RESEARCH ARTICLE                                                                OPEN ACCESS

# Thermal Stresses in a Limiting Thick Circular Plate with Internal Heat Generation

## Chetana Bhongade

*Department of Mathematics, Shree Shivaji Arts, Commerce & Science College,*
*Rajura, pin:442905, Maharashtra, India*

**ABSTRACT**

The present paper deals with the determination of quasi static thermal stresses in a limiting thick circular plate with internal heat generation subjected to arbitrary heat flux on upper and lower surface and the fixed circular edge is thermally insulated. Initially the plate is at zero temperature. Here we modify Kulkarni [1]and compute the effects of internal heat generation and heat supply in terms of stresses along radial direction. The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions. The results for temperature change, displacement and stresses have been computed numerically and illustrated graphically.

**KEYWORDS:** Thermal stresses, limiting thick circular plate, internal heat generation.

-----------------------------------------------------------------------------------------------------------------------------
-----------------------------------------------------------------------------------------------------------------------------

## I. INTRODUCTION

Progress in the field of air-craft and machine structures, mainly with gas and steam turbines and the emergence of new topics in chemical engineering have given rise to numerous problems in which thermal stresses play an important role and frequently even a primary role. Nowacki [2] has determined the temperature distribution on the upper face, with zero temperature on the lower face and the circular edge thermally insulated. C. M.Bhongade and M.H. Durge [3] studied an inverse steady state thermal stresses in a limiting thick circular plate with internal heat generation.

Most recently C. M. Bhongade and M.H. Durge [4] considered thick circular plate and discuss the effect of Michell function on steady state behavior of thick circular plate, now here we consider a thick circular plate with internal heat generation subjected to arbitrary heat flux on upper and lower surface and the fixed circular edge is thermally insulated. Initially the plate is at zero temperature. Here we modify Kulkarni [1] and compute the effects of internal heat generation and heat supply on the limiting thick circular plate in terms of stresses along radial direction. The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions. The results for temperature change, displacement and stresses have been

computed numerically and illustrated graphically. A mathematical model has been constructed with the help of numerical illustration by considering steel (0.5% carbon) limiting thick circular plate. No one previously studied such type of problem. This is new contribution to the field.

The direct problem is very important in view of its relevance to various industrial mechanics subjected to heating such as the main shaft of lathe, turbines and the role of rolling mill, base of furnace of boiler of a thermal power plant and gas power plant.

## II. FORMULATION OF THE PROBLEM

Consider a limiting thick circular plate of radius $a$ and thickness $2h$ defined by $0 \leq r \leq a, -h \leq z \leq h$. Initially the plate is at zero temperature. Let the plate be subjected to a arbitrary heat flux $\pm f(r,t)$ prescribed over the upper surface $(z = h)$ and the lower surface $(z = -h)$. The fixed circular edge $(r = a)$ is thermally insulated. Assume a limiting thick circular plate with internal heat generation is free from traction. Under these prescribed conditions, the quasi static transient thermal stresses are required to be determined.

The differential equation governing the displacement potential function $\phi(r,z,t)$ is given in [5] as

*Chetana Bhongade., International Journal of Engineering Research and Applications*
*www.ijera.com*
*ISSN: 2248-9622, Vol. 15, Issue 7, July 2025, pp 10-18*

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = K\tau \quad (1)$$

where $K$ is the restraint coefficient and temperature change $\tau = T - T_i$, $T_i$ is initial temperature. Displacement function $\phi$ is known as Goodier's thermoelastic displacement potential.

The temperature of the plate at time $t$ satisfying the heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$(2)$$

with the boundary conditions

$$T = \pm f(r,t) \; at \; z = \pm h, \; 0 \le r \le a$$

$$(3)$$

$$\frac{\partial T}{\partial r} = 0 \; at \; r = a, -h \le z \le h$$

$$(4)$$

$$q(r,z,t) = \delta(r - r_0)\sin(\beta_m z)(1 - e^{-t}), \; 0 < r_0 < a$$

$$(5)$$

and the initial condition

$$T = 0 \; at \; t = 0$$

$$(6)$$

where $\alpha$ is the thermal diffusivity of the material of the plate, $k$ is the thermal conductivity of the material of the plate, $q$ is the internal heat generation and $\delta(r)$ is well known dirac delta function of argument $r$.

The Michell's function $M$ must satisfy

$$\nabla^2 \nabla^2 M = 0$$

$$(7)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

$$(8)$$

The components of the stresses are represented by the thermoelastic displacement potential $\phi$ and Michell's function $M$ as

$$\sigma_{rr} = 2G\left\{\frac{\partial^2 \phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left[v\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right]\right\}$$

$$(9)$$

$$\sigma_{\theta\theta} = 2G\left\{\frac{1}{r}\frac{\partial \phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left[v\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right]\right\}$$

$$(10)$$

$$\sigma_{zz} = 2G\left\{\frac{\partial^2 \phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left[(2-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\}$$

$$(11)$$

and

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left[(1-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\}$$

$$(12)$$

where $G$ and $v$ are the shear modulus and Poisson's ratio respectively.

For traction free surface stress functions

$$\sigma_{rr} = \sigma_{rz} = 0 \; at \; z = h$$

$$(13)$$

Equations (1) to (13) constitute mathematical formulation of the problem.

### III. SOLUTION

To obtain the expression for temperature $T$ ($r, z, t$), we introduce the finite Hankel transform over the variable $r$ and its inverse transform defined by [6] as

$$\bar{T}(\beta_m, z, t) = \int_0^a r \, K_0(\beta_m, r) \, T(r, z, t) \, dr$$

$$(14)$$

$$T(r, z, t) = \sum_{m=1}^{\infty} K_0(\beta_m, r) \, \bar{T}(\beta_m, z, t)$$

$$(15)$$

where $K_0(\beta_m, r) = \frac{\sqrt{2}}{a}\frac{J_0(\beta_m r)}{J_0(\beta_m a)},$

$$(16)$$

$\beta_1, \beta_2 \ldots$ are roots of transcendental equation

$$J_1(\beta_m a) = 0$$

$$(17)$$

*Chetana Bhongade., International Journal of Engineering Research and Applications*
*www.ijera.com*
*ISSN: 2248-9622, Vol. 15, Issue 7, July 2025, pp 10-18*

where $J_n(x)$ is Bessel function of the first kind of order n.

On applying the finite Hankel transform defined in the Eq. (14), its inverse transform defined in (15)

and applying Laplace transform and its inverse by residue method successively to the Eq. (2), one obtains the expression for temperature as

$$T(\mathrm{r},\mathrm{z},\mathrm{t}) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sqrt{2}}{a}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}$$

$$\times \left\{ \left(\frac{n\pi\alpha}{2(-1)^n h^2}\right)\left[\sin[\frac{n\pi}{2h}(z+h)]+\sin[\frac{n\pi}{2h}(z-h)]\right]g(t)\right.$$

$$\left.+\left[\frac{1}{2\alpha\beta_m^2}+\frac{e^{-t}}{1-2\alpha\beta_m^2}+\frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2-1)}\right]\frac{\alpha\,D_m\sin(\beta_m z)}{k}\right\} \tag{18}$$

where

$$D_m = \frac{\sqrt{2}}{a}\frac{r_0 J_0(\beta_m r_0)}{J_0(\beta_m a)}$$

$$g(t) = \int_0^t e^{-\alpha\left[\beta_m^2+\frac{n^2\pi^2}{4h^2}\right](t-u)}$$

$$\times \left[\frac{\alpha\,D_m}{k}\sin(\beta_m h)\left(\frac{1}{2\alpha\beta_m^2}+\frac{e^{-u}}{1-2\alpha\beta_m^2}+\frac{e^{-2\alpha\beta_m^2 u}}{2\alpha\beta_m^2(2\alpha\beta_m^2-1)}-F(\beta_m,u)\right)\right] du$$

Since initial temperature $T_i = 0$, $\tau = T - T_i$

$$\tau = T \tag{19}$$

Michell's function $M$

Now let's assume that Michell's function $M$, which satisfy Eq.(7) is given by

$$M = \left(\frac{\sqrt{2}}{a}K\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}f(r,t)$$

$$\times [B_{mn}\sin h(\beta_m z)+C_{mn}\beta_m z\,\cos h(\beta_m z)]$$

(20)

where $B_{mn}$ and $C_{mn}$ are arbitrary functions, which can be determined by using condition (13).

Goodiers Thermoelastic Displacement Potential $\phi(r,z,t)$

Assuming the displacement function $\phi(r,z,t)$ which satisfies Eq. (1) as

$$\phi(r,z,t) = K\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sqrt{2}}{a}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}\left\{\left(\frac{-n\pi\alpha}{2(-1)^n h^2}\right)\frac{1}{\left(\beta_m^2+\frac{n^2\pi^2}{4h^2}\right)}\right.$$

$$\times \left[\sin[\frac{n\pi}{2h}(z+h)]+\sin[\frac{n\pi}{2h}(z-h)]\right]g(t)$$

$$\left.-\left(\frac{\alpha\,D_m}{2k\beta_m^2}\right)\left[\frac{1}{2\alpha\beta_m^2}+\frac{e^{-t}}{1-2\alpha\beta_m^2}+\frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2-1)}\right]\sin(\beta_m z)\right\}$$

(21)

Now using Eqs. (18), (20) and (21) in Eqs. (9), (10), (11) and (12), one obtains the expressions for stresses respectively as

*Chetana Bhongade., International Journal of Engineering Research and Applications*
*www.ijera.com*
*ISSN: 2248-9622, Vol. 15, Issue 7, July 2025, pp 10-18*

$$\frac{\sigma_{rr}}{K} = 2G \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sqrt{2}}{a} \frac{1}{J_0(\beta_m a)} \left\{ \left( \frac{n\pi\alpha}{2(-1)^n h^2} \right) \left[ \frac{\beta_m{}^2 J_1{}'(\beta_m r)}{\left( \beta_m{}^2 + \frac{n^2\pi^2}{4h^2} \right)} - J_0(\beta_m r) \right] \right.$$

$$\times \left[ \sin[\frac{n\pi}{2h}(z+h)] + \sin[\frac{n\pi}{2h}(z-h)] \right] g(t)$$

$$+ \frac{\alpha D_m \sin(\beta_m z)}{k} \left[ \frac{J_1{}'(\beta_m r)}{2\left( \beta_m{}^2 + \frac{n^2\pi^2}{4h^2} \right)} - J_0(\beta_m r) \right] \left[ \frac{1}{2\alpha\beta_m{}^2} + \frac{e^{-t}}{1-2\alpha\beta_m{}^2} + \frac{e^{-2\alpha\beta_m{}^2 t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)} \right]$$

$$+ J_1{}'(\beta_m r) \beta_m{}^2 [\, B_{mn} \beta_m F(\beta_m, t) \cosh(\beta_m z)]$$

$$+ C_{mn} \beta_m{}^2 \left[ \begin{array}{c} 2v \beta_m J_0(\beta_m r) \cosh(\beta_m z) + F(\beta_m, t) J_1{}'(\beta_m r) \\ \times (\beta_m \cosh(\beta_m z) + \beta_m{}^2 z \sinh(\beta_m z)) \end{array} \right] \right\} \tag{22}$$

$$\frac{\sigma_{\theta\theta}}{K} = 2G \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sqrt{2}}{a} \frac{1}{J_0(\beta_m a)} \left\{ \left( \frac{n\pi\alpha}{2(-1)^n h^2} \right) \left[ \frac{\beta_m J_1(\beta_m r)}{r\left( \beta_m{}^2 + \frac{n^2\pi^2}{4h^2} \right)} - J_0(\beta_m r) \right] \right.$$

$$\times \left[ \sin[\frac{n\pi}{2h}(z+h)] + \sin[\frac{n\pi}{2h}(z-h)] \right] g(t)$$

$$+ \frac{\alpha D_m \sin(\beta_m z)}{k} \left[ \frac{J_1(\beta_m r)}{2\beta_m r \left( \beta_m{}^2 + \frac{n^2\pi^2}{4h^2} \right)} - J_0(\beta_m r) \right] \left[ \frac{1}{2\alpha\beta_m{}^2} + \frac{e^{-t}}{1-2\alpha\beta_m{}^2} + \frac{e^{-2\alpha\beta_m{}^2 t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)} \right]$$

$$+ B_{mn} \beta_m{}^2 F(\beta_m, t) \cosh(\beta_m z) \frac{J_1(\beta_m r)}{r}$$

$$+ C_{mn} \beta_m{}^2 F(\beta_m, t) \left[ \begin{array}{c} 2v \beta_m J_0(\beta_m r) \cosh(\beta_m z) + \cosh(\beta_m z) \frac{J_1(\beta_m r)}{r} \\ \times \beta_m z \sinh(\beta_m z) \end{array} \right] \right\} \tag{23}$$

$$\frac{\sigma_{zz}}{K} = 2G \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sqrt{2}}{a} \frac{J_0(\beta_m r)}{J_0(\beta_m a)} \left\{ \left( \frac{n^3\pi^3\alpha}{8(-1)^n h^4} \right) \frac{1}{\left( \beta_m{}^2 + \frac{n^2\pi^2}{4h^2} \right)} \right.$$

$$\times \left[ \left( \sin[\frac{n\pi}{2h}(z+h)] + \sin[\frac{n\pi}{2h}(z-h)] \right) g(t) \right.$$

$$\left. + \frac{\alpha D_m \sin(\beta_m z)}{2k} \left[ \frac{1}{2\alpha\beta_m{}^2} + \frac{e^{-t}}{1-2\alpha\beta_m{}^2} + \frac{e^{-2\alpha\beta_m{}^2 t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)} \right] \right]$$

$$- \left( \frac{n\pi\alpha}{2(-1)^n h^2} \right) \left[ \sin[\frac{n\pi}{2h}(z+h)] + \sin[\frac{n\pi}{2h}(z-h)] \right] g(t)$$

$$- B_{mn} \beta_m{}^2 F(\beta_m, t) \sinh(\beta_m z) + C_{mn} \beta_m{}^2 F(\beta_m, t)$$

$$\times \left[ 2(1-v) \sinh(\beta_m z) - \beta_m z \cosh(\beta_m z) \right] \right\} \tag{24}$$

$$\frac{\sigma_{rz}}{K} = 2G \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sqrt{2}}{a} \frac{\beta_m J_1(\beta_m r)}{J_0(\beta_m a)} \left\{ \left( \frac{n^2\pi^2\alpha}{4(-1)^n h^3} \right) \frac{1}{\left( \beta_m{}^2 + \frac{n^2\pi^2}{4h^2} \right)} \right.$$

$$\times \left( \cos[\frac{n\pi}{2h}(z+h)] + \cos[\frac{n\pi}{2h}(z-h)] \right) g(t)$$

$$+ \frac{\alpha D_m \cos(\beta_m z)}{2k\beta_m} \left[ \frac{1}{2\alpha\beta_m{}^2} + \frac{e^{-t}}{1-2\alpha\beta_m{}^2} + \frac{e^{-2\alpha\beta_m{}^2 t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)} \right]$$

$$- B_{mn} \beta_m{}^2 F(\beta_m, t) \sinh(\beta_m z) + C_{mn} \beta_m{}^2 F(\beta_m, t) \right\}$$

*Chetana Bhongade., International Journal of Engineering Research and Applications*
*www.ijera.com*
*ISSN: 2248-9622, Vol. 15, Issue 7, July 2025, pp 10-18*

$$\times \left[2v \sinh(\beta_m z) - \beta_m z \cosh(\beta_m z)\right] \tag{25}$$

In order to satisfy condition Eq. (13), solving Eqs. (22) and (25) for $B_{mn}$ and $C_{mn}$ one obtains

$$C_{mn} = \frac{1}{\beta_m^2 R} \left\{ \left[-\frac{\alpha D_m}{k} \sin(\beta_m h) \sinh(\beta_m h)\right] \left[\frac{\beta_m^2 J_1'(\beta_m a)}{2\left(\beta_m^2 + \frac{n^2 \pi^2}{4h^2}\right)} - J_0(\beta_m a)\right] \right.$$

$$\times \left(\frac{1}{2\alpha\beta_m^2} + \frac{e^{-t}}{1-2\alpha\beta_m^2} + \frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2 - 1)}\right) + J_1'(\beta_m a)\,\beta_m \cosh(\beta_m h)$$

$$\left. \times \left[\left(\frac{-n^2 \pi^2 \alpha}{4(-1)^n h^3}\right)\frac{g(t)\langle(-1)^n + 1\rangle}{\left(\beta_m^2 + \frac{n^2\pi^2}{4h^2}\right)} - \frac{\alpha D_m}{2k\beta_m}\cos(\beta_m h)\left(\frac{1}{2\alpha\beta_m^2} + \frac{e^{-t}}{1-2\alpha\beta_m^2} + \frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2 - 1)}\right)\right]\right\} \tag{26}$$

$$B_{mn} = \frac{-1}{\beta_m^2 R} \left\{ \frac{\alpha D_m}{k}\sin(\beta_m h)\left[\frac{\beta_m^2 J_1'(\beta_m a)}{2\left(\beta_m^2 + \frac{n^2\pi^2}{4h^2}\right)} - J_0(\beta_m a)\right]\left(\frac{1}{2\alpha\beta_m^2} + \frac{e^{-t}}{1-2\alpha\beta_m^2} + \frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2 - 1)}\right) \right.$$

$$\times \left[2v \sinh(\beta_m h) - \beta_m h \cosh(\beta_m h)\right]$$

$$+ \frac{1}{F(\beta_m, t)}\begin{bmatrix} 2v\,\beta_m J_0(\beta_m a)\cosh(\beta_m h) + J_1'(\beta_m a)\,F(\beta_m, t) \\ \times\,(\beta_m \cosh(\beta_m h) + \beta_m^2 h \sinh(\beta_m h)) \end{bmatrix}$$

$$\left. \times \left[\left(\frac{-n^2\pi^2\alpha}{4(-1)^n h^3}\right)\frac{g(t)\langle(-1)^n + 1\rangle}{\left(\beta_m^2 + \frac{n^2\pi^2}{4h^2}\right)} - \frac{\alpha D_m}{2k\beta_m}\cos(\beta_m h)\left(\frac{1}{2\alpha\beta_m^2} + \frac{e^{-t}}{1-2\alpha\beta_m^2} + \frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2 - 1)}\right)\right]\right\} \tag{27}$$

where

$$R = J_1'(\beta_m a)\,\beta_m\,F(\beta_m, t)\,\sinh(\beta_m h)\,[2v\sinh(\beta_m h) - \beta_m h \cosh(\beta_m h)]$$

$$+ \sinh(\beta_m h)\begin{bmatrix} 2v\,\beta_m J_0(\beta_m a)\cosh(\beta_m h) + J_1'(\beta_m a)\,F(\beta_m, t) \\ \times\,(\beta_m \cosh(\beta_m h) + \beta_m^2 h \sinh(\beta_m h)) \end{bmatrix}$$

## IV. SPECIAL CASE AND NUMERICAL CALCULATIONS

Setting

$$f(r,t) = \delta(r - r_0)(1 - e^{-t})$$

$a = 2m,\ h = 0.200000000000000000000001m,\ r_0 = 1m, t = 2\ sec.$

where $\delta(r)$ is well known diract delta function of argument $r$.

Material Properties

The numerical calculation has been carried out for steel (0.5% carbon) limiting thick circular plate with the material properties defined as

Thermal diffusivity $\alpha = 14.74\times 10^{-6}\ m^2 s^{-1}$,
Specific heat $c_\rho = 465$ J/kg,

Thermal conductivity $k = 53.6$ W/m K,
Poisson ratio $\vartheta = 0.35$,
Young's modulus $E = 130\ G$ pa,
Lame constant $\mu = 26.67$,
Coefficient of linear thermal expansion
$a_t = 13\times 10^{-6}\ 1/K$

Roots of Transcendental Equation
The $\beta_1 = 1.9159, \beta_2 = 3.5078, \beta_3 = 5.0867, \beta_4 = 6.6618, \beta_5 = 8.2353, \beta_6 = 9.8079$ are the roots of transcendental equation $J_1(\beta_m a) = 0$. The numerical calculation and the graph has been carried out with the help of mathematical software Mat lab.

## V. DISCUSSION

In this paper a limiting thick circular plate with internal heat generation is considered and determined the expressions for temperature, displacement and stresses. We compute the effects of internal heat generation and heat supply in terms

*Chetana Bhongade., International Journal of Engineering Research and Applications*
*www.ijera.com*
*ISSN: 2248-9622, Vol. 15, Issue 7, July 2025, pp 10-18*

of stresses along radial direction by substituting $q = 0$ in Eqs. (18), (21), (22), (23), (24), (25), (26) and (27). We compare the results for $q = 0$ and $q \neq 0$. As a special case mathematical model is constructed

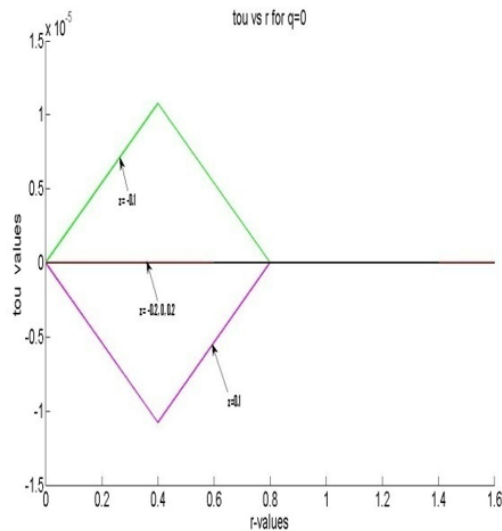by considering steel (0.5% carbon) limiting thick circular plate with the material properties specified above.



**Fig. 1** Temperature $\tau$ for ($q$=0).
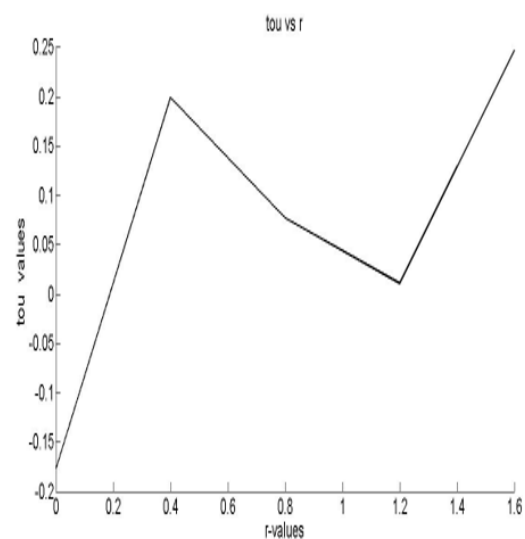


**Fig. 2** Temperature $\tau$ for ($q \neq 0$).
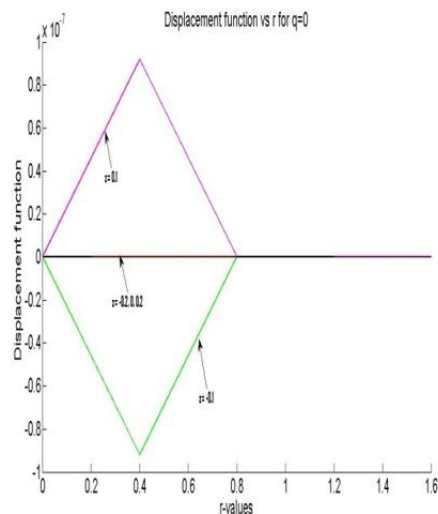


**Fig. 3** The displacement $\phi$ for ($q$= 0).
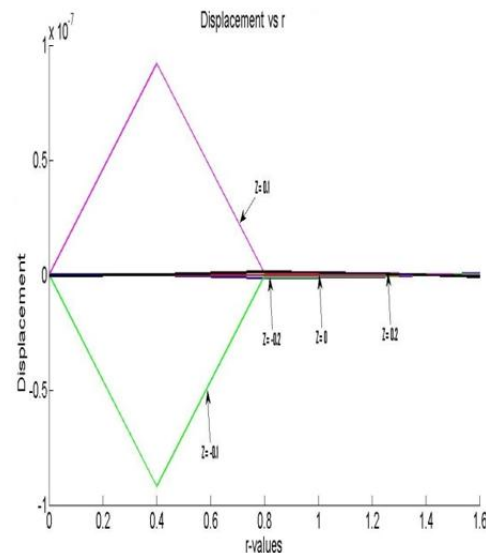


**Fig. 4** The displacement $\phi$ for ($q \neq 0$).
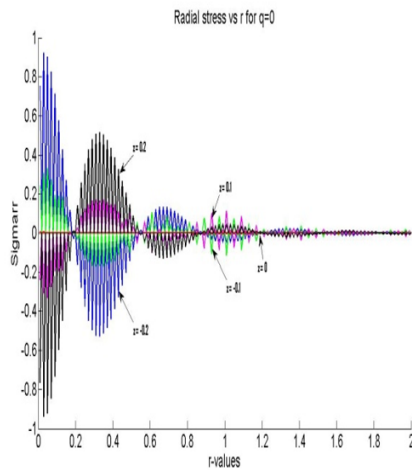
**Fig. 5** Radial stresses $\frac{\sigma_{rr}}{K}$ for ($q = 0$).
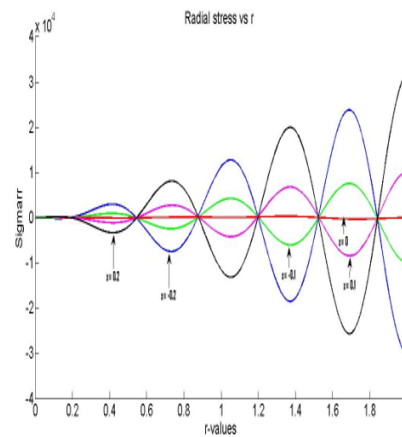


**Fig. 6** Radial stresses $\frac{\sigma_{rr}}{K}$ for ($q \neq 0$).
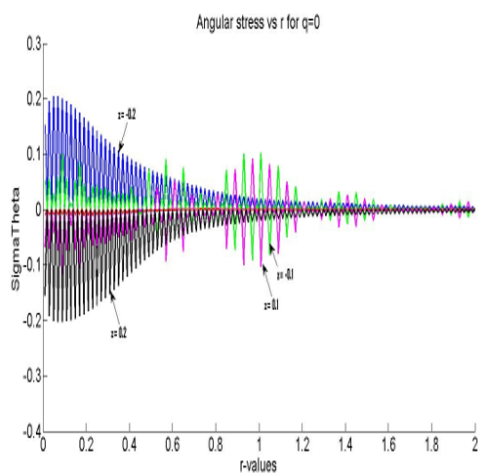


**Fig. 7** Angular stresses $\frac{\sigma_{\theta\theta}}{K}$ for ($q = 0$).
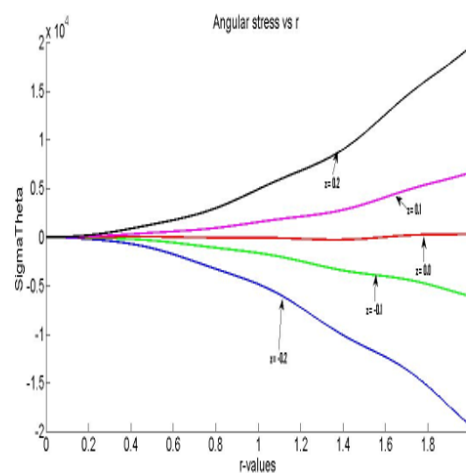


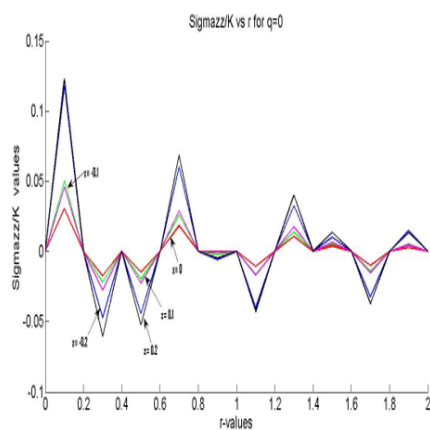**Fig. 8** Angular stresses $\frac{\sigma_{\theta\theta}}{K}$ for ($q \neq 0$).



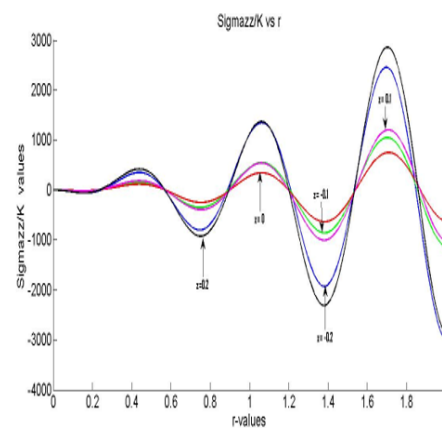**Fig. 9** Axial stresses $\frac{\sigma_{zz}}{K}$ for ($q = 0$).



**Fig. 10** Axial stresses $\frac{\sigma_{zz}}{K}$ for ($q \neq 0$).

*Chetana Bhongade., International Journal of Engineering Research and Applications*
*www.ijera.com*
*ISSN: 2248-9622, Vol. 15, Issue 7, July 2025, pp 10-18*

**Fig. 11** Stress function $\frac{\sigma_{rz}}{K}$ for ($q$= 0).



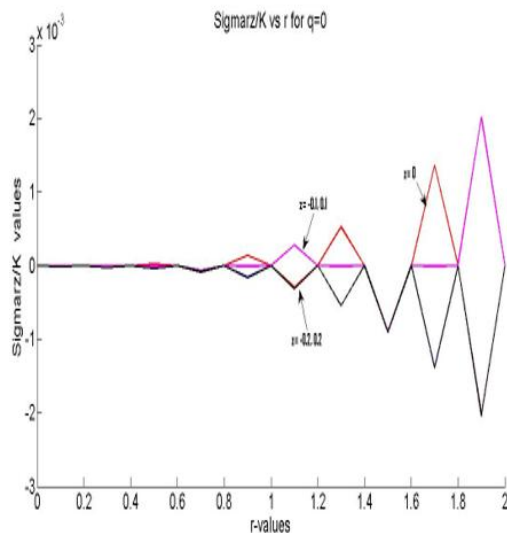**Fig. 12** Stress function $\frac{\sigma_{rz}}{K}$ for ($q\neq$ 0).

From fig. 1 and 2, it is observed that due to internal heat generation in limiting thick circular plate temperature increases along radial direction. Heat supply shows a negligible effect at the point of heat supply along radial direction.

From fig. 3 and 4, it is observed that the internal heat generation and heat supply shows a negligible effect on displacement in limiting thick circular plate at the point of heat supply along radial direction.

From fig. 5 and 6, it is observed that the internal heat generation and heat supply develops tensile radial stress $\frac{\sigma_{rr}}{K}$ towards the lateral surface of limiting thick circular plate along radial direction.

From fig. 7 and 8, it is observed that the internal heat generation and heat supply develops tensile angular stress $\frac{\sigma_{\theta\theta}}{K}$ towards the lateral surface of limiting thick circular plate along radial direction.

From fig. 9 and 10, it is observed that the internal heat generation and heat supply develops tensile axial stress $\frac{\sigma_{zz}}{K}$ towards the lateral surface of limiting thick circular plate along radial direction.

From fig. 11 and 12, it is observed that the internal heat generation and heat supply develops infinitesimal stress $\frac{\sigma_{rz}}{K}$ towards the lateral surface of limiting thick circular plate along radial direction.
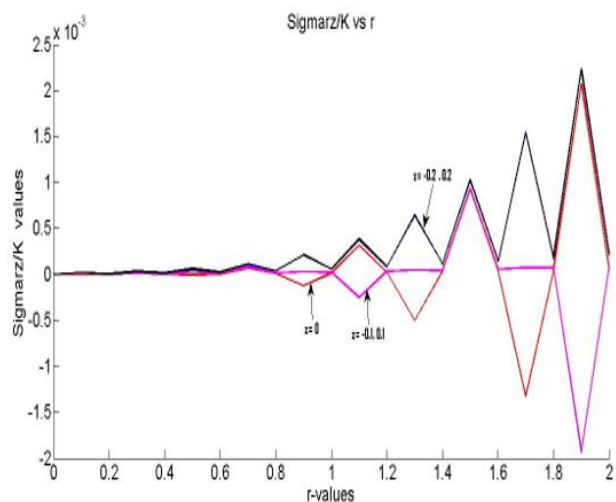
## VI.     CONCLUSION

We can conclude that due to internal heat generation in limiting thick circular plate temperature increases along radial direction. Heat supply shows a negligible effect on temperature, displacement at the point of heat supply in limiting thick circular plate along radial direction. The internal heat generation and heat supply develops tensile radial stress $\frac{\sigma_{rr}}{K}$ , angular stress $\frac{\sigma_{\theta\theta}}{K}$, the axial stress $\frac{\sigma_{zz}}{K}$ and the stress $\frac{\sigma_{rz}}{K}$ towards the lateral surface of limiting thick circular plate along radial direction.

The results obtained here are useful in engineering problems particularly in the determination of state of stress in a limiting thick circular plate and base of furnace of boiler of a thermal power plant and gas power plant.

### REFERENCES

[1]. V. S. Kulkarni and K. C. Deshmukh, Quasi static thermal stresses in a thick circular plate due to axisymmetric heat supply, Int. J. of Appl. Math and Mech. 5(6), 2009, 38-50.
[2]. W. Nowacki, The state of stresses in a thick circular disk due to temperature field, Bull. Acad. Polon. Sci., Scr. Scl. Tech., 5, 1957, 227.
[3]. C. M. Bhongade and M. H. Durge, An inverse steady state thermal stresses in a thin clamped circular plate with internal heat generation, American Journal of Engineering Research,2(10), 2013, 276-281.

[4]. C. M. Bhongade and M. H. Durge, Effect of Michell function on steady state behavior of thick circular plate, IOSR Journal of Mathematics,8(2), 2013, 55-60.

[5]. Naotake Noda, Richard B Hetnarski and Yoshinobu Tanigawa, Thermal Stresses, 2 nd edn., (Taylor and Francis New York, 2003), 259-261.

[6]. M. N. Ozisik, Boundary Value Problems of Heat Conduction, International Text Book Company, Scranton, Pennsylvania, 1968.