RESEARCH ARTICLE OPEN ACCESS

Numerical Solution of Klein–Gordon and Sine-Gordon Equations Using the Numerical Method of Gridless Lines

Mehrzad Ghorbani,
Assistant Professor, Department Of Mathematics, Qom University Of Technology, Qom, Iran.
Saeid Saki,
Master Student, Department Of Mathematics, Qom University Of Technology, Qom, Iran.

ABSTRACT
Over The Past 30 Years, Analytical, Laboratory And Numerical Studies Have Been Carried Out On The Equations With A Soliton Response, And Several Nonlinear Equations With Similar Characteristics Were Presented. Using The Mathematical Model Of The One-Dimensional Nonlinear Klein-Gordon Equation And The Sin-Gordon Two-Dimensional Equations By Using Meshless Lines Using Radial Basis Functions, Data Analysis Has Been Done. The Results Of The Research, As Well As Discussion Of The Results, Are Presented And, After A General Conclusion, Express The Limitations That The Researchers Encountered During The Implementation Of This Research. Ultimately, Suggestions In The Form Of Scientific And Applied Offers Are Presented.

Keywords: Sine Gordon And Klein Gordon Equations, Non-Linear Systems, Meshless Methods, Radial Base Functions, Special Functions, Optics

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Date of Submission: 19-04-2018 Date of acceptance: 04-05-2018

I. INTRODUCTION

Numerical Solution Of Klein-Gordon And Sine-Gordon Equations Using Numerical Methods Of Nonlinear Grids In The real World, Many Applications, And Physical Phenomena Are Studied By Mathematical Models.(Hussain et. Al. 2013) Many Of The Important Mathematical Models For Differential Concepts Are Differential Equations With Relative Derivatives .Movement Of Objects, Heat Transfer, Curvature And Corrosion Of Materials, Vibrations, Chemical And Physical Reactions Are All Modeled By Differential Equations, In Particular, Differential Equations With Relative Derivatives .Movement Of Objects, Convection, Curvature And Corrosion Of Materials, Vibrations, Chemical And Physical Reactions Are All Modeled By Differential Equations, Especially Differential Equations With Relative Derivatives. Always Obtaining Precise Answers For Differential Equations With Relative Derivatives Of The Physical Systems Has Been One Of The Major Concerns Of Scientists And Engineers.

In The Past Few Years, A Group Of Non-Networked Methods Based On Radial Basis Functions Have Attracted More Attention To The Numerical Solution Of Partial Differential Equations. At First, Radial Basis Functions Were Introduced To Interpolate Data In Multivariate Functions. However, The Feature Without The Network Was Motivated By The Researchers To Use Them To Solve The Numerical Partial Differential Equations (Qureshi Et Al., 2012).

The Differential Equation Represents The Relationship Between A Function Of One Or More Independent Variables And Derivatives Of This Function Relative To Them. Differential Equations With Partial Derivatives Have Important Applications In Various Fields Of Science (Bohmann, 2003).

In These Methods, The Domain Space Defined By The Partial Differential Equation Is Divided Into Networks. A Grid Is Defined In Terms Of Distance Or Open Space Between The Edges Formed By Connecting The Nodes. In The Finite Element Method, The Elements In The Finite Difference Method, The Networks Used Are Often Aligned Lines, And In The Finite-Volume Method Of Networks, Is Called Volume. These Denominations Contain Certain Physical Meanings Because They Are Used For Various Physical Problems. Although All These Names Can Be Called The Same Network (Alipanah, Et Al., 2011).

Differential Equations Are A Good Tool For Formulating Many Of The Phenomena In The Real World. Whenever There Is A Relationship Between Several Variables As Well As The Rates Of Their Variations In Different States And Times, Those Phenomena Are Expressed By Differential Equations. As The Simplest Example, The Motion Of An Object Can Be Described By Its Speed And Location At Different Times. Newton's Equation

Shows The Relationship Between Location, Velocity, Acceleration And Various Forces Imposed On The Object; In This Case, We Can Express The Motion Of A Body In The Domain Of Differential Equation, In Which The Place Is A Function Of Time (Hessamodini et al., 2011).

Since The Exact Solution Of Quantum Systems Plays An Important Role In Quantum Mechanics, Today, Many Of The Potentials In Non-Relativistic Quantum Mechanics Are Precisely Solved And Their Energy Spectrum And Their Wave Functions Are Accurately Obtained.In Recent Years, Considerable Efforts Have Been Made To Study Relativistic Equations And Study Their Relativistic Effect, Among Which The Solution Of The Klein-Gordon Equation Has Been Considered As One Of These Relativistic Equations, And This Equation Is Precisely Solved For Many Potentials.Also, The Klein-Gordon Equation With Effective Mass Dependent On Location Is Considered, Taking Into Account The Different Mass Distribution Function, Using Different Methods For Much Potential Such As Morse, Hulthen, Pöschl-Teller And Othershave Been Studied.One Of The Methods For Solving The Klein-Gordon Equation Is The Use Of Supersymmetry And Invariant Form In Non-Relativistic Quantum Mechanics (Hussain Et. Al. 2013, Panahi Et Al., 2011).

The Sine-Gordon Equation Was First Discovered In The Nineteenth Century In The Fixed-Curvature Constant Strands And Attracted Much Attention In 1970, Which Led To The Introduction Of Soliton Responses. The Soliton Is Introduced As A Solution To The Nonlinear Wave Equation.The Sine-Gordon Equation Has Many Applications, Including The Fundamental Particle Theory, Magnetic Flux Emission In Josephson Junctions, DNA Dynamics, And Displacement In Crystals (Behrouzi Far Et Al., 2016).

In This Numerical Research, We Examine The Klein-Gordon And Sine-Gordon Equations Using The Numerical Method Of Nonlinear Methods And Compare The Obtained Results.

II. METHODOLOGY

In Practice, Solving A First-Order Problem Or Boundary Value Involves Finding An Infinite Function That Applies To The Equation And The Lateral Conditions Of The Problem.

Approximation Of RBF

A Discrete In Particular Derivatives Reduce The Second-Order Differential-RBF Approximation By The Second-Order Differential Equation.To Reduce The First Ordinary Differential Equation, We Use U_t (X, T) = V (X, T) Instead.

We Have The Following Equations For The System:

$$U_T = VV_T = F(X,T) - AU_{Xx} - Bu - \Gamma U^2$$

By Choosing N Distinct Interpolation Points In The Domain [A,B], If RBF, $\psi(\|X-X_j\|)=\psi_j$ (X), , Then The U (X, T) And V (X, T) Functions Are Estimated By U (X, T) And V (X, T) Respectively.

$$u(x,t) = \sum_{j=1}^{n} y_j^1(t)\psi_j(x) = \psi^T(x)\Gamma^1(t)$$

$$V(X,T) = \sum_{j=1}^{N} \Gamma_j^2(T)\psi_j(X) = \psi^T(X)\Gamma^2(T)$$

In This Equation, $\psi(x) = [\psi_{j_{(x)}}]_{n\times 1} r^{(i)} = [Y_j^{(i)}(t)]_{n\times 1}$ Are Unknown.(Shen, 2009) For The Understanding Of These Unknowns, We Use The Displacement Method. To Move Point'sx_k We Will Have:

$$U(X_K,T) = \sum_{J=I}^{N} Y_I^1 \Psi_I(X_K)$$

$$V(X_K,T) = \sum_{J=I}^{N} \Gamma_J^2 \psi_J(X_K)$$

In This Case, The Matrix Form Is As Follows:

$Ar^{(1)} = U \qquad Ar^{(2)} = V$

$U = [U(X_J,T)]_{N\times 1} \qquad V = [V(X_J,T)]_{N\times 1}$

$$A = \begin{bmatrix} \psi^T(X_1) \\ \psi^T(X_1) \\ \cdot \\ \cdot \\ \cdot \\ \psi^T(X_N) \end{bmatrix} = \begin{bmatrix} \psi_1(X_1)\psi_2(X_1) & \dots & \psi_3(X_1) \\ \psi_1(X_1)\psi_2(X_1) & \dots & \psi_3(X_1) \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ \psi_1(X_1)\psi_2(X_1) & \dots & \psi_3(X_1) \end{bmatrix}$$

U(X,T) = $\psi^T(X)A^{-1}U = N(X)U$

V (X, T) = $\psi^T(x)A^{-1}v = N(X)v$

N(X)= $\psi^T \psi^T(X)A^{-1}$

We Use The Approximation Of Rbfs In The Equation. (Hardy 1971) The Second Derivative Of The Differential Equation And The ODES Device Are Obtained From Equations (1) And (11) (Lancaster Et. Al. 1981, Platte, Driscoll 2006).

$$U_{Tt} + Au_{Xx} + Bu + \Gamma u^2 = F(X,T)$$

$$\frac{D^2U}{Dt^2} = F(T) - AN_{Xx}U - Bu - \Gamma U^2$$

$$\frac{Du}{Dt} = V$$

III. DISCUSSION

Klein-Gordon's Equation Plays An Important Role In Many Applications Of Mathematical Physics Such As Solid State Physics, Nonlinear Optics, And Quantum Field Theory. In

This Study, We Consider A High-Precision Numerical Solution To A Non-Linear One-Dimensional Klein-Gordon Equation That Has The General Form:

$$\frac{\partial^2 u}{\partial t^2} + \alpha \frac{\partial^2 u}{\partial x^2} + g(u), a \le x \le b, 0 \le t \le T$$

U (A, T) = P_1(T), U(B, T) = P_2 (T), 0≤ T $\le T$

Operational Matrix Derivative And Integral

First, We Make The Necessary Preparations. A Number Of Chebyshev's I Degree Terms In The Definition Are Given By The Following Formula:

T_1 (X) = Cos (I Cos^{-1}(X)), I = 0,1,2,….

Where X Is In The Network Points gauss-Chebyshev-Lobatto (GCL)Will Be Evaluated, Clustered Through The Endpoints. Suppose That The Chebyshev Series Is Shortened From UX (X) And U (X) As Follows:

$U_x(X) \cong \sum_{I=0}^{N-3} a_i\, T_i(x), \quad u_x(x) \cong \cong \sum_{I=0}^{N-3} b_i\, T_i(x)$

Where The Coefficients Are Obtained Using The Fourier Series Transform (FFT), Using The O (N Log N) Operation. (Churchill Ruel, 1954)

$$\int^{x} T_0(y)dy = T_1(x)$$

$$\int^{x} T_1(y)dy = \frac{1}{4}T_2(x) + \frac{1}{4}T_0(x),$$

$$\check{U}_x = \int^{x} T_1(y)dy = \frac{T_{i+1} + (x)}{2(i+1)} - \frac{T_{i-1} - (x)}{2(i-1)}, i \ge 2$$

$$\check{U}_x = H_0\,\check{U}, \quad \check{U} = H_1\,\check{U},$$

Where H_0 And H_1 Are Integral Characteristic Matrices Of N-2 * N-1 Dimension And Are Presented In The Following Way.

$$H_0 = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & 0 & 1 & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix},$$

$$H_1 = \begin{pmatrix} 1 & 0 & 0 & & & & \\ & 1 & \ddots & & & & \\ & & \frac{1}{4} & \ddots & -\frac{1}{4} & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & \frac{1}{2(N-4)} & \ddots & -\frac{1}{2(N-4)} \\ & & & & & \frac{1}{2(N-3)} & 0 \end{pmatrix}.$$

$$\bar{U}_i^{(1)} = 2\sum_{p=i+1}^{\infty} p\bar{U}_p$$

$$\hat{U}_{xx} = D_1\hat{U}$$

$$D_1 = \begin{pmatrix} 0 & 0 & 2 & 0 & 6 & 0 & 10 & 0 & \dots \\ & 0 & 0 & 4 & 0 & 8 & 0 & 12 & \dots \\ & & \ddots & 0 & 6 & 0 & 10 & 0 & \dots \\ & & & \ddots & 0 & 8 & 0 & 12 & \dots \\ & & & & \ddots & 0 & 10 & 0 & \dots \\ & & & & & \ddots & 0 & 12 & \dots \\ & & & & & & \ddots & 0 & \dots \\ & & & & & & & \ddots & \end{pmatrix}$$

$$u_{xx}(x) \cong [T_0(x), \dots, T_{N-3}(x)]D_1\,\hat{U},$$
$$u_x(x) \cong [T_0(x), \dots, T_{N-3}(x)]H_0\,\hat{U},$$
$$u(x) \cong [T_0(x), \dots, T_{N-3}(x)]H_1\,\hat{U},$$

CTMMID Is The Complete Implementation Of The Methodology

$$u_{tt}^{n+1} + \alpha u_{xx}^{n+1} + \beta u^{n+1} = f(x,t) - \gamma(u^n)^k$$

$$u_{xt}(x,t) \cong \sum_{i=0}^{N-3}\sum_{j=0}^{M-3} aa_{i,j}\, T_i(x)T_j(t)$$

$= [\ T_0(x), \dots, T_{N-3}(x)]\ H_0\tilde{U}\ (H_0)^T \times [T_0(t), \dots, T_{M-3}(t)\ ]^T$

And It's In The Form Below, M-1 In N-1 In The Matrix $\tilde{U}$ That

$$\begin{bmatrix} bbo,o & ba_{0,0} & ba_{0,1} & \dots & \dots & ba_{0,M-3} \\ ab0,0 & aa_{0,0} & aa_{0,0} & \cdots & \dots & aa_{0,M-3} \\ ab1,0 & aa_{1,0} & aa_{1,1} & \cdots & \dots & aa_{1,M-3} \\ ab2,0 & aa_{2,0} & aa_{2,1} & \ddots & \ddots & aa_{2,M-3} \\ ab_{N-3,0} & aa_{N-3,0} & aa_{N-3,1} & \dots & \dots & a_{N-3,m-3} \end{bmatrix}$$

.

$$u\ \ (x,t) \cong \sum_{i=0}^{N-3}\sum_{j=0}^{M-3} bb_{i,j}\, T_i(x)T_j(t)$$

=

$T_0(x), \dots, T_{N-3}(x)]\ H_1\tilde{U}\ (H_1)^T \times$
$T_0(t), \dots, T_{M-3}(t)\ ]^T$

$$u_t(x,t) \cong \sum_{i=0}^{N-3}\sum_{j=0}^{M-3} ba_{i,j}\, T_i(x)T_j(t)$$

$$[T_0(x), \dots, T_{N-3}(x)]\ H_1\tilde{U}\ (H_0)^T \times [T_0(t), \dots, T_{M-3}(t)\ ]^T$$

$$U_{xx}(x,t), \dots, [T_0(x)\ T_{N-3}\ (x)]D_1\tilde{U}\ (H_1)^T \times\ [T_0(T), \dots, T_{N-3}(T)]^T$$

$$U_{tt}(x,t), \dots, [T_0(x)\ T_{N-3}\ (x)]H_1\tilde{U}\ (D_1)^T \times\ [T_0(t), \dots, T_{M-3}(t)]^T$$

Which Specifies:

F^n (X,T) = F(X,T) - $\gamma(u^n)^k$

F^N (X,T) ≅ $\sum_{i=0}^{N-3}\sum_{j=0}^{M-3} F_{i,j}^n\, T_i(x)T_j(t)$

We First Describe The Same As The First One:

$$p^{b_0}(t) = 1/2(\frac{\partial p_2(t)}{\partial t} + \frac{\partial p_1(t)}{\partial t}),$$

$$p^{a_0}(t) = 1/2(\frac{\partial p_2(t)}{\partial t} - \frac{\partial p_1(t)}{\partial t}),$$

And The Matrix $\tilde{U}_{12}, \tilde{U}_{22}$ Equations Are Obtained By The Following Form:

$\hat{U}_{12} = \begin{bmatrix} Q_2 \\ Q_1 \end{bmatrix} \hat{U}_{22} + [\, \tilde{P}\, b_0 \,, \tilde{P} a_0 \,]^T$

Using The Equations Given Below For The Functions (T)$p^{a0} and\ (t) p^{b0}$

$$\tilde{P}\, b_0 = [p_i^{b_0}]_{i=1}^{M-3}$$
$$\tilde{P}\, a_0 = [p_i^{a_0}]_{i=1}^{M-3}$$

Thus:

$$Vec(\hat{U}_{12}) = W_{12} V_{ec}\,(\hat{U}_{22}) + g_{12}$$

Having The Following Values And Characteristic Matrix In The Form Below

$W_{12} = I_{M-1} \times \begin{bmatrix} Q_2 \\ Q_1 \end{bmatrix}$

$G_{12} = Vec\ ([\tilde{P}\, b_0 \,, \tilde{P}\, a_0])^T$

The Next Variable Of The Proposed Equation Is:

$$\theta^{b0}\ \ (x) = \varphi_2(x) + \varphi_1(x), \theta^{a0}(x)$$

Then We Have:

$$Vec\ \ (\hat{U}_{12}) = W_{21} Vec\ \ (\hat{U}_{22}) + g_{21}$$

$g_{21} = V_{ec}\,([\tilde{\theta}_x^{b0}, \tilde{\ } \theta_x^{a0}])$

$\tilde{\theta}_x^{b0} = [\tilde{\theta}_x^{b0},\ \ I]_{i=1}^{N-3}$

$$\tilde{\ }\theta_x^{a0} = [\tilde{\theta}_x^{a0}, \qquad I]_{i=1}^{N-3}$$

In This Case, We Will Have:

$$Vec\ \ (\hat{U}_{11}) = W_{11} V\ \ ec\,(\hat{U}_{12}) + g_{11}$$

Where In:

$$W_{11} = \begin{bmatrix} Q_4 \\ Q_3 \end{bmatrix} \times I_2$$

$$\overline{g_{11}} = V\ \ ec \begin{bmatrix} \emptyset_0^{b0} & \emptyset_0^{a0} \\ \emptyset_{x,0}^{b0} & \emptyset_{x,0}^{a0} \end{bmatrix}$$

Description Of CTMMID-DDM

The Second Method For Spacing Large Space Is As Follows.

Two Sub-Domains:

A Simple Domain Parsing For A Rectangular Domain Is Shown In Figure 1 (Block 1),Which Includes Two Non-Overlapping subdomain s and An interface Γ^{ib} . We Will Convert Each Subset To $[-1,\ 1]^T$, And Introduce It As $\Omega_m, m = 1,2$. The Klein Gordon Universal Equation (3.1) Is Then Converted Into Two Subsets:

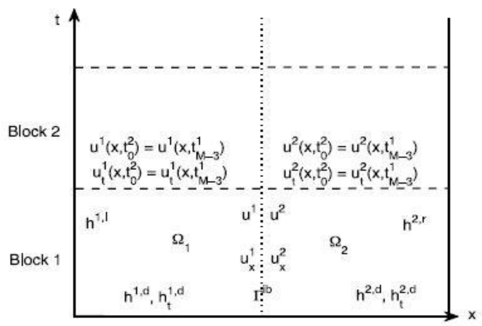

Figure 1 Two Sub-Domains And A Time Block For A One-Dimensional Problem

According To The Initial Conditions:

$$u^m(x,-1) = h^{m,d}(x), u_t^m(x,-1) = h^{m,d}(x)$$

By Establishing The Following Equations:

$$u^m(x,-1) = h^{m,d}(x), u_t^m(x,-1) = h^{m,d}(x)$$
$$h^{m,d} = \theta_1^m(x), h_t^{m,d}(x) = \theta_2^m(x)$$

The Permitted Boundary Conditions Will Be As Follows.

$$u^1(-1,t) = p_1(t), \qquad u^2(1,t) = p_2(t),$$
$$ux^1(-1,t) = s_1(t), \qquad u_x^2(1,t) = s_2(t),$$

And Boundary Conditions:

$$u^1 | r^{ib} = u^2 | r^{ib}$$

Similar To Equation (3.7), Each Optional Subset In The Matrix Is As Follows:

$$D_1 \times H_1 + \alpha^m H_1 \times \beta^m H_1)\, Vec\,(\tilde{U}^{m,n+1})$$
$$= Vec\,(\tilde{F}^{M,N}\,)$$

Border Reduction Technique

To Simplify The Markup, We Delete Them Mark In The U^m Mark And Understand That The Following Statements Are In Each Subset Ω_m

IV. CONCLUSION

In Connection With The analysis Of The Stability And Convergence Of The Proposed Marching Method In equations, We Study The Following Theorems.

Theorem: It Is Proposed For Marching Method In Constant Equations, For Example, A, In Such A Way That

Max $= \{|U_0|\,, |Q_0|\,, |W_0|\} \ll \text{Max}\{|u_M|\,, |Q_M|\,, |W_M|\}$

Proof: Suppose $M_s = max_{t\in[0,1]}\{s(t)\}$ And $M = \text{Max}|\Delta| - \infty = \text{Min}(\delta_i, \acute{\delta}_i)$

$K_s = max_{t\in[0,1]}\{s(t)\}$ From Theorem 3 Of [7] We Will Have:

$$|D_t(Q_{i,n})| \leq \frac{C}{|\delta_{-\infty}|} |Q_{i,n}|$$

Where C Is A Positive Constant, We Will Have:

$$|W_{i-1,n}| \leq \left(1 + h \frac{C}{|\delta_{-\infty}|}\right) \text{Max}\{|Q_{i,n}|, |W_{i,n}|\}$$

Also, We Will Have:

$$|U_{i-1,n}| \leq (1+h)\,\text{Max}\{|U_{i,n}|, |Q_{i,n}|\}$$

$|Q_{i-1,n}| \leq |Q_{i,n}| + h(M_s^2 |W_{i,n}| + \quad M_s K_s |Q_{i,n}| + |M_f| + M_f$

$$\leq (1 + (1 + M_s^2 + M_s K_s)\,\text{Max}\{|Q_{i,n}|, |W_{i,n}|, M_f\}$$

$$C_\delta = \text{Max}\left\{1{,}1 + M_s^2 + M_S K_S, \frac{C}{|\delta|_{-\infty}}\right\}$$

In This Case, We Will Come To The Conclusions:

$\text{Max}\{|u_{i-1}|, |Q_{i-1}|, |W_{i-1}|\}$

$$\leq (1 + hC_\delta)\,\text{Max}\{|u_i|, |Q_i|, |W_i|\}$$

With M Times Repetition Of The Last Equation, We Will Have

$$\mathrm{Max}\{|u_0|,|Q_0|,|W_0|\} \le (1 + hC_\delta)^M \mathrm{Max}\{|u_M|,|Q_M|,|W_M|\}$$

This Concludes:

$$\mathrm{Max}\{|u_0|,|Q_0|,|W_0|\} \le \mathrm{Exp}(C_\delta)\mathrm{Max}\{|u_M|,|Q_M|,|W_M|\}$$

By Puttinga= Exp(C_δ), The Proof Ends.

Theorm2: It Is Proposed For Marching Method, For The δ Constant, When H And εtends To Zero, Themollified Response Responds To The Points Of Interest.

Proof: From The Definitions Of The Discrete Error Function, We Will Have:

$$\nabla u_{i,n} = u_{i,n} - v(ih,nk), \nabla Q_{i,n} = Q_{i,n} - v_x(ih,nk), \nabla w_{i,n} = w_{i,n} - v_t(ih,nk)$$

We Will Use The Taylor Series:

$$v\big((\mathrm{I}-1)\mathrm{H},\mathrm{Nk}\big) = v(ih,nk) - hv_x(ih,nk) + o(h^2)$$

$$v_x\big((\mathrm{I}-1)\mathrm{H},\mathrm{Nk}\big) = v_x(ih,nk) - h[\big(s(nk)\big)^2 v_t(ih,nk) - s(nk)(ih)s_t(nk) v_x(ih.nk) + o(h^2)$$

$$v_x\big((\mathrm{I}-1)\mathrm{H},\mathrm{Nk}\big) = v_t(\mathrm{Ih},\mathrm{Nk}) - \mathrm{H}\left(\frac{d}{dt}\right) v_x(\mathrm{Ih},\mathrm{Nk}) + \mathrm{O}(h^2)$$

On The Other Hand, We Will Have

$$\nabla u_{i-1,n} = \nabla u_{i,n} + \big(u_{i-1,n} - u_{i,n}\big) - \Big(v\big((\mathrm{I}-1)\mathrm{H},\mathrm{Nk}\big) - v(\mathrm{Ih},\mathrm{Nk})\Big) = \nabla u_{i,n} - h\nabla Q_{i,n} + O(h^2)$$

$$\nabla Q_{i-1,n} = \nabla Q_{i,n} + \big(Q_{i-1,n} - Q_{i,n}\big) - \Big(v_x\big((\mathrm{I}-1)\mathrm{H},\mathrm{Nk}\big) - v_x(\mathrm{Ih},\mathrm{Nk})\Big)$$

$$= \nabla Q_{i,n} + \mathrm{H}\Big[\big(\mathrm{S}(\mathrm{Nk})\big)^2 \nabla W_{i,n} - (s(nk))(ih)s_t(nk)\nabla Q_{i,n}\Big] + \mathrm{O}(h^2)$$

$$\nabla w_{i-1,n} = \nabla w_{i,n} + \big(w_{i-1,n} - w_{i-n}\big) - \Big(v_t\big((i-1)h,nk\big) - v_t(ih,nk)\Big)$$

$$= \nabla W_{i,n} - h\Big(D_t\big(J_{\nabla i}Q_{i,n}\big) - v_t(ih,nk)\Big) + o(h^2)$$

$$|\nabla u_{i+1,n}| \le \big||\nabla u_{i,n} + h|\nabla Q_{i,n}| + o(h^2)$$

$$|\nabla Q_{i+1,n}| \le \big||\nabla Q_{i,n} + h(M_s^2|\nabla w_{i,n}| + M_s K_s|\nabla Q_{i,n}|o(h^2)$$

$|\nabla w_{i+1,n}|$ ≤|(|∇W_(I,N) |+H(C (∇Q_(I,N))/|∆|_(-∞) +Cδ K^2)|$o(h^2)$

Suppose:

$$\nabla_i = MAX\{|\nabla U_{i,n}|,|\nabla W_{i,n}|,|\nabla Q_{i,n}|\}$$

$$C_0 = \mathrm{Max}\left\{1, M_s^2 + M_S K_s, \frac{C}{|\delta|_{-\infty}}\right\},$$

$$C_1 = \frac{CK}{|\delta|_{-\infty}} + C_\delta K^2$$

In This Case, We Will Have:

After The L We Will Have:

$$\nabla_L \le \mathrm{Exp}(\mathrm{C}_0)\,(\nabla_M + C_1)$$

According To The Below Equations, We See That When

ε, H, K Tends To Zero, In This Case, ∇_M And C_1Tend To Zero, As A Result $\nabla_M + C_1$And Also∇_0 Tends To Zero.

$$|\nabla u_{M,n}| \le c(\varepsilon + k)$$

$$|\nabla Q_{M,n}| \le c(\varepsilon + k)$$

$$|\nabla W_{M,n}| \le c\delta_M(\varepsilon + k) + C_\delta K^2$$

By Using The Mathematical Model Of Klein Gordon's Nonlinear One- Dimensional Equation And Two- Dimensional Sine-Gordon Equation, The Data Analysis Was Performed Using Radial Basis Functions. The Results Of The Research And After A General Conclusion, As Well As The Discussion Of The Results, Express The Limitations That The Researcher Encountered During The Implementation Of This Research. Finally, Suggestions Are Presented In The Form Of Scientific And Practical Proposals.

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Mehrzad Ghorbani "Numerical Solution of Klein–Gordon and Sine-Gordon Equations Using the Numerical Method of Gridless Lines "International Journal of Engineering Research and Applications (IJERA) , vol. 8, no.5, 2018, pp. 66-71