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# On Some Double Integral Transformation of Aleph-Function

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**ABSTRACT**

In this paper, the author will establish a double integral transform of Aleph $(\aleph)$-function which leads to yet another interesting process of augmenting the parameters in the Aleph $(\aleph)$-function. The result is given of general character and on specializing the parameters suitably, yields several interesting results as special cases.
**Key words:** Aleph $(\aleph)$-function, Euler Transformation, Hypergeometric Function, Integral Transformation.
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## I. INTRODUCTION

Many researchers like Rainville [6, p.104], Abdul Halim and Al-Salam [1] have shown that the single and double Euler transformations of the hypergeometric function are effective tools for augmenting its parameters. Srivastava and Singhal [9] and Srivastava and Joshi [10] have discussed some similar interesting properties of $_pF_q$ in double $\aleph$-function and double Whittaker transforms respectively.

We have used the symbols $(a_r, A_r), \Delta(r,a), \Delta(r, \pm a), \Delta((r, a_p))$ to denote the set of parameters

$$(a_1, A_1),...,(a_r, A_r); \frac{a}{r}, \frac{a+1}{r},..., \frac{a+r-1}{r}; \Delta(r,a), \Delta(r,-a)$$

and $\Delta(r, a_1), \Delta(r, a_2),..., \Delta(r, a_p)$ respectively.

The $\aleph$-function introduced by Suland et.al. [11] will be represented and defined as follows:

$$\aleph[x] = \aleph^{m,n}_{p_i,q_i:\tau_i r}[x] = \aleph^{m,n}_{p_i,q_i:\tau_i r}\left[ x \Big|^{(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i}}_{(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i}} \right] = \frac{1}{2\pi i}\int_L \theta(s)ds \qquad (1.1)$$

where $i = \sqrt{-1}$

$$\theta(s) = \frac{\prod\limits_{j=1}^{m}\Gamma(b_j - B_j s)\prod\limits_{j=1}^{n}\Gamma(1-a_j+A_j s)}{\sum\limits_{i=1}^{r}\tau_i\left\{\prod\limits_{j=m+1}^{q_i}\Gamma(1-b_{ji}+B_{ji}s)\prod\limits_{j=n+1}^{p_i}\Gamma(a_{ji}-A_{ji}s)\right\}} \qquad (1.2)$$

$p_i, q_i (i=1,...,r), m, n$ are integers satisfying $0 \le n \le p_i$, $0 \le m \le q_i, (i=1,...,r), r$ is finite

$A_j, B_j, A_{ij}, B_{ji}$ are real and $a_j, b_j, a_{ji}, b_{ji}$ are complex numbers such that

$A_j(b_h + v) \ne B_h(a_j - v - k)$ for $v, k = 0 1, 2, ...$

We shall use the following notations:

$A^* = (a_j, A_j)_{1,n}, [\tau_i(a_{ji}, A_{ji})]_{n+1,p_i}; B^* = (b_j, B_j)_{1,m}, [\tau_i(b_{ji}, B_{ji})]_{m+1,q_i}$

$A^{**} = (c_j, C_j)_{1,r}, [\tau_i(c_{ji}, C_{ji})]_{r+1,u_i}; B^{**} = (d_j, D_j)_{1,s}, [\tau_i(d_{ji}, D_{ji})]_{s+1,v_i}$

If we take $\tau_i = 1, r = 1$ in (1.1), the $\aleph$ - function reduces to the Fox's H-function [4].

## II.    MAIN RESULT

In this section, we have established the following double integral transform of $\aleph$-function:
If $s, k$ and $r$ are positive integers, then

$$\int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^\mu \, \aleph_{u_i,v_i:\tau_i:r}^{f,g} \left[ \gamma(x+y) \Big|_{D^*}^{C^*} \right]$$

$$\aleph_{p_i,q_i:\tau_i:r}^{m,n} \left[ t x^s y^k (x+y)^r \Big|_{B^*}^{A^*} \right] dx\,dy =$$

$$(2\pi)^{(1-D)\left(f+g-\frac{1}{2}u-\frac{1}{2}v\right)+\frac{1}{2}} D^{\sum_1^v d_j - \sum_1^u c_j + \left(A-\frac{1}{2}\right)(u-v)}$$

$$\frac{s^{\alpha-\frac{1}{2}} k^{\beta-\frac{1}{2}}}{\lambda^{\alpha+\beta+\mu}(s+k)^{\alpha+\beta-\frac{1}{2}}} \aleph_{p_i+\rho+Dv_i,q_i+\rho+Du_i:\tau_i:r}^{m+Dg,n+\rho+Df}$$

$$\left[ \frac{t\delta D^{D(v-u)}}{\gamma^D} \Bigg|_{\Delta((D,1-A-C_g)),B^*,\Delta(k+s,1-\alpha-\beta),\Delta(D,1-A-c_{g+1}),...,\Delta(D,1-A-c_v)}^{\Delta((D,1-A-D_f)),\Delta(s,1-\alpha),\Delta(k,1-\beta),A^*,\Delta(D,1-A-d_{f+1}),...,\Delta(D,1-A-d_u)} \right] \tag{2.1}$$

Where $\lambda = \dfrac{s^s k^k}{(s+k)^{(s+k)}}, \rho = s+k, D = s+k+r, A = \alpha+\beta+\mu,$

$0 \le Dg \le Du \le Dv < Du + q - p, u + v - 2g \le 2f \le 2v, 0 \le n \le p,$

$p + q - 2n < 2m \le 2q,$

$$\mathrm{Re}\left( \min \frac{d_i}{\delta_i} + D \min \frac{b_{ji}}{\beta_{ji}} \right) > \mathrm{Re}(-A) > \mathrm{Re}\left[ D\left( \frac{s-\alpha}{s}, \frac{k-\beta}{k}, a_l \right) + C_t - D - 1 \right]$$

$i = 1, 2, ..., f; j = 1, 2, ..., m; l = 1, 2, ...n; t = 1, 2, ..., g; u, \mathrm{Re}(\min C_i + A) - v,$

$$\mathrm{Re}\left( \max \frac{d_j}{\delta_j} + A \right) - uD + v + \frac{1}{2} D\left( Dv - Du + 1 \right) > D(Dv - Du),$$

$$\mathrm{Re}\max\left( \frac{s-\alpha}{s}, \frac{k-\beta}{k}, a_l \right),$$

$i = 1, 2, ..., u; j = 1, 2, ..., v; l = 1, 2, ..., u; |\arg\gamma| \le \left( f + g - \frac{1}{2}u - \frac{1}{2}v \right)\pi,$

$|\arg t| < \left( m + n - \frac{1}{2}p - \frac{1}{2}q \right)\pi, \mathrm{Re}\left( \alpha + s\frac{b_{ji}}{\beta_{ji}} \right) > 0, \mathrm{Re}\left( \beta + k\frac{b_{ji}}{\beta_{ji}} \right) > 0, j = 1, 2, ..., m$

And the double integral converges.
Proof: To prove (2.1), we start with the following known result [2, p. 177]

$$\int_0^\infty \int_0^\infty \theta(x+y) x^{\alpha-1} y^{\beta-1} dx\,dy = B(\alpha, \beta) \int_0^\infty \theta(z) z^{\alpha+\beta-1} dz \tag{2.2}$$

Which is valid for $\mathrm{Re}(\alpha) > 0$ and $\mathrm{Re}(\beta) > 0$.
It is easy to prove by following the technique of reversing the order of integrations, that

$$\int_0^\infty \int_0^\infty \theta(x+y) x^{\alpha-1} y^{\beta-1} \aleph_{p_i,q_i:\tau_i:r}^{m,n} \left[ t x^s y^k (x+y)^r \Big|_{B^*}^{A^*} \right] dx\,dy =$$

$$\sqrt{2\pi} \, \frac{s^{\alpha-\frac{1}{2}} k^{\beta-\frac{1}{2}}}{(s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$\int_0^\infty \phi(z) z^{\alpha+\beta-1} \aleph_{p_i+\rho, q_i+\rho:\tau_i:r}^{m,n+\rho} \left[ t\delta z^D \Big|_{B^*,\Delta(k+s,1-\alpha-\beta)}^{\Delta(s,1-\alpha),\Delta(k,1-\beta),A^*} \right] dz \qquad (2.3)$$

Where $s, k$ and $r$ are positive integers,

$$\delta = \frac{s^s k^k}{(s+k)^{s+k}}, \rho = s+k, D = s+k+r, p+q < 2(m+n),$$

$$|\arg t| < \left( m+n-\frac{1}{2}p-\frac{1}{2}q \right)\pi,$$

$$\mathrm{Re}\left( \alpha + s\frac{b_j}{\beta_j} \right) > 0, \mathrm{Re}\left( \beta + k\frac{b_j}{\beta_j} \right) > 0, j = 1, 2, ..., m.$$

In (2.3), taking

$$\theta(x) = z^\mu \aleph_{u_i, v_i:\tau_i:r}^{f,g} \left[ \gamma x \Big|_{B^{**}}^{A^{**}} \right]$$

And evaluating the integral on the right hand side using [7, p.401] the result (2.1) follows.

## III.    PARTICULAR CASES

On choosing the parameters suitably in (2.1), several known and unknown results are obtained as particular cases. However, we mention some of the interesting results here.

(a) Taking

$$f = v = 2, g = 0, u = 1, c_1 = \frac{1}{2}, d_1 = v, d_2 = -v, \mu = \sigma + \frac{1}{2}, \alpha_j = \beta_j = \delta_j = \lambda_j = 1, r = 1, \tau_i = 1$$

in (2.1) and using [3, p.216, (5)]

$$H_{1,2}^{2,0}\left[ z \Big|_{(b,1),(-b,1)}^{\left(\frac{1}{2},1\right)} \right] = \pi^{-\frac{1}{2}} e^{-\frac{1}{2}x} K_b\left( -\frac{1}{2}z \right),$$

We obtain

$$\int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^{\sigma+\frac{1}{2}} a^{-\frac{1}{2}\gamma(x+y)} K_v\left\{ -\frac{1}{2}\gamma(x+y) \right\}$$

$$H_{p_i,q_i:1}^{m,n}\left[ tx^s y^k (x+y)^r \Big|_{B^*}^{A^*} \right] dxdy =$$

$$(2\pi)^{-\frac{1}{2}(2-D)\left( f+g-\frac{1}{2}u-\frac{1}{2}v \right)+\frac{1}{2}} D^{A-1} \sqrt{\pi} \frac{s^{\alpha-\frac{1}{2}} k^{\beta-\frac{1}{2}}}{\lambda^{\alpha+\beta+\mu}(s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$H_{p+\rho+2D, q+\rho+D}^{m,n+\rho+2D}\left[ \frac{t\delta D^D}{\gamma^D} \Big|_{B^*,\Delta((D,\frac{1}{2}-A)),\Delta(k+s,1-\alpha-\beta)}^{\Delta((D,1-A\mp v)),\Delta(s,1-\alpha),\Delta(k,1-\beta),A^*} \right], \qquad (3.1)$$

Where $\delta, D$ and $\gamma$ have the same value as (2.1) and

$$A = \sigma + \alpha + \beta + \frac{1}{2}; p+q < 2(m+n), \mathrm{Re}(\alpha + s\frac{b_{ji}}{\beta_{ji}} \pm v) > 0, \mathrm{Re}(\beta + s\frac{b_{ji}}{\beta_{ji}} \pm v) > 0,$$

$$\mathrm{Re}\left( \alpha + \beta + \mu \pm v + D\frac{b_{ji}}{\beta_{ji}} + \frac{1}{2} \right) > 0, j = 1, 2, ..., m; \mathrm{Re}(\lambda) > 0,$$

$$|\arg t| < \left( m+n-\frac{1}{2}p-\frac{1}{2}q \right)\pi$$

(b) Further, replacing $q, t$ and $(a_p, \alpha_p)$ by $q+1, -t$ and $(1-a_p, \alpha_p)$ respectively and then putting

$$\int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^{\mu+\frac{1}{2}} a^{-\frac{1}{2}\lambda(x+y)} K_v \left\{ -\frac{1}{2}\gamma(x+y) \right\}$$

$$_pF_q \left[ \begin{matrix} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{matrix} : tx^s y^k (x+y)^r \right] dxdy =$$

$$\frac{\sqrt{\pi}\,\Gamma\left(\alpha+\beta+\sigma \pm v + \frac{1}{2}\right)}{\gamma^{\alpha+\beta+\sigma+\frac{1}{2}}\Gamma\left(\alpha+\beta+\sigma+1\right)} B(\alpha, \beta)$$

$$_{p+3s+3k+2r}F_{q+2s+2k+r} \left[ t\delta \left( \frac{s+k+r}{\gamma} \right)^{s+k+r} \Bigg| \begin{matrix} \Delta(s+k+r, \alpha+\beta+\sigma \pm v+\frac{1}{2},), \Delta(s,\alpha), \Delta(k,\beta), (a_p,1) \\ (b_q,1), \Delta(s+k,\alpha+\beta), \Delta(k+s,\alpha+\beta+\sigma+1) \end{matrix} \right], \qquad (3.2)$$

provided $\mathrm{Re}(\sigma + \alpha + \beta \pm v + \frac{1}{2}) > 0, \mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0.$

(c) Setting $v = f = 2, g = 0, u = 0, c_1 = \sigma, d_1 = -\frac{1}{2} - v, d_2 = -\frac{1}{2} + v, r = 1, \tau_i = 1$ in (2.1) and using the known formula [3,p.216,(6)]

$$H_{1,2}^{2,0}\left[ z \Bigg| \begin{matrix} (1-k,1) \\ (\frac{1}{2}+m,1),(\frac{1}{2}-m,1) \end{matrix} \right] = e^{-\frac{1}{2}z} W_{k,m}(z),$$

We have

$$\int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^{\mu} e^{-\frac{1}{2}\gamma(x+y)} W_{\lambda,v}[\gamma(x+y)]$$

$$H_{p,q}^{m,n}\left[ tx^s y^k (x+y)^r \Big|_{B^*}^{A^*} \right] dxdy =$$

$$(2\pi)^{\frac{1}{2}(2-D)} D^{\sigma+A-\frac{1}{2}} \frac{s^{\alpha-\frac{1}{2}} k^{\beta-\frac{1}{2}}}{\lambda^{\alpha+\beta+\gamma}(s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$H_{p+\rho+2D, q+\rho+D}^{m,n+\rho+2D}\left[ \frac{t\delta D^D}{\gamma^D} \Bigg| \begin{matrix} \Delta(D, \frac{1}{2}-A \pm v), \Delta(s, 1-\alpha), \Delta(k, 1-\beta), A^* \\ B^*, \Delta(k+s, 1-\alpha-\beta), \Delta(D, \sigma-A) \end{matrix} \right], \qquad (3.3)$$

Where
$D, \rho, \delta$ and $A$ are given in (2.1);

$$p+q < 2(m+n), |\arg t| < \left( m+n-\frac{1}{2}p-\frac{1}{2}q \right)\pi, \mathrm{Re}(\gamma) > 0, \mathrm{Re}(\alpha+sb_j > 0,$$

$$\mathrm{Re}(k+sb_j) > 0, \mathrm{Re}\left( m+n+\mu+Db_j \pm v+\frac{1}{2} \right) > 0, j=1,2,...,m.$$

(d) Further, replacing $q, t$ and $(a_p, \alpha_p)$ by $q+1, -t$ and $(1-a_p, \alpha_p)$ respectively and then putting

$m = 1, n = p, b_1 = 1, b_{j+1} = b_j (j=1,2,...q)$,

using the result [3,p. 215, (1)] and [3,p. 4, (11)], we obtain an interesting result obtained by Srivastava and Singhal [9]:

$m = 1, n = p, b_1 = 1, b_{j+1} = b_j (j=1,2,...,q)$ and using the result [p.215,(1)], (3.3) reduces to a result due to Srivastava and Joshi [9,p.19,(2.3)]

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$$\int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^\eta e^{-\frac{1}{2}\gamma(x+y)} W_{\mu,v}\{\gamma(x+y)\}$$

$$_pF_q\left[\begin{matrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{matrix}:tx^s y^k (x+y)^r\right]dxdy =$$

$$\frac{\Gamma\left(\alpha+\beta+\eta\pm v+\frac{1}{2}\right)}{\lambda^{\alpha+\beta+\eta}\Gamma\left(\alpha+\beta+\eta-\mu+1\right)}B(\alpha,\beta)$$

$$_{p+3s+3k+2r}F_{q+2s+2k+r}\left[t\delta\delta'\left|\begin{matrix}\Delta(s+k+r,\alpha+\beta+\eta\pm v+\frac{1}{2},),\Delta(s,\alpha),\Delta(k,\beta),(a_p,1)\\(b_q,1),\Delta(s+k,\alpha+\beta),\Delta(k+s+r,\alpha+\beta+\eta-\mu+1)\end{matrix}\right.\right] \quad (3.4)$$

Where

$$\delta = \frac{s^s k^k}{(s+k)^{s+k}}, \delta' = \left(\frac{s+k+r}{\gamma}\right)^{s+k+r}$$

$\mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0, \mathrm{Re}(\gamma) > 0, \mathrm{Re}\left(\alpha+\beta+\eta\pm v+\frac{1}{2}\right) > 0$ and the resulting hypergeometric series converges.

With $\mu = 0, v = \pm\frac{1}{2}$ and $\eta = -\frac{1}{2}$ ,(3.4) reduces to the earlier results of Jain [5] and Singh [8].

(e) Choosing

$$f = g = u = 1, v = 2, c_1 = 1-k, d_1 = \frac{1}{2}+M, d_2 = \frac{1}{2}-M, \alpha_j = \beta_j = \delta_j = \gamma_j = 1, r = 1, \tau_i = 1$$

in (2.1) and using the known result

$$H_{1,2}^{1,1}\left[z\left|\begin{matrix}(1-k,1)\\(\frac{1}{2}+m,1),(\frac{1}{2}-m,1)\end{matrix}\right.\right] = \frac{\Gamma\left(\frac{1}{2}+k+m\right)}{\Gamma(2m+1)}e^{-\frac{1}{2}z}M_{k,m}(z),$$

We obtain

$$\int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^\eta e^{-\frac{1}{2}\gamma(x+y)} M_{k,m}[\gamma(x+y)]$$

$$H_{p_i,q_i:1}^{m,n}\left[tx^s y^k (x+y)^r \left|\begin{matrix}A^*\\B^*\end{matrix}\right.\right]dxdy =$$

$$(2\pi)^{\frac{1}{2}(2-D)} D^{k+A-\frac{1}{2}}\frac{\Gamma(2m+1)}{\Gamma\left(k+m+\frac{1}{2}\right)}\frac{s^{\alpha-\frac{1}{2}}k^{\beta-\frac{1}{2}}}{\lambda^{\alpha+\beta+\eta}(s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$H_{p+\rho+2D,q+\rho+D}^{m+D,n+\rho+D}\left[\frac{t\delta D^D}{\gamma^D}\left|\begin{matrix}\Delta(D,\frac{1}{2}-A-m),\Delta(s,1-\alpha),\Delta(k,1-\beta),\Delta(D,\frac{1}{2}-A+m)\\\Delta(k+s,1-\alpha-\beta),\Delta(D,k-A)\end{matrix}\right.\right], \quad (3.5)$$

Where

$D, \rho, \delta$ and $A$ are given in (2.1);

$$p+q < 2(m+n), |\arg t| < \left(m+n-\frac{1}{2}p-\frac{1}{2}q\right)\pi, \mathrm{Re}(\gamma) > 0, \mathrm{Re}(\alpha+sb_j) > 0,$$

$$\mathrm{Re}(\beta+kb_j) > 0, \mathrm{Re}\left(\alpha+\beta+\eta+Db_j+m+\frac{1}{2}\right) > 0, j = 1,2,...,m.$$

(f) Substituting $f=1, g=u=0, v=2, d_1=\frac{1}{2}v, d_2=-\frac{1}{2}v, r=1, \tau_i=1$

and using the result [3,p.216,(3)]

$$H_{0,2}^{1,0}\left[z\left|\begin{matrix}-\\\left(\frac{1}{2}\mu,1\right),\left(-\frac{1}{2}\mu,1\right)\end{matrix}\right.\right]=J_\mu(2\sqrt{z}),$$

(2.1) reduces to

$$\int_0^\infty\int_0^\infty x^{\alpha-1}y^{\beta-1}(x+y)^\sigma J_\mu(2\sqrt{\gamma(x+y)})H_{p,q}^{m,n}\left[tx^s y^k\left|\begin{matrix}A^*\\B^*\end{matrix}\right.\right]dxdy$$

$$=\sqrt{2\pi}\frac{D^{2A-1}s^{\alpha-\frac{1}{2}}k^{\beta-\frac{1}{2}}}{\gamma^{\alpha+\beta+\eta}(s+k)^{\alpha+\beta-\frac{1}{2}}}$$

$$H_{p+\rho+2D,q+p}^{m,n+\rho+D}\left[t\delta\left(\frac{D}{\gamma}\right)^D\left|\begin{matrix}\Delta\left(D,1-A-\frac{1}{2}\mu\right),\Delta(s,1-\alpha),\Delta(k,1-\beta),A^*,\Delta\left(D,1-A+\frac{1}{2}\mu\right)\\B^*,\Delta(k+s,1-\alpha-\beta)\end{matrix}\right.\right]$$ (3.6)

Where $\delta, D, \rho$ and $A$ have the same values given in (2.1);

$$p+q<2(m+n),|\arg t|<\left(m+n-\frac{1}{2}p-\frac{1}{2}q\right)\pi, \operatorname{Re}(\gamma)>0, \operatorname{Re}(\alpha+sb_j)>0,$$

$$\operatorname{Re}(\beta+kb_j)>0, \operatorname{Re}\left(\alpha+\beta+\eta+\frac{1}{2}\mu+Db_j\right)>0, j=1,2,...,m;$$

$$\operatorname{Re}\left(\alpha+\beta+\eta-D+Da_i\right)<\frac{1}{4}, i=1,2,...,n.$$

In view of the numerous properties of $\aleph$-function, on specializing the parameters suitably, a large number of interesting results may be obtained as particular case.

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