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On the Study of Expansive and Compressive Acceleration Waves in Planar and Non-planar van der Waals Gas Flows

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Abstract
The objective of this research paper is to investigate the evolution and attenuation behaviour of expansive and compressive acceleration waves in one-dimensional compressible, instable, inviscid (1D-CII) van der Waals (vdW) gas with generalized geometric framework for the planar and non-planar flows. The solution of the problem of propagating of acceleration waves is obtained along the path of characteristic by means of the characteristics of the regulatory system of "quasilinear hyperbolic partial differential equations (QHPDEs)" as the system of reference coordinates. It is shown that the non-linear nature of expansive and compressive acceleration waves in the physical plane is revealed through a linear solution in the characteristic plane. The research also derives the transport equation that regulates the manner in which the acceleration waves evolve and attenuate in vdW gas. Further, it has also been discussed how vdW parameters affect the formation of shock and the behavior of amplitude of the acceleration waves not only in planar geometry but also in cylindrically symmetric and spherically symmetric geometries. Also, the cases where the compressive wave fronts (whatever their beginning strength may be) may end up as shock waves, are discussed. The study further elaborates and compares the complexities of the behaviour of solutions in vdW gas with that of an ideal gas.
Keywords: Acceleration wave, van der Waals gas (vdW gas), Shock wave, generalized geometry.

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I. Introduction

The term "acceleration wave" refers to a discrete geometric surface that has discontinuous acceleration (but not velocity) and moves with respect to the material. Acceleration waves basically observed in the physical systems where the rate of change of velocity propagates through a medium. Acceleration waves are a fascinating aspect of wave dynamics. In case of acceleration waves, the disturbance is related to changes in velocity, rather than the velocity itself. In simple words, an acceleration wave is a disturbance in which the acceleration of particles, within a medium, changes over time. Acceleration waves are generally produced by abrupt changes of speed and force in the medium. These waves can take place in different physical systems fluids, solids and gases. Acceleration waves can arise from sudden impacts, explosions, or other transient events that produce high levels of acceleration. Depending on the physical system in which acceleration waves occur, acceleration waves can be seen in several forms, for example, shock waves in fluids (in fluid dynamics), electromagnetic acceleration waves (in electromagnetism), gravitational waves (in general relativity). Acceleration waves can be classified into expansive and compressive types based on their effect on the medium. Expansive acceleration Waves cause an increase in velocity and a decrease in pressure within the medium and they are widely related to rarefaction phenomena. Unlike shock waves, which cause abrupt changes, expansive waves are continuous and smooth. In case of compressive acceleration waves, there is an increase in the pressure and a decrease in the velocity. These waves often result in shock waves when the amplitude surpasses critical threshold and are commonly observed in high-speed aerodynamics and material deformation processes.

In fluid dynamics, the existence of acceleration wave in shock waves plays an important role in understanding supersonic flight, explosions, and the behavior of materials under high-pressure conditions. Moreover, the study of these waves can help scientists to understand very complex mysteries of general relativity, geophysics, aerospace engineering, fluid dynamics, material science, shockwave therapy and dark universe of

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physics in aspects of mathematics. By studying the behavior and effects of acceleration waves, scientists can understand complex aspects of physics and mathematics, improve technological systems, and expand their understanding and knowledge about the dark universe. The study of acceleration waves helps to develop systems where high velocities and rapid changes in velocity occur. In industrial applications, the implications of acceleration waves are also seen in areas such as shockwave analysis, which plays a role in mining, demolition, and medical treatments (such as shockwave therapy).The propagation of acceleration waves can be described using "Partial Differential Equations (PDEs)". Systems of QHPDEs are utilized as mathematical models for various physical phenomena, including nuclear and chemical explosions, as well as supersonic flow [1-4]. It is remarkable that acceleration waves are crucial for studying material behaviour under dynamic loading conditions. Focusing on unsteady compressible fluid flow, acceleration waves-characterized by discontinuities in fluid particle acceleration—have drawn considerable attention for both their mathematical and physical characteristics. These waves epitomize a distinctive form of nonlinear wave processes that can be comprehensively analyzed using both analytical and numerical approaches. The unique feature of acceleration waves is that their propagation conditions are compatible with algebraic equations. For nonlinear systems, it is quite intriguing to look for analytic solutions and discuss its physics in gasdynamics.

Weak discontinuities arise along the characteristic(C) curves in the nonlinear system of hyperbolic equations. When the flow variables have unbounded derivatives all the way through the wave front, this weak discontinuity is said to be a shock wave and the propagation no longer takes place along the leading characteristics. The physical process of wave steepening or breaking belongs to the evolution of shock waves. Over the last few years, many researchers have worked on the increasingly compressive waves giving rise to shock waves. Many researchers have studied the wave propagation problems including nonlinear effects. Some of them are Menon et al. [5], Thomas [6], Coleman and Gurtin [7], Anile and Russo [8], and Jordan [9]. Shyam et al. [10] used singular surface theory to investigate how weak shock waves develop and decay in radiating gases. Nonetheless, the techniques of Whitham [1], Courant and Friedrichs [4], Jeffrey and Taniuti [11] can anticipate nearly the entire history of the shock for only relatively weak shocks. Shankar [13] effectively applied Keller's [12] ray theory to investigate the shock wave propagation in the radiative magnetogasdynamics.

Bowen and Chen [14] conducted a significant study on acceleration waves having arbitrary shape. They also improved one of his main theorems, and their specialized asymptotic formulas were adapted to fit acceleration waves of various shapes. Ram R. [15] explored how acceleration waves travel along C paths. By using the governing quasilinear system's characteristics as a reference framework, he was able to understand the behaviour of these waves better. His work also investigated how radiative heat transfer intensifies these acceleration waves, providing deeper insights into their propagation. Singh. et al. [16] tackled the problem of developing low-intensity shock waves of planar and curved categories in media with non-ideal (n-idl) properties. The role of the n-idl parameter and the curved geometry of the wave front in n-idl interference, pulse weakening and shock wave creation are elaborately covered. Nath and Kadam [17] study the evolution of acceleration waves with planar and cylindrical symmetry in a n-idl relaxing gas affected by the transverse magnetic field. Singh et al. [18] investigated the behavior of acceleration waves in unsteady, inviscid under finite magnetic pressure. The assessment examines shock formation under n-idl gas flows using planar and cylindrical core geometries that centre all non-inertial fluxes. They also analysed the differences in the behavior of solutions between ideal and n-idl magnetogasdynamic regimes, providing insights into how these conditions affect the outcomes. Further, Nath et al. [19] applied the similar approach to examine the more primary attributes of weak nonlinear waves and developed the transport equation which bears the essence of edifying the evolution and diminution of shocks in magnetogasdynamics environments with n-idl properties. Singh et al. [17-18, 20-21] uses the same technique in their study of the development and the diminution of the acceleration waves in various media such as Solids, Liquids and Gases. The group theoretic approach was used by Jena and Sharma [22] to determine "self-similar solutions" for analyzing shock wave movement in dusty gas environments. Similarly, using the transformations of Lie group, Chadha and Jena [23] found a "self-similar solution to the problem of shock wave propagation in dusty n-idl gases". Anand [24] was able to present the shock jump relation for dusty gas media. Chaturvedi et al. [25] studied the behaviour of low-intensity shock waves as they move through an instable unidirectional flow in a radiating gas of n-idl nature. Their study particularly focused on how these shock waves are influenced by a magnetic field of transverse type. In a more recent study, Khapra D. and Patel A. [26] explored the composition of normal shock waves in planar stationary flow involving n-idl low concentrated gas mixtures. Making the use of the Navier–Stokes–Fourier framework to guide their analysis, they looked into how varying viscosity and thermal conductivity influenced these shock waves,

The research about the shock wave production in vdW gases has gained interests from researchers and practitioners of various fields due to its importance as applied in space, physical sciences, chemical processes, nuclear reactions, among others and aerospace engineering [27-29] because when the gas temperature rises significantly and the density drops, the ideal gas model ceases to be applicable. Zhao et al. [30] performed comprehensive categorization of shock waves and occurrences of shock wave bifurcation accompanied by the acceptance in vdW fluids. In her study, Ambika [31] applied the principles of progressive waves in analyzing the disturbances of small amplitude in vdW gases. Nath et al. [32] have analysed the characteristics of wave features in vdW gases using the same approach. Sharma et al. [33] employed the multiple time scales technique to investigate and analyse wave interactions in non-equilibrium gas flows. Likewise, Arora et al. [34-35] used this method to obtain asymptotic forms of solutions for systems of hyperbolic equations in various media. Moreover, high-frequency fast magnetosonic waves in an axisymmetric equilibrium plasma have been investigated using weakly nonlinear geometrical optics, and the fast magnetosonic razor geometry has been evaluated by Manuel [36-37]

In the present study, we have well thought-out the problem of acceleration waves propagating in the “one-dimensional compressible, instable, inviscid (1D-CII)”flow of a vdW gas in the context of generalized geometric framework, however, it is more intricate to study the nature of shock-linked phenomena in vdW gases or real gases as compared to that in ideal gas. Using the basic physical properties of controlling system as reference coordinates; we have investigated the movement of acceleration waves following the directions of characteristic trajectories. Our findings reveal that the linear type solution, in the plane of characteristic, corresponds to the nonlinear pattern of acceleration waves in the physical plane. Furthermore, we have derived a transport equation that characterizes the evolution and attenuation of acceleration waves in1D-CII flows with planar, cylindrical and spherical symmetry in vdW gas. It is observed that the transition to shock waves occurs in both “planar and non-planar (P/N-P)” flows for all compression waves, whatever their initial intensity is. Also, the implications of vdW parameters on the evolution and attenuation behaviour of acceleration waves are investigated and explained. We also have compared the behaviour of solution in vdW gas with that of in ideal gas.

II. Governing equations with characteristics

In the general family of real gases, the vdW gas equation of state, may be represented as

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT \qquad (1)$$

where p represents pressure, V denotes volume, R indicates to the gas constant and T stands for absolute temperature. The strength "a" of intermolecular attraction among the vdW gas particles and "b", the excluded volume of the vdW gas, are used as parameters which specify the non-idealness. We know well that under low temperature and high pressure, gases behave like real gases. In terms of the equation (1), the internal energy expression, through the assistance of $R=(\gamma-1)C_v$,where C_v is the specific heat at fixed volume, may be formulated as follows [1,4, 31-32]

$$E=\frac{(p+a\rho^2)(1-b\rho)-a\rho^2(\gamma-1)}{(\gamma-1)\rho} \qquad (2)$$

Where ρ specifies the gas density, and γ, the adiabatic exponent, given by the ratio $\frac{C_p}{C_v}$, which is the specific heat capacity at fixed pressure (C_p) divided by that at fixed volume (C_v).If we set $a=0$ and $b=0$ in (1), the vdW gas equation of state reduces to that of an ideal gas equation. Thus, the equations that describe the 1D-CII-P/N-P flows in vdW gas can be taken as

$$\rho_t+u\rho_x+\rho u_x+\frac{N\rho u}{x}=0, \qquad (3a)$$

$$u_t+uu_x+p_x/\rho=0\,, \qquad (3b)$$

$$p_t+up_x+\rho A^2\left(u_x+\frac{Nu}{x}\right)=0\,, \qquad (3c)$$

All of these equations align with the formulations presented by Whitham[1], Courant and Hilbert [38], and Thompson [39]. In this particular context, u represents the velocity of gas, t denotes the duration of time, and x signifies the position in space. The parameter N distinguishes different types of flows: $N=0$ indicates to the planar flow, $N=1$ indicates to the cylindrically symmetric(CS) flow, and $N=2$ indicates to the spherically symmetric(SS) flow. Also, A denotes the velocity of sound given by

$$A = \sqrt{\left(\frac{\gamma p + a\rho^2(\gamma - 2 + 2b\rho)}{(1-b\rho)\rho}\right)}, \tag{4}$$

Now writing the system of equations (3a), (3b) & (3c) in matrix form, we get

$$U_t + PU_x + Q = 0, \tag{5}$$

where U is a 3×1 column matrix having elements,U_{ij}, where $U_{11} = \rho, U_{21} = u, U_{31} = p$,and the matrices P(square matrix) and Q(column matrix) are respectively given by

$$P= \begin{bmatrix} u & \rho & 0 \\ 0 & u & \rho^{-1} \\ 0 & \rho A^2 & u \end{bmatrix}_{3\times3} \quad \text{and} Q= \begin{bmatrix} N\rho u/_x \\ 0 \\ \rho A^2 Nu/_x \end{bmatrix}_{3\times1} \tag{6}$$

The eigen values of the system (5) obtained by us are,

$\lambda_1 = u + A, \lambda_2 = u, \lambda_3 = u - A$, which are real and distinct and corresponding eigenvectors respectively are

$$v_1 = \begin{pmatrix} \frac{1}{A^2} \\ \frac{1}{A\rho} \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \; v_3 = \begin{pmatrix} \frac{1}{A^2} \\ -\frac{1}{A\rho} \\ 1 \end{pmatrix}.$$

Hence, the system (5) of PDEs is hyperbolic and yields three C curves. Two of these curves represent waves moving in the $+x$ and$-x$ directions with shock speeds of $u + A$ and $u - A$ respectively. The third curve describes the particle path moving at the shock velocity u. Acceleration waves propagate along these characteristics. Here, we are focusing on the waves that propagate via the ground (unchanging) state U_0 defined by $U_0 = (\rho_0, 0, p_0)$. The C speeds at the ground state U_0 are expressed by $\lambda'_1 = A_0, \lambda'_2 = 0$and,$\lambda'_3 = -A_0$. In this case, the subscript "0" specifies the evaluation at $U = U_0$.. Thus, where $U(x,t)$ is continuous, but its derivatives U_t and U_x of first order and its derivatives higher order suffer from finite jumps (discontinuities) over $F(t)$ and the function $U(x,t)$ fulfils (5) everywhere, excluding the C function $F(t)$. These jump discontinuities are referred to as "acceleration waves" or "weak discontinuities".

Now, let $[U]$ represents the jump of the quantity U across $F(t)$, then along curve $F(t)$, we have

$$\frac{\partial}{\partial t}[U] = [U_t] + \frac{dF(t)}{dt}[U_x], \tag{7}$$

where $\frac{\partial}{\partial t}$ specifies the time derivatives seen from the perspective of the wave front. With reference to equation (7), taking jump in (5) and the continuity condition $[U] = 0$, we find

$$\left(P - \frac{dF}{dt}I\right)[U_x] = 0, \tag{8}$$

Where I is the 3×3 identity matrix.

Based on equation (8), it can be noted that a limited break in the continuity of acceleration waves following the C trajectory results in the characteristics speed of propagation $\frac{dF}{dt}$ being the C values of the matrix P. As a result, it can be immediately inferred that 3families of C curves exist, with two of them being explicitly provided

$$\frac{dx}{dt} = u \pm A \tag{9}$$

where A is the sound speed and is given by (4).

In equation(9),$\frac{dx}{dt} = u + A$ represents the outgoing wavelet and $\frac{dx}{dt} = u - A$ represents incoming wavelet in the direction of x-axis travelling with the speed A of sound in vdW gas medium and the remaining one which represents the trajectory of the fluid particles, is

$$\frac{dx}{dt} = u \tag{10}$$

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III. Characteristic transformation

For solving the problems involving QHPDEs, it is highly beneficial to utilize coordinates of C as our system of reference. Consequently, we define ϕ and ψ as two C variables such that (Singh et al. [21]):

(i) ϕ is a "**wave tag**" so that $\frac{dx}{dt} = u + A$ (the outgoing C) maintains a constant value of ϕ in the (x,t)-plane and this will be labelled by $\phi = t^*$, if a wave moving outward is produced at a time t^*.

(ii) ψ is the "**particle tag**" which ensures that ψremains unchanged along the fluid particle's path described by $\frac{dx}{dt} = u$ in the (x,t)-plane. Whenever the C wave front moves over a particle at time t', the particle along with its trajectory shall be designated by a unique identifier $\phi = t'$.It is evident that for each pair of coordinates (ϕ, ψ), we have an associated pair of coordinates(x,t) such that$x = x(\phi,\psi)$ and t$= t(\phi,\psi)$ and the relationship between these pairs is established. Utilizing the characteristics of ϕ and ψ, the derived PDEs, are as follows

$$\left.\begin{aligned}\frac{\partial x}{\partial \phi} &= u\frac{\partial t}{\partial \phi}\\ \text{i.e,}\ \ x_\phi &= ut_\phi\end{aligned}\right\}\text{and}\quad \left.\begin{aligned}\frac{\partial x}{\partial \psi} &= (u+A)\frac{\partial t}{\partial \psi}\\ \text{i.e,}\ \ x_\psi &= (u+A)t_\psi\end{aligned}\right\} \tag{11}$$

With this in mind as for the above transformations, we have

$$U_t = \frac{\partial U}{\partial t} = \frac{\frac{\partial U}{\partial \psi}\frac{\partial x}{\partial \phi} - \frac{\partial U}{\partial \phi}\frac{\partial x}{\partial \psi}}{\frac{\partial (x,t)}{\partial (\phi,\psi)}} \tag{12}$$

$$U_x = \frac{\partial U}{\partial x} = \frac{\frac{\partial U}{\partial \phi}\frac{\partial t}{\partial \psi} - \frac{\partial U}{\partial \psi}\frac{\partial t}{\partial \phi}}{\frac{\partial (x,t)}{\partial (\phi,\psi)}} \tag{13}$$

where the Jacobian of transformation $\frac{\partial (x,t)}{\partial (\phi,\psi)}$ plays a pivotal role in solving issues related to shock formation and is given as

$\frac{\partial (x,t)}{\partial (\phi,\psi)} = -A\frac{\partial t}{\partial \phi}\frac{\partial t}{\partial \psi}$, i.e., in short notation, $\frac{\partial (x,t)}{\partial (\phi,\psi)} = -At_\phi t_\psi$.

The above relations clearly show that $\frac{\partial (x,t)}{\partial (\phi,\psi)} = 0$ occurs if and only if $\frac{\partial t}{\partial \phi} = 0$, while two adjacent characteristics conclude into a shock wave. This is because the doubling or over-
lapping of gas particles is physically impossible. Therefore, $\frac{\partial t}{\partial \psi} \neq 0$. Hence, to steepen the wave front or to form a shock wave, $\frac{\partial (x,t)}{\partial (\phi,\psi)} = 0$ will give us a required condition.

Now using equation (12) and (13) in equations (3a), (3b) and (3c), and using (11), we obtain

$$A\rho_\phi t_\psi - \rho u_\phi t_\psi + \rho u_\psi t_\phi + \frac{N\rho u A t_\phi t_\psi}{x} = 0, \tag{14}$$

$$\rho A u_\phi t_\psi - p_\phi t_\psi + p_\psi t_\phi = 0 \tag{15}$$

$$A p_\phi t_\psi - \rho A^2 u_\phi t_\psi + \rho A^2 u_\psi t_\phi + \rho A^2\frac{NuAt_\phi t_\psi}{x} = 0 \tag{16}$$

Now substituting (15) and (16) in (14), we obtain

$$p_\psi + \rho A u_\psi = -\frac{N\rho u A^2 t_\psi}{x} \tag{17}$$

IV. Boundary conditions

At the shock front, the constraints at the boundaries can be given as

$$[\rho] = 0, [u] = 0, [T] = 0, [p] = 0,\ \text{with } t = \psi \text{ and } \phi = 0. \tag{18}$$

As we presume that the flow of the fluid just before the shock front is both uniform and stationary, therefore, according to equation (18),we have

$$\rho_\psi = 0, u_\psi = 0, p_\psi = 0,\ T_\psi = 0, t_\psi = 1 \text{ at } \phi = 0. \tag{19}$$

V. Solution of the problem

Using the equations (18) and (19) in equations (14), (15) & (11), we obtain

$$\rho_\phi = \frac{\rho_0 u_\phi}{A_0}, \text{ at } \phi = 0, \tag{20}$$

$$p_\phi = \rho_0 A_0 u_\phi, \text{ at } \phi = 0, \tag{21}$$

$$x_\phi = 0, x_\psi = A_0, \text{ at } \phi = 0, \tag{22}$$

Here the subscript "0" represent the fluid flow parameters just preceding the shock front (i.e. region at rest).
To determine the amplitude of the acceleration wave at the shock front, we need to substitute equation (19) into equation (13). Thus, doing so, we have

$$\left[\frac{\partial u}{\partial x}\right] = Y = -\frac{u_\phi}{A_0 t_\phi}\ \ , \text{at } \phi = 0, \tag{23}$$

in which Y signifies the amplitude of the acceleration wave.
Now we will discuss the dependency of u_ϕ and t_ϕ on time. For this we differentiate equations (11),(17) and (21) w.r.t. ψ and ϕ, we get

$$x_{\phi\psi} = ut_{\phi\psi} + u_\psi t_\phi\ \ , x_{\psi\psi} = (u+A)t_{\psi\psi} + (u_\psi + A_\psi)t_\psi \tag{24a}$$

$$x_{\phi\phi} = u_\phi t_\phi + ut_{\phi\phi}\ \ , x_{\psi\phi} = (u+A)t_{\psi\phi} + (u_\phi + A_\phi)t_\psi \tag{24b}$$

$$p_{\psi\psi} + \rho A u_{\psi\psi} + u_\psi(\rho A_\psi + \rho_\psi A) = -\frac{N[x\{\rho_\psi uA^2 t_\psi + \rho u_\psi A^2 t_\psi + 2\rho u A A_\psi t_\psi + \rho u A^2 t_{\psi\psi}\} - \rho u A^2 t_\psi x_\psi]}{x^2} \tag{25a}$$

$$p_{\psi\phi} + \rho A u_{\psi\phi} + u_\phi(\rho A_\phi + \rho_\phi A) = -\frac{N[x\{\rho_\phi uA^2 t_\psi + \rho u_\phi A^2 t_\psi + 2\rho u A A_\phi t_\psi + \rho u A^2 t_{\psi\phi}\} - \rho u A^2 t_\psi x_\phi]}{x^2} \tag{25b}$$

$p_{\phi\psi} = \rho_0 A_0 u_{\phi\psi}$, at $\phi = 0$, (26a)

$p_{\phi\phi} = \rho_0 A_0 u_{\phi\phi}$, at $\phi = 0$, (26b)

Solving equations (24a),(24b), (25a), (25b), (26a) and (26b) and using (23),we get

$$\frac{u_{\phi\psi}}{t_\phi} = \frac{NA_0}{2\psi} Y, \text{at } \phi = 0, \quad (27)$$

$$\frac{t_{\phi\psi}}{t_\phi} = \left[1 + \left\{\frac{(\gamma-1+2b\rho_0}{2(1-b\rho_0)}\right\} + \left\{\frac{2a\rho_0^2\gamma+4ab\rho_0^3-4a\rho_0^2+2ab\rho_0^2}{2A_0^2(1-b\rho_0)}\right\}\right] Y, \text{ at } \phi = 0. \quad (28)$$

Now, we will proceed to find the transport equation that controls the evolution and attenuation behavior of expansive and compressive acceleration waves. For this, we differentiate equation (23) with respect to ψ and then use equations (27) and (28) to obtain

$$\frac{dY}{d\psi} + \frac{N}{2\psi} Y + \left[1 + \left\{\frac{(\gamma-1+2b\rho_0}{2(1-b\rho_0)}\right\} + \left\{\frac{2a\rho_0^2\gamma+4ab\rho_0^3-4a\rho_0^2+2ab\rho_0^2}{2A_0^2(1-b\rho_0)}\right\}\right] Y^2 = 0, \text{ at } \phi = 0. \quad (29)$$

As in Ram [1978], let us consider the following dimensionless parameters:

$$\zeta = \frac{Y}{Y^*}, \eta = \frac{\psi-\psi^*}{2\psi^*}, \omega = Y^*\psi^* \quad (30)$$

where the parametersζ, η, ω are dimensionless and stand for the associated wave amplitude, time and beginning acceleration, respectively, and"*" marks the beginning wave level.

Taking the equation (30) into consideration, the equation (29) can be written as

$$\frac{d\zeta}{d\eta} + \frac{N}{(2\,\eta+1)}\zeta + 2\left[1 + \left\{\frac{(\gamma-1+2b\rho_0}{2(1-b\rho_0)}\right\} + \left\{\frac{2a\rho_0^2\gamma+4ab\rho_0^3-4a\rho_0^2+2ab\rho_0^2}{2A_0^2(1-b\rho_0)}\right\}\right]\omega\zeta^2 = 0 \quad (31)$$

Note that equation (31) cannot be solved in present form. Therefore, reducing it into linear Differential equation form and then solving it, we get the analytical solution as

$$\zeta = \left[\left(1 + 2\omega\left[1 + \left\{\frac{(\gamma-1+2b\rho_0}{2(1-b\rho_0)}\right\} + \left\{\frac{2a\rho_0^2\gamma+4ab\rho_0^3-4a\rho_0^2+2ab\rho_0^2}{2A_0^2(1-b\rho_0)}\right\}\right]\Theta(\eta)\right)(1+2\eta)^{N/2}\right]^{-1} \quad (32)$$

where $A_0 = \sqrt{\left(\frac{\gamma p_0 + a\rho_0^2(\gamma-2+2b\rho_0)}{(1-b\rho_0)\rho_0}\right)}and\Theta(\eta) = \int_0^\eta (1+2\eta)^{N/2} d\eta$.

The function$\Theta(\eta)$holds a crucial position and has a major involvement in the failure of the C solution. From equations (32) and (23), we observe that the shock wave emerges when $t_\phi$becomes zero, i.e.

$$\left(1 + 2\omega\left[1 + \left\{\frac{(\gamma-1+2b\rho_0}{2(1-b\rho_0)}\right\} + \left\{\frac{2a\rho_0^2\gamma+4ab\rho_0^3-4a\rho_0^2+2ab\rho_0^2}{2A_0^2(1-b\rho_0)}\right\}\right]\Theta(\eta)\right) = 0 \quad (33)$$

Since the value of $\Theta(\eta) \geq 0$, therefore from equation (33), we note that the only compressive wave fronts($\omega < 0$) may concludes into shock waves.

VI. Results and discussion

In the present paper we investigated the evolution and attenuation behaviour of expansive and compressive acceleration waves in one-dimensional compressible, instable, inviscid van der Waals (vdW) gas for generalized geometry. The transport equation that controls the evolution and attenuation behaviour of compressive and expansive acceleration waves in 1D-CII-P/N-P flows in vdW gas is derived. If we put $a = 0, b = 0$ in equation (32), the outcomes obtained here are analogous with the outcomes as found previously by Ram [15], Nath and Kadam [17] and Singh et al. [18, 20-21] in absence of magnetic fields and radiative effects.

We have assigned the values to the constants used in the computation as

$\gamma = 1.67, \rho_0 = 1.0\,, p_0 = 1.0\,, \omega = -0.3, 0.3$ with the values of vdW gas parameters a & b as tabulated below:

Values of vdW gas parameters a & b					
Values of a	0.0	1.0	1.0	2.5	3.0
Values of b	0.0	0.02	0.04	0.06	0.06

Figures (1-3) represents the variation of amplitude of the compressive wave 'ζ' versus η for different values of vdW parameters 'a' and 'b' for $N = 0, 1, 2$ respectively. From these figures, we noticed that all of the compressive waves, regardless of their beginning strength, eventually end up as shock waves as is presented by Schmitt [40] in absence of magnetic field. This is different from what occurs with ideal radiating gas where a critical amplitude can steadily be found and it confirms that compressive waves with beginning amplitude greater than this value eventually end up as shock waves, while beginning amplitude below the critical threshold always leads to the decline of disturbance [15]. In figure (1), the different lines represent the solution profile for different values of vdW parameters 'a' and 'b'. Here, the vertical line represents the position and time of shock formation in vdW gas medium having n-idl character. From figure (1), we observe that keeping the value of vdW parameter 'a' fixed, if we have an upward trend in the value of 'b', the position of shock production time is decreasing and consequently, shock formation will occur early while at a constant value of 'b' an upward trend in the value of 'a' results in the extension of the shock production time. Thus, we have found the opposite results as obtained above. Further, as we move from planar symmetry to cylindrical symmetry and then from CS flow to SS flow, the shock formation time is decreased i.e. early formation of shock will happen. From figures (1-3), we observe that the alteration in the steepness or flatness of compressive waves in case of CS flow $(N = 1)$ and SS flow$(N = 2)$ is more rapid as compare to the planar flow$(N = 0)$.Figures (4)-(6) represent the attenuation of expansive wave front $(\omega > 0)$ in planar, CS and SS flows in vdW gas respectively. In all the three cases, the amplitude attained by the expansive waves diminishes and finally damped out. Further, the effect of variation of vdW parameters'a' and 'b' on the attenuation of expansive wave fronts is presented. It is noticed that for a fixed value of vdW parameter 'a'an upward trend in the value of 'b'causes a drop in the attenuation process. Also, similar effect of vdW parameter 'a' for a fixed value of 'b' on the attenuation of expansive wave fronts is noted. The solution curves of compressive waves and expansive waves corresponding to the CS and SS flows are shown in figures (2), (3), (5) and (6). From these curves, we see that the solution profile aligns with that of planar flow. However, there is a minor difference in that the process of becoming compressive waves steeper (or flatter) is less significant in comparison to the case of planar flow. These findings also agree closely with the results established earlier by some distinguished authors such as Ram [15], Schmitt [40] and Singh et al. [18, 20-21].

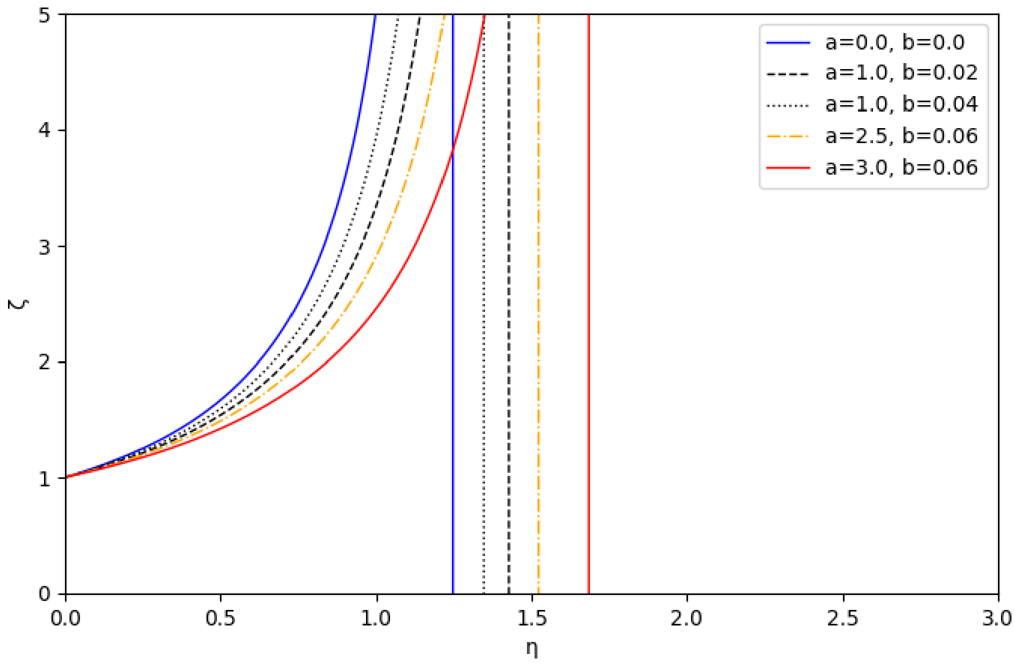

Figure-1: The effect of vdW parameters 'a' and 'b'on the evolution of compressive waves for planar flow $(N = 0)$ with $\omega = -0.3$.

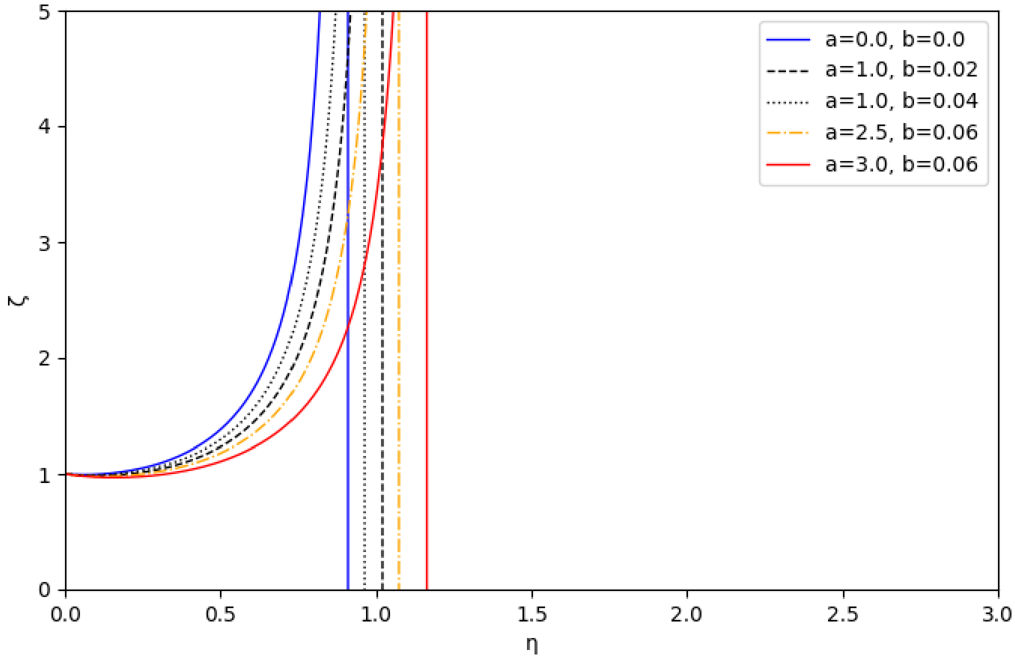

Figure-2: The effect of vdW parameters ‘a’ and ‘b’ on the evolution of compressive waves for CS flow ($N = 1$) with $\omega = -0.3$.

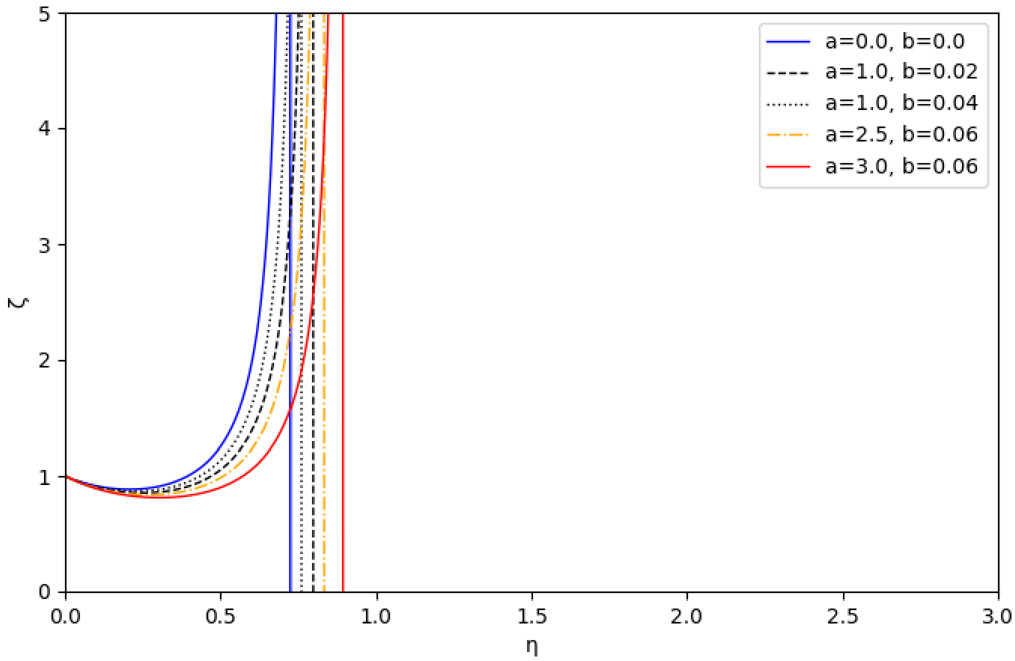

Figure-3: The effect of vdW parameters ‘a’ and ‘b’on the evolution of compressive waves for SS flow ($N = 2$) with $\omega = -0.3$.

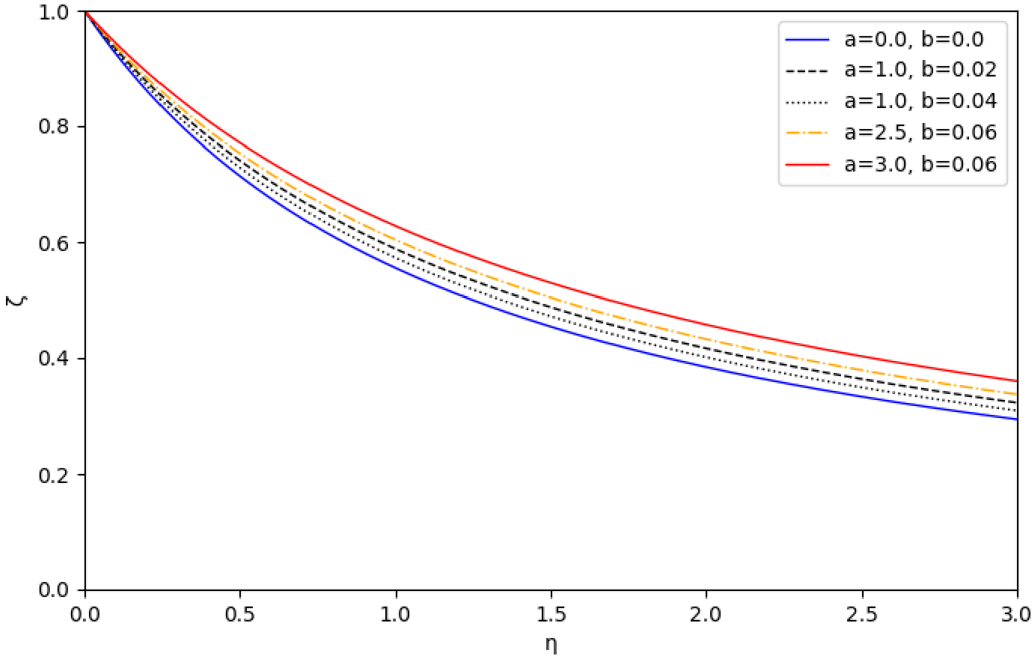

Figure-4: The effect of vdW parameters 'a' and 'b' on the attenuation of expansive waves for planar flow $(N = 0)$ with $\omega = 0.3$.

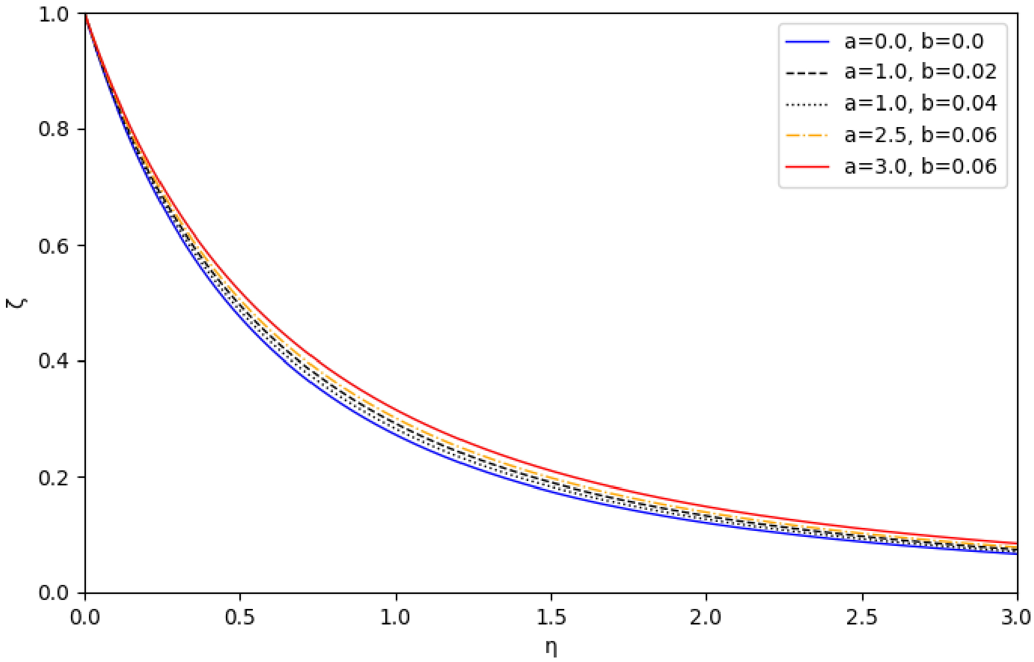

Figure-5: The effect of vdW parameters 'a' and 'b'on the attenuation of expansive waves for CS flow $(N = 1)$ with $\omega = 0.3$.

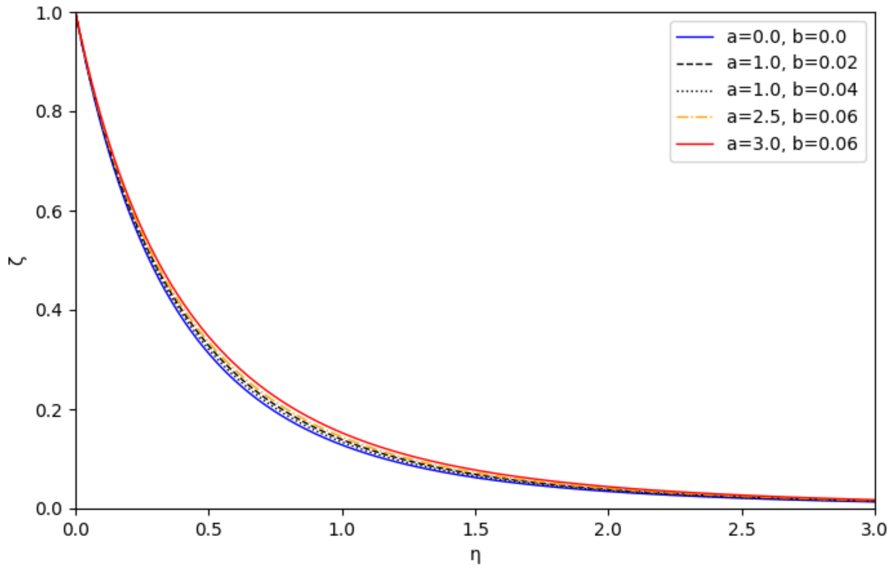

Figure-6: The effect of vdW parameters 'a' and 'b'on the attenuation of expansive waves for SS flow ($N = 2$) with $\omega = 0.3$.

VII. Conclusion:

The study examines the evolution and attenuation behaviour of expansive and compressive acceleration waves in vdW gas under "one-dimensional compressible, instable, inviscid(1D-CII)" for planar and non-planar (P/N-P) flows. Using the attributes of the regulatory system as reference coordinates, the solution of the problem of acceleration waves, propagating along the directions of C path, is derived. The research indicates that the underlying nonlinear nature of acceleration waves in the physical plane is revealed through a linear solution in the C plane. The transport equation that regulates the manner in which the expansive and compressive acceleration waves evolve and attenuate in vdW gas, is derived. Observation shows that compressive waves, regardless of their beginning strength, eventually end up as shock waves. It has been observed that as the vdW parameter "a" increases, the formation of shocks is delayed. Also, a similar effect of the increasing values of "b"was observed on the shock formation time. Our findings confirm that in planar, CS and SS flows, the amplitude of the expansive wave fronts diminishes. Moreover, vdW gas, being more complex than ideal gas, causes slower attenuation of expansive waves than what is observed in dust free gas.

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