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A New Approach of the Estimation of the Parameters of the Linear Muskingum Model for River Flow Simulation: Application to Bafing, Senegal River Flow Tributary between Dakka Saidou and Bafing Makana Gauges Stations

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ABSTRACT
Flood routing is a non-structural method in flood warning. Muskingum Model is a lumped flood routing model based on the continuity equation and a linear discharge storage relation. Its parameters $K$and $x$of are graphically estimated through subjective trial and error procedures. A new approach of parameters $K$and x estimation based on observations is proposed. Least square method is used to estimate parameters for every year of the calibration period. Parameters not satisfying the criteria for stability or corresponding to negative K value are eliminated. Remaining parameters are applied separately to the whole hydrographs of the calibration period. Those with the lowest value of the RMSE are selected. An application is made on the Bafing, Senegal River tributary upstream Manantali Dam. River Reach limited by Dakka Saidou and Bafing-Makana stream gauges is considered. The flood hydrographs observed on the 1961-2013 period are used for calibration. The RMSE over this period for the year 1984 is minimal and the Best Linear Muskingum parameters for 1984 are representative for this reach. These parameters can be used for reconstruction of missing data and can be integrated in a decision support system of the Manantali dam.

Keywords: Muskingum Model, Best Parameters Calibration, Senegal River, Bafing River, Bafing-Makana, Dakka Saidou and Manantali Dam.

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I. INTRODUCTION

Flood wave generally come from dam break, rainfall from upstream catchment, releases from reservoirs or dam spills. Flood wave propagates through rivers channels and reservoirs and changes of magnitude shape and velocity. Studies on flood waves are of two kinds: forecasting or routing. River stages or discharges forecasting allows to warn in advance the approaching flood. Forecasting at a given time uses observations of a variable available up to that time to calculate the future value of that variable. A forecasting model can be very complex when computation facilities information about the catchment and enough data are available. Flood routing estimates shape and magnitude of a flood as it propagates down through river and channels or through reservoirs. Flood routing is used in design protection to test the proposed measures against the flood wave in reservoir operation for flood control [1]. Flood routing is either "hydrologic routing'' or 'hydraulic routing''. Hydraulic routing models solve one dimensional continuity and momentum Saint-Venant equation governing river flow in channel using appropriate discretization scheme and numerical solution procedure with initial and boundary conditions [2]. They are very expensive and need lots of data related to channel geometry and specifications and measurements of flows what prevents their use. Hydrologic routing models are lumped. They don't involve momentum equation. They only use storage – continuity equation. to capture the dynamics of flood through calibration of parameters [3]; [4]. Continuity equation equates the rate of storage in a reach to the difference between inflow and outflow of the river reach. Storage can be either linear or nonlinear. Muskingum model belongs to that class of hydrologic routing model. This model is a very popular procedure among hydrologists [6]. Muskingum method was first developed by McCarthy in the Muskingum River in Ohio for flood control. In Linear Muskingum Model storage is linear. There is an intensive research in Linear Muskingum model parameters $K$and $x$calibration. McCarthy proposed a fitting curve method using trial and error graphical procedure. Many optimization techniques have been proposed to solve the problem of minimization of objective function such as sum of square of deviation between observed and calculated outflows. Among them the least square method is very popular [6]; [7]. Linear Muskingum fitting curve shows a looping aspect what is a consequence of nonlinear behavior of parameters and results on their highly variation. Lot of author have preferred nonlinear methods to solve for the parameters: improved backtracking search algorithm [8] combination of the Modified Honey Bee Mating Optimization and the Generation Reduced Gradient Algorithms [9], state space representation [10]; ([11], Particle Swarm Optimization Algorithm (Moghaddam et al. 2016); hybrid algorithm and the Nelder – Mead simplex algorithm [12]. Though these methods have shown efficiency in nonlinear estimation of Muskingum parameters according to their authors, their implementation are very complicated. In this paper we present a new approach of Muskingum $K$and x parameters based on observations of inflow and outflow discharges for nonlinear parameters. This new method has revealed itself for very efficient when applied to Senegal River basin Dakka Saidou – Bafing Makana reach.

II. MATERIAL AND METHODS

1.1. Linear Muskingum Method

Muskingum Method has first been proposed by McCarthy for flood control of the River Muskingum in Ohio. It involves the Continuity "Equation 1" and a Linear Storage "Equation 2":

$$\frac{dS_t}{dt} = I_t - O_t \qquad (1)$$

$$S_t = K[xI_t + (1-x)O_t] \qquad (2)$$

S_t is the channel absolute storage at time t $I_t$and $O_t$are the rate of inflow and outflow at time $t$$K$is the storage time constant of the reach; value of K is reasonably close to the flow travel within the river reach x is a weighting factor varying between 0 and 0.5 ; " Equation 1" is the time rate of change in the channel storage at time t

1.1.1. Linear Muskingum Parameter$K$ and x Estimation

1.1.2. Muskingum-Cunge Method

Cunge has shown that Muskingum is second order approximation of convection diffusion equation and has related x to physical features of the channel [13]

$$x = \frac{i-1}{N} \qquad (3)$$

In many papers K is considered as travel time of the flood and estimated according to the following equation:

$$K = \frac{L}{V_m} \qquad (4)$$

Where$L$ is the length of the reach and V_m the mean of the velocity of the flood between along the reach.

1.1.3. Graphical procedure

McCarthy proposed a trial – and – errors graphical procedure for calibrating the two parameters K and x using inflows and outflows. Once weighting parameter coefficient x is assumed. values of $xI_t + (1-x)O_t$ are computed using recorded data and plotted against accumulated storage S_t. The value of x that minimizes the width of the plotted loop can be chosen as the correct value of x and the line slope of the correct value of x is taken as K.

1.1.4. Least square Method

The graphical procedure has been shown to be subjective and inefficient. Alternative methods which have no physical base have been developed; these methods utilize curve fitting techniques. One of these methods is least square method. Integrating equation leads to:

$$\int_t^{t+\Delta t} dS = \int_t^{t+\Delta t} (I_t - O_t)dt \qquad (5)$$

Using the trapeze's method for integration:

$$S_{t+\Delta t} - S_t = \frac{\Delta t}{2}[(I_t + I_{t+\Delta t}) - (O_t + O_{t+\Delta t})] \qquad (6)$$

Let's set:

$$Y_t = S_{t+\Delta t} - S_t \qquad (7)$$

from equation we can write:

$$Y_t = K(x)X_t \qquad (8)$$

Where

$$X_t = \Delta t\{[xI_{t+\Delta t} + (1-x)O_{t+\Delta t}] - [xI_t + (1-x)O_t]\} \qquad (9)$$

In least square method objective function to minimize for a given value of x is the Sum of Squares of Errors (SSE):

$$f(x) = \sum_{t=1}^{N}(Y_t - KX_t)^2 \qquad (10)$$

Setting to zero the derivative of f(x) versus K gives:

$$\sum_{t=1}^{N} Y_t X_t - K(x)\sum_{t=1}^{N}(X_t)^2 = 0 \qquad (11)$$

We obtain the following estimation for K:

$$\mathrm{K(x)} = \frac{\sum_{t=1}^{N} Y_t X_t}{\sum_{t=1}^{N}(X_t)^2} \qquad (12)$$

Once K and x have been estimated the following relations should be verified:

$$\Delta t \geq 2K \qquad (13)$$

$$\Delta t < 2K \qquad (14)$$

1.2. Linear Muskingum flood routing equation

Equalization of "Equations 6 and 8" give equation:

$$\frac{\Delta t}{2}[(I_t + I_{t+\Delta t}) - (O_t + O_{t+\Delta t})] = K\Delta t\{[xI_{t+\Delta t} + (1-x)O_{t+\Delta t}] - [xI_t + (1-x)O_t]\} \qquad (15)$$

Rearranging the equation leads to the following linear Muskingum flood routing equation:

$$O_{t+\Delta t} = C_0 I_t + C_1 I_{t+\Delta t} + C_2 O_t \qquad (16)$$

Where

$$C_0 = \frac{\Delta t + 2K}{\Delta t + 2K(1-x)} \qquad (17)$$

$$C_1 = \frac{\Delta t - 2K}{\Delta t + 2K(1-x)} \qquad (18)$$

$$C_2 = \frac{-\Delta t + 2K(1-x)}{\Delta t + 2K(1-x)} \qquad (19)$$

Where

$$C_0 + C_1 + C_2 = 1 \qquad (20)$$

1.3. Linear Muskingum Parameter estimation Method

1.3.1. Generation of set of *K* and *x* parameters

We use the least square estimation method as presented in paragraph using the following algorithm for any year of the:

Step 1: For year 1 to N

Step 2: for x varying from 0 to 1 with a fixed step 0.01

Step 3: calculate $\widehat{K}(x)$ (equation)

Step 4: use x and $\widehat{K}(x)$ to calculate coefficients $\hat{C}_0(x), \hat{C}_1(x), \hat{C}_2(x)$ (equations)

Step 5: Next year

1.3.2. Best set of Kand x parameters

Step 4 use equation to calculate $\widehat{O}_{t+\Delta t}$ on the period of calibration

Step 5 calculate the Root Mean Square Error RMSE below:

$$RMSE = \sqrt{\sum_{t1}^{N}\frac{\left(\hat{O}_{t+\Delta t} - O_{t+\Delta t}\right)^2}{N-1}} \qquad (21)$$

Where $\widehat{O}_{t+\Delta t}$ is the calculated flow downstream of the river reach at time $t+\Delta t.O_{t+\Delta t}$ observed downstream if the river reach at time $t+\Delta t$

Step 6: best value ofx and $K(x)$ corresponds to the lower value of RMSE

Step 7: repeat steps Step 4andStep 5for every year of the calibration period

1.3.3. Best value of Kand x parameters for the river reach

Step 7 allows to have a series of parameters (K, x). Negative values of K and values of K that doesn't respect equations are eliminated. Remaining values (K, x) coefficients are applied to each year of the calibration period and the RMSE calculated.

Finally the best parameters for the river reach are those for which the RMSE criterion is the lowest.

III. APPLICATION TO SENEGAL RIVER REACH BETWEEN DAKKA-SAIDOU AND BAFING-MAKANA

The preceding procedure has been applied to Senegal River reach between Dakka Saidou and Bafing-Makana. Bafing-Makana station is upstream Manantali Dam and controls inflow at this dam.

Table 1 Geographical location and characteristics of annual flows

	Long	Lat	Mean	SD	C_v	C_a	C_f
BM	-10.28°	12.55°	244	71	0.29	1.1	1.05
DS	-10.61°	11.95°	215	53	0.24	0.8	0.8

Standard deviation: SD; Relative standard deviation: C_v; Asymmetry coefficient: C_a; Flattening coefficient: C_f; Bafing Makana : BM ; Dakka Saidou: DS.

According to Table 1 dispersion around the mean is low (column 6). For both stream gauges the distribution of the flows is spread to the right because Cs >0. For Bafing Makana stream gauge (column8. line 1), $C_a > 1$ that is the probability distribution is above that of Gauss. For Dakka Saidou stream gauge (column 8. line 2), $C_a < 1$ the probability distribution is below that of Gauss.

1.4. Area of study

The Senegal River 1790 km long has its source in Guinea at an altitude of 750 m. It crosses between four countries Mali, Mauritania the Republic of Guinea and Senegal to jump into the Atlantic Ocean in St. Louis (SENEGAL). The climate is very varied in the basin: Guinean in the South. Sudanese in the center and Sahelian in the North. It includes two seasons: a warm and rainy season that goes from July to October a dry and cold season from November to February and a dry and hot season from March to June [14]. Leaving the South going North reliefs are found at about 800 m altitude. The highest point of the basin is at 1446 m altitude and the average altitude of the southern part is 1000 m (Fig. 1). Its area is 337000 km^2.The Senegal River is the result of the joinof Bakoye River and Bafing River at Bafoulabé. The river has two large dams: the Manantali Dam in Mali and the Maka-Diama Dam on the Mauritania-Senegal border near the sea.The Manantali Dam makes a reservoir. The Maka-Diama Dam stops salt water going deep into the countryside. In 1972 Mali, Mauritania and Senegal made the Organization for the Development of the Senegal River. This organization manages the land around the river Guinea joined this organization in 2005.

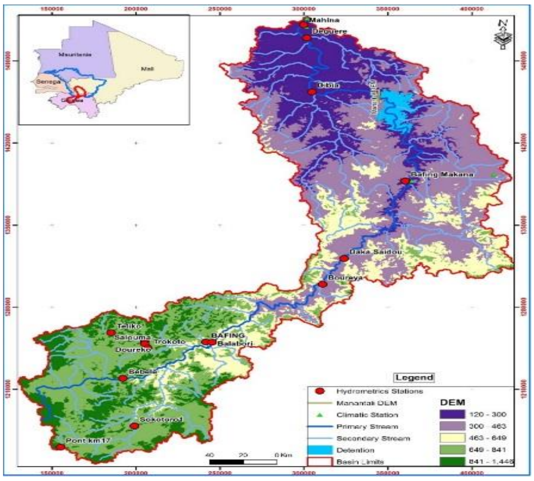

Figure. 1 Study area

1.5. Presentation of Data

Daily discharges during the period 1961-2013 are utilized for best parameters estimation of Senegal River. In Table1 we present the statics of the observations. In Fig.1 we indicate the geographical positions of these stream gauges.

1.6. K and x parameters set generation

Best Parameter Estimation of calibration are first calculated for every year of the period of calibration using linear least squares estimation the reach. These data come from the database of the Organization for the Development of the

(Table 2). Procedure described in paragraph previous has been applied Table 2 shows that parameters are highly variable probably due to the nonlinearity of the storage relation. Column 2 and 3 show that negative values of K correspond to inacceptable values for x (x=1).

Table 2 Linear Muskingum Parameter in the period of calibration

Year	**K (j)**	**X**	C_0	C_1	C_2	RMSE
1961	0.06	0.01	0.889	0.887	-0.776	238.2
1962	0.27	0	0.646	0.646	-0.293	122.1
1963	-0.11	1	0.774	1.226	-1.000	231.2
1964	0.16	0.21	0.850	0.742	-	223.9

					0.592	
1965	0.58	0.27	0.710	0.370	-0.081	124.5
1966	0.16	0.09	0.793	0.747	-0.54	202.3
1967	0.41	0	0.550	0.550	-0.101	238.2
1968	0.41	0.17	0.677	0.510	-0.187	52.6
1969	0.23	0.04	0.888	0.442	-0.331	178.3
1970	0.22	0.3	0.864	0.659	-0.523	184.4
1971	-0.14	1	0.713	1.287	-1.000	226.6
1972	0.34	0.12	0.678	0.576	-0.254	40.6
1973	0.22	0.3	0.866	0.666	-0.532	81.7
1974	-0.23	1	0.546	1.454	-1.000	278.0
1975	0.08	0.3	0.940	0.849	-0.789	187.7
1976	0.32	0	0.607	0.607	-0.213	45.5
1977	0.47	0	0.514	0.514	-0.029	24.1
1978	0.23	0	0.686	0.686	-0.372	44.8
1979	1.00	0.24	0.587	0.206	0.206	16.6
1980	0.16	0	0.763	0.763	-0.526	66.6
1981	0.17	0	0.749	0.749	-0.497	48.9
1982	0.17	0	0.749	0.749	-0.497	19.7
1983	0.20	0	0.715	0.715	-0.429	41.6
1984	1.00	0.13	0.460	0.270	0.270	14.7
1985	0.09	0	0.852	0.852	-0.704	43.5
1986	0.28	0.16	0.743	0.622	-0.365	84.6
1987	0.43	0	0.539	0.539	-0.078	25.1
1988	0.11	0	0.816	0.816	-0.632	91.4
1989	0.23	0.17	0.784	0.672	-0.456	44.0
1990	0.26	0	0.656	0.656	-0.313	38.2
1991	0.19	0.22	0.836	0.707	-0.543	81.9
1992	0.34	0.4	0.904	0.519	-0.423	75.8
1993	0.50	0.27	0.734	0.421	-0.155	33.4
1994	0.07	0.5	1.000	0.867	-	196.1

					0.867	
1995	0.42	0.3	0.790	0.474	-0.264	89.2
1996	0.23	0.42	0.942	0.640	-0.583	76.8
1997	0.40	0.26	0.758	0.495	-0.252	51.9
1998	0.22	0.39	0.924	0.654	-0.578	129.5
1999	0.36	0.3	0.808	0.520	-0.328	79.0
2000	0.03	0	0.939	0.939	-0.878	294.1
2001	0.12	1	1.244	0.756	-1.000	219.4
2002	0.06	0.19	0.929	0.886	-0.816	109.6
2003	0.64	0.32	0.754	0.317	-0.071	118.8
2004	0.36	0.56	1.066	0.451	-0.517	68.7
2005	0.27	0.2	0.772	0.620	-0.393	83.3
2006	0.33	0.17	0.717	0.571	-0.287	45.7
2007	0.21	0.35	0.900	0.666	-0.566	143.1
2008	0.53	0.33	0.790	0.383	-0.174	78.8
2009	-0.07	1	0.857	1.146	-1.003	276.3
2010	1.01	0.46	0.923	0.034	0.043	206.7
2011	0.19	0.02	0.729	0.718	-0.448	59.6
2012	0.04	1	1.071	0.929	-1.000	154.9
2013	0.19	0.52	1.013	0.683	-0.696	148.0

In Table 3 we present K and x coefficients for which where criterion of stability is verified. These correspond to years 1979, 1984, 2010. For these years the values of coefficients $\mathbf{C_1}$ and $\mathbf{C_2}$ are quite the sameand their sum is about equal to $\mathbf{C_0}$. But this is not true for year 2010 whose RMSE is more important.

Table 3 Values of K and x with stability criterion verified

Year	K (j)	X	C_0	C_1	C_2	RMSE
1979	1	0.24	0.587	0.206	0.206	16.6
1984	1	0.13	0.459	0.270	0.270	14.7
2010	1.01	0.46	0.922	0.033	0.043	206.7

1.7. Best Linear Muskingum Models Parameters selection.

We have divided the period of observation into three samples: 1961-1978 for calibration. 1979-1996 for verification and 1997-2013 for validation. The parameters of Table 3 have been applied to each of the three periods and the RMSE calculated. The parameters of the year 1984 are the best suited for these periods according to the RMSE (Table 4). Then the same parameters are applied to the whole of the table 3 are applied to the whole period of observations. Once again parameters of year 1984 correspond to the lowest value of RMSE (Table 4). These coefficients can be considered as best Linear Muskingum Model Parameters for the

Senegal River Reach upstream Manantali Dam between Dakka Saidou and Bafing Makana.

Table 4 : Comparaison chart RMSE

Period	Parameters of year	RMSE
1961-1978	1979	92.5
	1984	86.1
	2010	110.2
1978-1996	1979	44.4
	1984	42.1
	2010	51.1
1997-2013	1979	89
	1984	82.9
	2010	107
1961-2013	1979	78.3
	1984	73.1
	2010	93.5

As illustration the plot of Cumulative Distribution Function of RMSE is presented in fig. 2. It shows that RMSE corresponds to coefficients of year 1984.

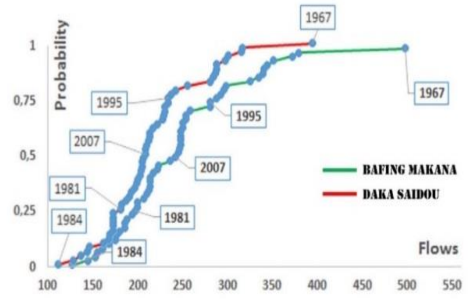

Figure. 2 Cumulative Distribution Function

The Cumulative Distribution Function of the annual discharges is plotted (Fig. 3). Analysis of fig shows that annual discharge is the lowest of the period during year 1984 and the highest during year 1967. For year 1984, river flow is located in the minor bed of the river and there is not outflow. This method seems to be efficient for river with no floodplain. year 1984 corresponding flow in minor bed.

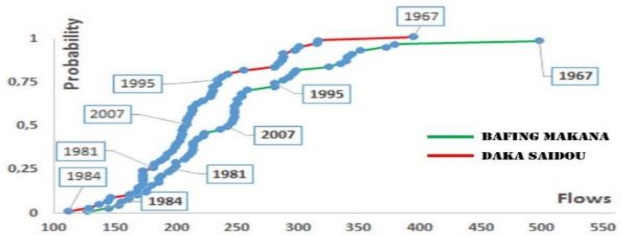

Figure.3 Cumulative Distribution Function of annual flows

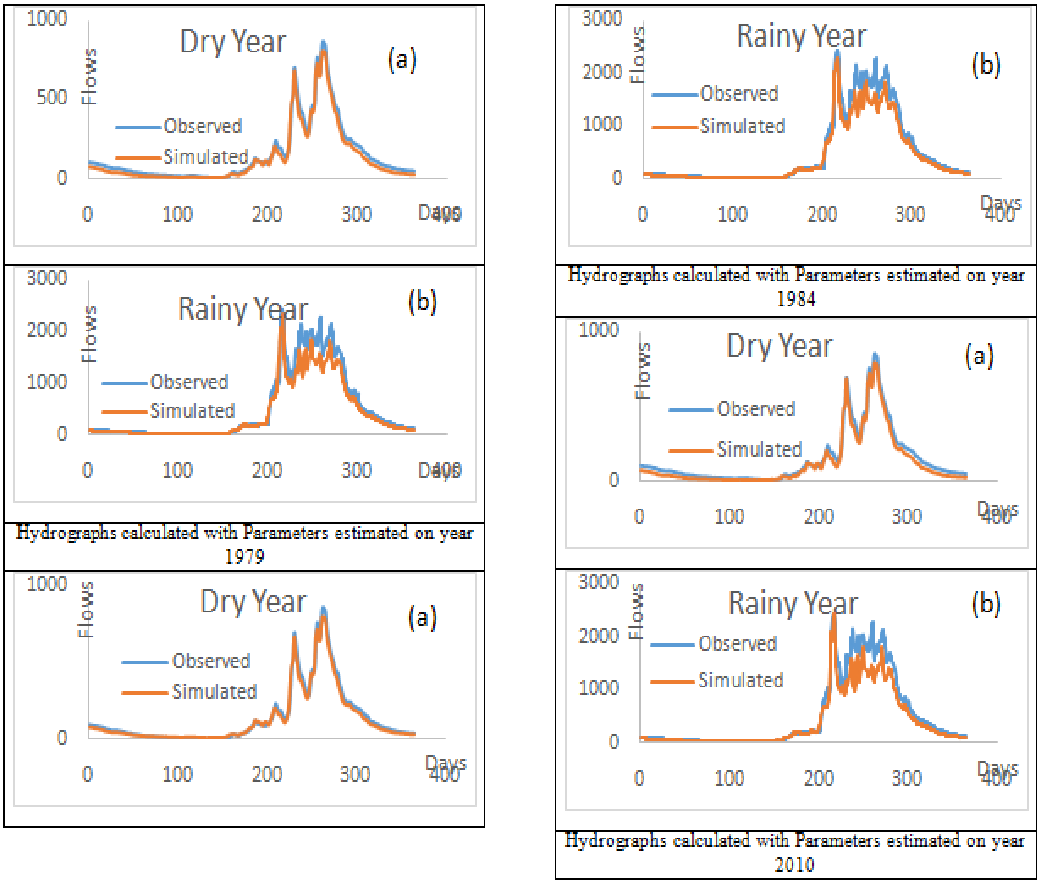

Figure. 4 Observed and calculated hydrographs for dry year 1984 (a) and rainy year 1967 (b)

IV. CONCLUSION

The main objective of this paper was to find the optimal parameters of the Muskingum method for the Bafing Makana-Dakka Saidou section of the Bafing River, a tributary of Senegal River upstream the Manantali multipurpose Dam. Daily flows observed at both Bafing Makana and Dakka Saidou stream gauges over the period from 1961 - 2013 were used. The parameters K and x of the Muskingum method were generated for each of these years using least square method. We found that the stability criteria of the Muskingum Method were verified only for three of these years: 1979, 1984 and 2010. The coefficients $\mathbf{C_0}$,$\mathbf{C_1}$and $\mathbf{C_0}$ calculated from each of the pairs of coefficients (K, x) of these three years were used to calculate daily flows over the entire period. We have noted that the Root Mean Square is minimal for the coefficients calculated with the daily flows of year 1984. These coefficients can thus be considered as the optimal coefficients for the Dakka Saidou-Bafing Makana section. They can be applied for Manantali Dam reservoir operation (i.e flood routing) or missing data filling.

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