33	184	1005	85	920	1.4	1.05
2000 VW Golf	107	797	85	712	1.21	105	586	85	501	1.43	1.18

TABLE 2

IV. CONCLUSION

As discussed in this paper, ride frequency of the suspension system plays a major role in the design of the vehicle. By considering this parameter the comfort levels and performance levels of the vehicle are largely impacted.

However, the determination of the ride frequency is largely based on assumptions and is done by choosing a figure in the ball park range and hence, requires in depth study and analysis.

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Nandan Rajeev" Natural Frequency, Ride Frequency and their Influence inSuspension System Design" International Journal of Engineering Research and Applications (IJERA), Vol. 09, No.03, 2019, pp. 60-64