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Hinge Domination Number of a Graph

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ABSTRACT: We introduce a new parameter hinged dominating set defined as, a set S of vertices in a graph G is a hinged dominating set if for every vertex u in V-S is adjacent to some vertex v in S and a vertex w in V-S such that (v,w) is not an edge in E. Hence for every vertex in V-S The hinged domination number $\gamma_h(G)$ is the minimum size of a hinged dominating set. In this paper, we obtain the exact values of $\gamma_h(G)$ for some standard graphs like path, cycle, star, wheel etc and establish lower bound on hinge domination number for as tree.

Keywords: Domination number, standard graphs
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I. INTRODUCTION

The study of domination in graphs has found rapid growth in the recent years. It is a highly flourishing area of research in graph theory. So far, hundreds of research articles have appeared on this topic of research in view of its growing real life.

All graphs considered here are finite, nontrivial, undirected with no loops, multiple edges and isolated vertices. The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it. Many variants of dominating sets are available in the existing literature. A set D of vertices in a graph G(V, E) is a dominating set of G if every vertex in V–D is adjacent to some vertex in D. In other words D is a dominating set if the closed neighborhood N[D] = V. A dominating set is called a minimal dominating set (MDS) if no proper subset D' of D is a dominating set. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set.

A company which has different departments like manufacturing, dispatch, accounts etc has outsourced customer care. So the customer care personals need to have contact with company departments as well as customers, and depending on the requirement of customer contact respective department and complete the communication. Based on this concept of two side communication requirement, we have introduced a new domination parameter called hinge domination number. A dominating set S of a graph G(V, E) is a hinge dominating set, if for every vertex u ϵV-S is adjacent to some vertex v ϵ S and a vertex w ϵ V-S such that (v, w) is not an edge in E. The hinged domination number $\gamma_h(G)$ of G is the minimum cardinality of a hinged dominating set.

By hinge domination definition it is clear that there is a V shape edge formation between three vertices v, u, w. So if vertex u is hinge dominated it is essential that $d(u) \geq 2$. Hence all vertices of degree one, pendants, in a graph should be included into hinge dominating set.

II. MAIN RESULTS

Proposition 2.1 : Hinge domination number of a path graph P_n is

$$\gamma_h(P_n) = 2 \quad \text{if } n = 2$$
$$= k + 2 \quad \text{if } n = 3k$$
$$= \left\lceil \frac{n-1}{3} \right\rceil + 1 \quad \text{if } n \neq 3k$$

Proof : A path graph P_n has n vertices and n-1 edges with two end vertices of degree one and all internal vertices of degree 2. For n=2, 3 to satisfy the hinge domination condition we need to include all n vertices to hinge dominating set D. As we need to include both the end vertices in to D, for n=4, as shown in figure 1, hinge domination number is 2. For n > 4, for minimal hinge domination, include the middle vertex of each set of three consecutive vertices from v_3 to v_{n-2}. Suppose we denote vertices of the path graph as $V=\{v_1, u_1, w_1, v_2, u_2, w_2, v_3, u_3, w_3 \ldots\ldots\}$, then $D=\{v_1, v_2, v_3, \ldots\ldots\}$ forms a minimal hinge dominating set of P_n and so as to satisfy hinge domination condition we need to add some extra vertices to D as follows.

i) If n=3k+1 then the end vertex of path P_n need to be added to D.
ii) If n=3k+2 then last two vertices of path P_n need to be added to D.
iii) If n=3k then last set of three vertices of P_n should be added to D

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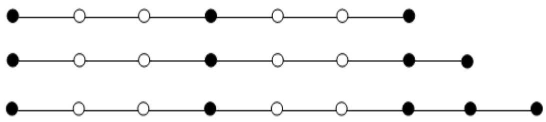

Figure 1 : Hinge dominating set of P_7, P_8, P_9

Thus

$$\gamma_h(P_n) = 2 \quad \text{if } n = 2$$
$$= k + 2 \quad \text{if } n = 3k$$
$$= \left\lceil \frac{n-1}{3} \right\rceil + 1 \quad \text{if } n \neq 3k$$

Proposition 2.2 : Hinge domination number of a cycle graph C_n is

$$\gamma_h(C_n) = k \quad \text{if } n = 3k$$
$$= k + 1 \quad \text{if } n = 3k + 1$$
$$= k + 2 \quad \text{if } n = 3k + 2$$

Proof : A cycle graph C_n has n vertices and n edges with every vertex of degree 2. For n= 3 to satisfy the hinge domination condition we need to include any one vertex to hinge dominating set D. Suppose we denote vertices of the cycle graph as V={v_1, u_1, w_1, v_2, u_2, w_2, v_3, u_3, w_3……..}, then D={v_1, v_2, v_3, ……. v_k} forms a minimal hinge dominating set of C_n and so as to satisfy hinge domination condition we need to add some extra vertices to D as follows.

i) If n=3k+1 then the last vertex of cycle C_n need to be added to D.

ii) If n=3k+2 then last two vertices of cycle C_n need to be added to D.

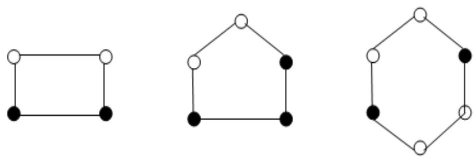

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Figure 2 : Hinge dominating set of C_4, C_5, C_6

Proposition 2.3 : Hinge domination number of a star graph S_{n+1} is n+1.

Proof : Star graph S_{n+1} has n pendant vertices and a center vertex connected to all these pendants. As all the pendants need to be included to hinge dominating set, the center vertex being sole vertex in V-D need to be included to D as hinge domination condition will not be valid for it otherwise. Therefore $\gamma_h(S_{n+1}) = n + 1$.

Proposition 2.4 : Hinge domination number of a wheel graph W_{n+1} is $\gamma_h(W_{n+1}) = \gamma_h(C_n)$.

Proof : Wheel graph W_{n+1} is a join of cycle graph C_n and a star graph S_{n+1}. It has n vertices of degree 3 and a center vertex of degree n connected to all vertices of the cycle graph. As the center vertex is connected to all the vertices of cycle graph it cannot be included to hinge dominating set. Now for the remaining n vertices using proposition 2.2 it is easy to prove that $\gamma_h(W_{n+1}) = \gamma_h(C_n)$.

Proposition 2.5 : Hinge domination number of a complete graph K_n is $\gamma_h(K_n) = n$.

Proof : Complete graph K_n has n vertices, where every vertex is connected to all the remaining vertices. Thus the condition for hinge domination can not be satisfied by any vertex in V-D, so all vertices should be included to D, giving $\gamma_h(K_n) = n$.

Proposition 2.6 : Hinge domination number of a complete bipartite graph $K_{m,n}$ is

$$\gamma_h(K_{m,n}) = 2$$

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Proof : Complete bipartite graph $K_{m,n}$ has vertex set V = V_1 U V_2, with m and n vertices each such that every vertex of V_1 is connected to all the vertices of V_2. Thus the selection of one vertex each from V_1 and V_2 included in to D will satisfy the condition for hinge domination by any vertex in V-D, so $\gamma_h(K_{m,n}) = 2$.

Proposition 2.7 : Hinge domination number of a tree T with p pendants $\gamma_h(T) \geq p$

Proof : Let T be a tree on n vertices, n-1 edges and p pendants. As every pendant vertex need to be added to hinge dominating set it is clear that $\gamma_h(T) \geq p$. The equality holds for a bi-star graph.

III. CONCLUSION

Exact values of hinge domination number of standard graphs are found and lower bound for hinge domination number of tree is established in the paper.

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