

## Total Time Minimizing Transportation Problem

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**ABSTRACT:** This paper shows the total transportation time problem regarding the time of the active transportation routes. If the multiple optimal solutions exist, it is possible to include other criteria as second level of criteria and find the corresponding solutions. Furthermore, if there is a multiple solution, again, the third objective can be optimized in lexicographic order. The methods of generation of the optimal solution in selected cases are developed. The numerical example is included.

**Keywords:** Transportation problem, total time as criteria, multicriteria optimization, lexicographic ordered criteria's.

### I. INTRODUCTION

The problem of minimization of the total transportation cost is commonly treated in literature as a basic single objective linear transportation model. The transportation time is relevant in a variety of real transportation problems, too. There are two types of problems regarding the transportation time [10]: (i) minimization of the total transportation time (linear function, as aggregate the products of transportation time and quantity), called minimization of 1st transportation time, and (ii) minimization of the transportation time of the longest active transporting route (nonlinear function), called minimization of 2nd transportation time or problem of Barasov [2]. For (ii), the total number of units on transportation operation with longest time is minimized in [4]. An

important variant of the total transportation time problem is formulated and resolved in [5] which are on whole included in [11] pp. 258-260.

The transportation time of the longest active transportation route(s) in problems where all destinations do not have the same importance are

$$X = \{x_{ij} \mid a_i \leq \sum_{j \in J} x_{ij} \leq A_i, \sum_{i \in I} x_{ij} \leq b_j \leq B_j, x_{ij} \geq 0, (i, j) \in I \times J\}$$

$$\left\{ \begin{array}{l} \sum_{j \in J} x_{ij} \leq a_i \leq A_i \\ \sum_{i \in I} x_{ij} \leq b_j \leq B_j \end{array} \right\}$$

Suppose  $t_{ij}$  = the time required for transporting all  $x_{ij}$  units using corresponding routes (i,j) for all  $(i, j) \in I \times J$ .  
 $F(x) = \sum_{(i,j) \in I \times J} t_{ij} x_{ij}$

But, we prefer that in many real life problems is

analyzed as the three classes single criteria and multi criteria problems of the transportation time. The corresponding algorithms are developed in case of problems with priority according to demands of the subset of the destinations [7]. Some bicriteria problems of transportation problems are shown, too [1], [6]. Another typical transportation problems are exposed in [8] and a research directive in [3]. Theoretical approach of Multi-Objective Programming and Goal Programming are shown in [9].

In this paper are formulated some variants of the total transportation time problem. The algorithms developed for determination their optimal solutions are presented and implementations are illustrated by means of a numerical example.

### FORMULATION OF THE PROBLEM

Let us consider the standard balanced transportation problem, with  $m$  sources  $A_i$  (with supplies  $a_i$ ),  $i \in I = \{1, 2, \dots, m\}$ , and  $n$  destinations  $B_j$  (with demands  $b_j$ ),  $j \in J = \{1, 2, \dots, n\}$ . If  $x_{ij}$  = the number of load units moving from  $A_i$  to  $B_j$ , the feasible solution  $(x)$  and set of feasible solutions  $(X)$  is:

$$X = \{x \mid x_{ij} \geq 0, (i, j) \in I \times J\}$$

(1)

and  $j \in J$ .

In the literature, instead of the total transportation time, often, is observed the "transportation efficiency" and minimized as following criterion:

(2)

mostly natural to focus to minimization only the

time of active transportation routes (i,j), as next  
 $T(x) = \sum_{ij} t_{ij} x_{ij}$   
 $i \in I, j \in J$   
 if  $x_{ij} > 0$  if  $x_{ij} = 0$

Indicative and clear example of this kind of total transportation time is in a military problem where is of primary importance to analyze the total time of all means of transportation which may be exposed to danger of the enemy attacks. This two types of

measure of the transportation efficiency (2) and (3) will be called Variant A (linear function) and Variant B (nonlinear function) of the total time transportation problem, respectively.

If multiple optimal solutions exist with  $T^*$  as minimal value of (3), it is recommended to optimize an another criteria retain  $T^*$ , like the transportation efficiency (2), the time of the longest  $q^{(k)} = t_{ij}$  is

The characteristic  $q_{ij}$  is the change of the transportation time in problem (3).

Then the solution  $x^{(k+1)}$  has:

$$\begin{cases} T(k+1) = T(k) + \sum_{ij} q_{ij} x_{ij} \\ \text{if } q_{ij} > 0 \\ \text{if } q_{ij} < 0 \end{cases}$$

Let  $T^*$  is minimum value of  $T(x)$ ,  $x^*$  is the optimal solution of (3) and  $X_T$  is a set of multiple optimal solutions of(3):

th

objective  
 active transportation operation, the number of units on transportation operation with longest time, the total transportation cost e.t.c.

**SOLUTIONMETHODS**

Let  $x^{(k)}$  and  $x^{(k+1)}$  are two basjs neighbouring feasible solutions, where  $x^{(k)}$  is entering basis variable and  $x^{(k)}$  is leaving basis variable for  $x^{(k)}$ :

$$\begin{aligned} x^{(k)} \text{ is contain: } & x^{(k)} = 0 \text{ and } x^{(k+1)} > 0 \\ & \text{contain: } x^{(k+1)} > 0 \text{ and } x^{(k+1)} = 0 \end{aligned}$$

there is:  $x^{(k+1)} = x^{(k)}$ .  
 In moving from  $x^{(k)}$  to  $x^{(k+1)}$  the total time  $T(x)$  given as (3) will be changed with the following values:

$$T(k+1) = T(k) + q^{(k)}$$

$$T(k+1) = T(k) - q^{(k)}$$

Clearly, the total time  $T^{(k+1)}$  is determined by values  $q^{(k)}$  as following:

$$\text{if } q^{(k)} > 0$$

$$(7)$$

$$T^* = \min_{x_{ij}} \left\{ \sum_{i \in I} \sum_{j \in J} x_{ij} t_{ij} \right\} \quad (8)$$

$$X_T^* = \{x_{ij}^* \mid T(x^*) = T^*\} \quad (10)$$

Above discussion makes possible to develop the solving methods for defined transportation problem (3). If this problem has multiple optimal solution (10), undoubtedly, maybe required to optimize some of next criteria:

$$T(x) = \max_{i \in I, j \in J} \{t_{ij} x_{ij}\} \quad (9)$$

Minimize the transpiration efficiency from (2)

$$\text{Minimize } T(x) = \max_{i \in I, j \in J} \{t_{ij} x_{ij}\} \quad (11)$$

Minimize the time of the longest active transportation operation

$$\text{Minimize } t(x) = \max_{i \in I, j \in J} \{t_{ij} x_{ij}\} \quad (12)$$

Minimize the number of units on transportation operation with longest time

$$\begin{cases} \text{Min} \\ Q(x) \end{cases} \quad (13)$$

$$\begin{cases} T(x) \leq T^* \\ t_{ij} \leq t(x) \\ ij \end{cases}$$

Minimize the total transportation cost

$$\begin{cases} \text{Min} \\ F(x) = cx \end{cases} \quad (14)$$

$$\left\{ \begin{array}{l} \\ \square \square ijij \end{array} \right\}$$

$$\begin{cases} T(x) \leq T^* \\ i \in I, j \in J \end{cases}$$

where  $c_{ij}$  = the units transportation costs.  
 Algorithm 1 finds the optimal solution and minimum total transportation time (3). Algorithm 2 continue the solving process using the multiple optimal solution of problem (3), if exist, and minimize chosen criteria (11) to (14).

**Algorithm 1.**  
**Step 0:** Find the basic feasible solution  $x^{(1)}$ . Set number of iteration  $k = 1$ .  
**Step 1:** Determine the indicators  $h^{(k)}$  of active transportation routes  $x^{(k)} > 0$ , and the total time  $T^{(k)} = T(x^{(k)})$ .

$$\begin{cases} h^{(k)} \leq 1 \\ \text{if } x^{(k)} \leq 0 \end{cases} \quad (15)$$

$$\begin{cases} T^{(k)} \\ \square \square \\ i \in I, j \in J \end{cases} \quad (16)$$

**Step 2:** Determine the characteristics  $q^{(k)}$  for all nonbasic variables  $x^{(k)} = 0$  using (5). Use the changing path of the basic solution (as in a Stepping-Stone method) and corresponding leaving basic variable, e.g.  $x^{(k)} > 0$  become  $x^{(k+1)} = 0$ , if entering basic variable would be  $x^{(k+1)} > 0$ .

**Step 3:** Check the optimality of the total time (3), using (7). If all  $q^{(k)} \leq 0$ , the optimal solution  $x^*$  is found. Stop. Otherwise, go to Step 4.

**Step 4:** Determine next basic solution, using entering variable  $x_{ij}$  with minimum  $q^{(k)}$ , regarding  $q^{(k)} < 0$ . Set  $k = k + 1$  and goto Step 1.

If the optimal solution  $x^*$  in last Step 3 has  $q^{(k)} = 0$  for nonbasic variables  $x^{(k)} = 0$ , there is no unique optimal solution and exist a set of multiple optimal solutions  $X_T^*$ . Each of these variables gives an alternative optimal solution for (3), go to optimize other criteria.

**Algorithm 2.**

**Step 0:** Chose one of criteria from (11) to (14) and calculate his increase  $\square^{(k)}$  for each nonbasic variable  $x^{(k)} = 0$  with  $q^{(k)} = 0$  in multiple optimal solution on end of

Algorithm 1. For  $\square^{(k)}$  use the known solving process for regarded criteria.

**Step 1:** If there are negative increase,  $\square^{(k)} < 0$ , for regarded criterion, chose minimum of them and minimize this criterion in set of multiple optimal solution for (3).

**Step 2:** Repeat Step 1 with each of negative increase for regarded criterion and choose solution with minimum criterion value.

**ANEXAMPLE**

Let us consider the following transportation problem with  $m = 4$  sources  $A_i$ ,  $i \in I = \{1, 2, 3, 4\}$ , and  $n = 5$  destinations  $B_j$ ,  $j \in J = \{1, 2, 3, 4, 5\}$ . The initial data are presented in TABLE 1. Each row corresponds to a supply point and each column to a demand point. The total supply 65 is equal to the total demand. In each cell (i,j), top left corner represents the time  $t_{ij}$  required

for transporting  $x_{ij}$  units from source  $A_i$  to destination  $B_j$ . The basic variables  $x_{ij}$  are presented in the middle of corresponding cells and the increase  $q_{ij}$  of time in bottom right corner of each cell  $(i,j)$  with nonbasic variable.

First basic feasible solution in the Step 0 of Algorithm 1 maybe determined using north-west corner method or another method. In TABLE 1 is chosed the optimal solution  $x^{(1)}$  of criteria  $F(x)$  with minimum value  $F^* = F^{(1)} = 222$  and increases  $d^{(1)}$

$$d^{(1)} = t$$

$$-t + t - t$$

$$x^{(2)} > 0, x^{(2)} = \min \{x\}$$

$$x^{(1)} = \min \{1, 2\} = 1 = x$$

$$x^{(1)} = x$$

$$x^{(2)} = 0$$

in TABLE 2 which verify the unique optimal solution. The corresponding total time  $T^{(1)} = 32$  is calculated in Step 1 with indicator  $h^{(1)}$  (TABLE 1 show only indicators with value 1 for active transportation routes).

In Step 2 are calculated increases  $q^{(1)}$  of the total transportation time  $T(x)$  for nonbasic variables. Let demonstrate them for cell  $(1,1)$ .

$$q^{(1)} = 0, L^{(1)} = \{ (1,1), (1,5), (4,5), (4,1) \}$$

$$= 11 - 5 + 5 - 9 = 2$$

11 11  
 15 41  
 15 15

$$q^{(1)} = t - t$$

$$= 11 - 5 = 6.$$

11 1115

The changing path L

<sup>(1)</sup>for Stepping-Stone method is used for calculate d  
 (1)

and indicate that x

11  
<sup>(1)</sup>is leaving variable if x  
<sup>(1)</sup>is entering variable. Then is q

11  
 (1) = t -

15 11  
 $t_{15} = 11 - 5 = 6.$

11 11

On changing path L

25  
 is no optimal, too, because for no basic variable x

(4,5) exists x

$$^{(3)} = \min \square x$$

(2), x

44

$$^{(2)} \square = \min \square 10, 1 \square = 1 = x$$

44 14 45

45 44

-2. After entering this variable, the optimal solution  $x^* = x^{(3)}$  is finding (TABLE 4).<sup>23</sup>

solution  $x^{(3)}$  with d

(3)

21

optimize some another criteria using Algorithm 2 regarding corresponding changing paths with their entering and leaving variables:

$$x^{(3)} = 0, L^{(3)} = \square (2,1), (2,5), (1,5), (1,4), (4,4) \square$$

23 23 21 21

$x^{(5)} > 0, x$

$$^{(5)} = \min \square x$$

(3), x

23 23

33 12

12 12

$$^{(1)} = \square (2,5), (4,5), (4,1), (2,1) \square \text{ for } x$$

<sup>(1)</sup> = 0 there are x

(2)

$$= \min \square x$$

(1), x

25

$$^{(1)} \square = \min \square 14, 13 \square = 13 = x$$

(1) and x

25

<sup>(2)</sup> = 0. Value q

$$^{(1)} = t - t$$

25

$$= 1 - 2$$

15 21

21 21

2525 21

= -1 shown that  $x^{(1)}$  is no optimal solution for T(x).

Entering basic variable  $x_{25}$  with

solution  $x^{(2)}$  decrease  $T^{(1)} = 32$  to  $T^{(2)} = T^{(1)} + t^{(1)} = 32 - 1 = 31$  (TABLE 3). This solution

<sup>(2)</sup> on path L

$$^{(2)} = \square (4,4), (1,4), (1,5),$$

44

<sup>(2)</sup> and t

$$^{(2)} = t - t$$

$$= 3 - 5 =$$

44 45

Minimum of total transportation time  $T^* = T^{(2)} = 29$  has multiple optimal

<sup>(3)</sup> = 0. Keeping minimum of T(x) maybe it is possible to

$$x^{(4)} > 0, x^{(4)} = \min \square x^{(3)}, x^{(3)}, x^{(3)} \square = \min \square 13, 9, 15 \square$$

$$= 9 = x^{(3)}, x^{(4)} = 0$$

$$x^{(3)} = 0, L^{(3)} = \square (2,3), (3,3), (3,2), (1,2) \square$$

$$^{(3)} \square = \min \square 15, 3 \square = 3 = x$$

(3), x

$$^{(5)} = 0.$$

**Table 1:** Initial data, optimal solution  $x^{(1)}$  for  $F(x)$  and indicators  $q^{(1)}$  for  $T(x)$

|  |       | Destinations |        |         |         |             | Supplies, $a_i$ |
|--|-------|--------------|--------|---------|---------|-------------|-----------------|
|  |       | $B_1$        | $B_2$  | $B_3$   | $B_4$   | $B_5$       |                 |
| Sources  | $A_1$ | 11<br>6      | 3<br>3 | 10<br>7 | 2<br>10 | 5<br>1      | 14              |
|  | $A_2$ | 2 (-)<br>13  | 7<br>4 | 3<br>0  | 8<br>6  | 1 (+)<br>-1 | 13              |
|  | $A_3$ | 12<br>7      | 2<br>7 | 4<br>15 | 5<br>3  | 7<br>2      | 22              |
|  | $A_4$ | 9 (+)<br>2   | 4<br>1 | 6<br>3  | 3<br>1  | 5 (-)<br>14 | 16              |
| Demands, $b_j$   |       | 15           | 10     | 15      | 10      | 15          | 65 \ 65         |
| $F^{(1)} = 3 \times 3 + 2 \times 10 + 5 \times 1 + 2 \times 13 + 2 \times 7 + 4 \times 15 + 9 \times 2 + 5 \times 14 = 222$<br>$h^{(1)} = h^{(1)} = h^{(1)} = h^{(1)} = h^{(1)} = h^{(1)} = 1$<br>12    14    15    21    32    33    41    45<br>$T^{(1)} = 3 \times 1 + 2 \times 1 + 5 \times 1 + 2 \times 1 + 2 \times 1 + 4 \times 1 + 9 \times 1 + 5 \times 1 = 32$ |       |              |        |         |         |             |                 |

Let analyze the longest time on the separable active transportation routes (12) and corresponding number of transported units (13):

$$t^{(3)} = \max_{ij} x_{ij} = 0$$

$$t_{ij} = \max(t_{12}, t_{14}, t_{15}, t_{25}, t_{32}, t_{33}, t_{41}, t_{44})$$

$$= \max(3, 5, 1, 2, 4, 9, 3) = 9 = t_{41}$$

$$Q^{(3)} = 15$$

$t_{ij} = 9$   
 Solution  $x^{(4)}$  keep  $t^{(4)} = t^{(3)} = 9$  and decrease  $Q^{(3)}$  to value  $Q^{(4)}$   
 $= Q^{(3)} - x$

$$Q^{(4)} = 15 - 6$$

$= 9$  (TABLE 5). So,  $x^{(4)}$  is better than  $x^{(3)}$ . With  $x$

$^{(5)}$  in  $x^{(5)}$  (TABLE 6) both of criteria has

$$23$$

same values as  $x^{(3)}$ , and  $x^{(5)}$  is not better solution.

**Table 2.**

| Nonbasic variables, $x_{ij}^{(1)} = 0$ | Indicators $d_{ij}^{(1)}$ for $F(x)$ | Indicators $q_{ij}^{(1)}$ for $T(x)$          |
|--|--------------------------------------|---|
| $x_{11}^{(1)} = 0$                     | $d_{11}^{(1)} = 2$                   | $q_{11}^{(1)} = t_{11} - t_{11} = 11 - 5 = 6$ |
| $x_{13}^{(1)} = 0$                     | $d_{13}^{(1)} = 5$                   | $q_{13}^{(1)} = t_{13} - t_{12} = 10 - 3 = 7$ |
| $x_{22}^{(1)} = 0$                     | $d_{22}^{(1)} = 11$                  | $q_{22}^{(1)} = t_{22} - t_{12} = 7 - 3 = 4$  |
| $x_{23}^{(1)} = 0$                     | $d_{23}^{(1)} = 5$                   | $q_{23}^{(1)} = t_{23} - t_{13} = 3 - 3 = 0$  |
| $x_{24}^{(1)} = 0$                     | $d_{24}^{(1)} = 13$                  | $q_{24}^{(1)} = t_{24} - t_{14} = 8 - 2 = 6$  |
| $x_{25}^{(1)} = 0$                     | $d_{25}^{(1)} = 3$                   | $q_{25}^{(1)} = t_{25} - t_{21} = 1 - 2 = -1$ |

|                     |   |   |
|---------------------|---|---|
| 25                  | 25  |   |
| $x^{(1)} = 0$<br>31 | $d^{(1)} = 4$<br>31                                 | $q_{31}^{(1)} = t_{31} - t_{14} = 12 - 5 = 7$ |
| $x^{(1)} = 0$<br>34 | $d^{(1)} = 4$<br>34                                 | $q_{31}^{(1)} = t_{31} - t_{14} = 12 - 5 = 7$ |
| $x^{(1)} = 0$<br>35 | $d^{(1)} = 3$<br>35                                 | $q_{35}^{(1)} = t_{35} - t_{15} = 7 - 5 = 2$  |
| $x^{(1)} = 0$<br>42 | $d^{(1)} = 1$<br>42                                 | $q_{42}^{(1)} = t_{42} - t_{12} = 4 - 3 = 1$  |
| $x^{(1)} = 0$<br>43 | $d^{(1)} = 1$<br>43                                 | $q_{43}^{(1)} = t_{43} - t_{12} = 6 - 3 = 3$  |
| $x^{(1)} = 0$<br>44 | $d^{(1)} = 1$<br>44                                 | $q_{44}^{(1)} = t_{44} - t_{14} = 3 - 2 = 1$  |
|                     | Unique optimal solution,<br>all $d^{(1)} > 0$<br>ij | No optimal solution, $q^{(1)} = -1$<br>25     |

Table 3: Solution  $x^{(2)}, T^{(2)} > T^*$

Table 4: Solution  $x^{(3)}, T^*$

|           |          |           |           |           |
|-----------|----------|-----------|-----------|-----------|
| 11        | 3        | 10        | 2 (-)     | 5 (+)     |
| 6         | <b>3</b> | 7         | <b>10</b> | <b>1</b>  |
| 2         | 7        | 3         | 8         | 1         |
| 1         | 4        | 0         | 6         | <b>13</b> |
| 12        | 2        | 4         | 5         | 7         |
| 7         | <b>7</b> | <b>15</b> | 3         | 2         |
| 9         | 4        | 6         | 3 (+)     | 5 (-)     |
| <b>15</b> | -1       | 1         | -2        | <b>1</b>  |

|           |          |           |          |           |
|-----------|----------|-----------|----------|-----------|
| 11        | 3        | 10        | 2 (-)    | 5 (+)     |
| 9         | <b>3</b> | 7         | <b>9</b> | <b>2</b>  |
| 2 (+)     | 7        | 3         | 8        | 1 (-)     |
| 0         | 4        | 0         | 6        | <b>13</b> |
| 12        | 2        | 4         | 5        | 7         |
| 10        | <b>7</b> | <b>15</b> | 3        | 2         |
| 9 (-)     | 4        | 6         | 3 (+)    | 5         |
| <b>15</b> | 1        | 3         | <b>1</b> | 2         |

$$T^{(2)} = T^{(1)} + t^{(1)} = 32 - 1 = 31 \quad T^{(3)} = T^{(2)} + t^{(2)} = 31 - 2 = 29$$

Regarding in  $x^{(3)}$  on total cost  $C(x)$  defined as (14) and unique cost in following matrix C, the total cost are  $C^{(3)} = 428$ . The increase of total cost of variables for multiple

optimal solution in minimum total time  $T^* = 29$  are  $C^{(3)} = -1$  (for  $x^{(3)} = 0$  with  $q^{(3)} = 0$ )

21      21      21

and  $C^{(k)} = 4$  (for  $x^{(3)} = 0$  with  $q^{(3)} = 0$ ). If  $x$  is entering basic variable, solution is  $x^{(4)}$

23      23      23      21  
 with minimum total cost  $C^{(5)} = 419$  for minimum total time  $T^* = 29$ . However, the minimum total cost  $C^* = 383$  without minimum total time  $T(x)$  would be obtain with  $x^{(6)}$  when the total time is  $T^{(6)} = 46$ .

The optimal solutions of some basic single criteria cost and time in transportation problems are compared in TABLE 7. Using the optimal solution of each of them, the values of others criteria are calculated. All solutions with time contain an

identical time of the longest route (12) in this trivial example. Clearly, it is not a general rule.

After conditional optimization for objective (12), the objective (13) was minimized for the total quantity of goods which is transported with longest time  $t^*$ , in the sense of Hammers "real" solution for optimal solution (12). However, objective (13) is

optimized against two conditions, keeping minimal total time  $T^* = 29$  and the time of the longest transport operation  $t^* = 9$  for  $T^* = 29$ .

**Table 5:** Solution  $x^{(4)}, T^*$       **Table 6:** Solution  $x^{(5)}, T^*$

|    |   |    |    |    |
|----|---|----|----|----|
| 11 | 3 | 10 | 2  | 5  |
|    | 3 | 7  |    | 11 |
| 2  | 7 | 3  | 8  | 1  |
| 9  |   |    |    | 4  |
| 12 | 2 | 4  | 5  | 7  |
|    | 7 | 15 |    |    |
| 9  | 4 | 6  | 3  | 5  |
| 6  |   |    | 10 |    |

|    |    |    |   |    |
|----|----|----|---|----|
| 11 | 3  | 10 | 2 | 5  |
|    |    |    | 9 | 5  |
| 2  | 7  | 3  | 8 | 1  |
|    |    | 3  |   | 10 |
| 12 | 2  | 4  | 5 | 7  |
|    | 10 | 12 |   |    |
| 9  | 4  | 6  | 3 | 5  |
| 15 |    |    | 1 |    |

$T^{(4)} = T^{(3)} + t^{(3)} = 29 + 0 = 29$        $T^{(5)} = T^{(3)} + t^{(3)} = 29 + 0 = 29$

$$C = \begin{bmatrix} 4 & 7 & 10 & 9 & 5 \\ 6 & 8 & 16 & 9 & 8 \\ 3 & 0 & 10 & 0 & 0 \end{bmatrix}, \quad x^{(6)} = \begin{bmatrix} 14 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 & 6 & 10 & 7 \\ 5 & 8 & 9 & 10 & 6 \end{bmatrix} \leq f$$

$$\begin{bmatrix} 0 & 7 & 15 & 0 & 0 \\ 1 & 0 & 0 & 0 & 15 \end{bmatrix} \leq f$$

**Table 7:** Optimal solutions of basic single criteria transportation problems

| Minimization criteria | Solutions   |  |  |   |
|-----------------------|---|--|--|---|
|                       | $x^{(1)} =$   | $x^{(3)} =$  | $x^{(4)} =$  | $x^{(6)} =$   |
|                       | $\begin{bmatrix} 0 & 3 & 0 & 10 & 1 \\ 13 & 0 & 0 & 0 & 0 \\ 0 & 7 & 15 & 0 & 0 \\ \leq 0 & 0 & 0 & 14 & f \end{bmatrix}$ | $\begin{bmatrix} 0 & 3 & 0 & 0 & 11 \\ 9 & 0 & 0 & 0 & 4 \\ 0 & 7 & 15 & 0 & 0 \\ \leq 6 & 0 & 0 & 10 & f \end{bmatrix}$ | $\begin{bmatrix} 0 & 3 & 0 & 0 & 11 \\ 9 & 0 & 0 & 0 & 4 \\ 0 & 7 & 15 & 0 & 0 \\ \leq 6 & 0 & 0 & 10 & f \end{bmatrix}$ | $\begin{bmatrix} 14 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 10 & 0 \\ 0 & 7 & 15 & 0 & 0 \\ \leq 1 & 0 & 0 & 0 & 15 & f \end{bmatrix}$ |
| F(x)                  | $F_{\min} = 222$  | 262  | 244  | 333   |
| T(x)                  | 31  | $T_{\min} = 29$  | $T_{\min} = 29$  | 48  |
| t(x)                  | $t_{\min} = 9$  | $t_{\min} = 9$   | $t_{\min} = 9$   | 11  |
| Q(x)                  | $Q_{\min} = 2$  | 15   | 6  | 14  |
| C(x)                  | 406   | 428  | 419  | $C_{\min} = 383$  |

**II. CONCLUSIONS**

The time of transport might be significant factor in several transportation problems. The efficiency of transportation have been introduced as an aggregate of the time and the quantity of goods on active transport operations, longest time on single active transport operation, total time on all active transport operations, total quantity of goods with a longest time of transport etc. The question arises from this whether to perform optimization at the same time for more objectives, with eventual priorities for each, or to treat single objective problems, which are of the great significance for particular problem. In this paper, the single

objective problem of total time minimization has been exposed having in mind active transport operations and is defined as Algorithm

1 for its solution. To start from the point of multiple optimal solution for specific total time of transport, it is further suggested the minimization of some other objective while keeping minimal total time of transport (Algorithm 2). In that way, it is passed to the two objective problems with strong lexicographic order for the determined objectives where the absolute priority is given to the total time of transport. Furthermore, if there is a multiple solution, the third objective can be optimized which

bears the next level of significance etc.

Defined process of solving the problem of minimization of total time on active transport operations is necessary even for solving each multi objective transport problem which involves mentioned objective. At the first step of solving any multi objective problem, it is necessary to determine optimal solutions for each objective separately, that is to treat single objective problems and then to keep searching for pareto-optimal solutions for each multi objective problem.

In hypothetical example of small dimensions two variants of problem were illustrated for the total time of transport with objectives of "total efficiency of transport from the time view"  $F(x)$  and total time of transport  $T(x)$ . Meanwhile, in the set of optimal solutions for  $T(x)$ , the conditional minimization was performed separately for the longest time on active transport operations with  $t(x)$  and separately for total cost of transport with  $C(x)$ .

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