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Convection in a Maxwellian Viscoelastic Fluid Layer Through Porous Medium

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ABSTRACT
The triply-diffusive convection in a Maxwellian viscoelastic fluid layer is mathematically investigated through porous medium. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For the case of stationary convection, the solute gradients play stabilizing role in the system while medium permeability plays destabilizing role. The dispersion relation is also analyzed numerically. Further, the solute gradients are found to introduce oscillatory modes, which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

Keywords: Maxwellian viscoelastic fluid, Porous medium, Solute gradients, Triply diffusive convection

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I. INTRODUCTION

The study of a layer of fluid heated from below in porous media is motivated both by theoretical and practical applications in engineering. Among the applications in the engineering discipline one can find the food process industry, chemical process industry, solidification, and centrifugal casting of metals. The development of geothermal power resources has increased the general interest in the properties of convection in porous media. The formation and derivation of the basic equations of a layer of fluid heated from below in a porous medium, using Boussinesq approximation, has been given in a treatise by Joseph [1]. When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. An extensive and updated account of convection in porous media has been given by Nield and Bejan [2].

The theoretical and experimental results on thermal convection in a fluid layer under varying assumptions of hydrodynamics have been given by Chandrasekhar [3] in his monograph. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient was investigated by Veronis [4]. Nield [5] considered the thermohaline convection in a horizontal layer of viscous fluid heated from below and salted from above. There has been considerable interest in the study of the breakdown of the stability of a layer of fluid subject to a vertical temperature gradient in a porous medium and the possibility of convective flow. The stability of flow of a single component fluid through a porous medium taking into account the Darcy resistance has been studied by Lapwood [6] and Wooding [7]. Conditions under which convective motions through porous medium are important in geophysics are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore it is desirable to consider a two component fluid and free boundaries.

In the standard Be'nard problem, the instability is driven by a density difference caused by a temperature difference between the upper and lower planes bounding the fluid. If the fluid, additionally has salt dissolved in it then there are potentially two destabilizing sources for the density difference, the temperature field and salt field. The solution behavior in the double-diffusive convection problem is more interesting than that of the single component situation in so much as new instability phenomena may occur which is not present in the classical Benard problem. When temperature and two or more component agencies, or three different salts, are present then the physical and mathematical situation becomes increasingly richer. For example, Degens et al. [8] reported that the saline waters of geothermally heated Lake Kivu are strongly stratified by temperature and a salinity which is the sum of comparable concentrations of many salts, while the oceans contain many salts in concentrations less than a few per cent of the sodium chloride concentration. It has been recognized previously that there are important fluid mechanical systems in which the density depends on three or more stratifying agencies having different diffusivities, which can be called multiply diffusive convection [9]. By analogy with the doubly diffusive case in which the density depends on two independent diffusing stratifying agencies, we refer to the isothermal quaternary and non-isothermal ternary (i.e. three components) cases as being triply-diffusive. Very interesting results in triply diffusive convection have been obtained by Pearlstein et al. [9]. They demonstrate that for triple diffusive convection linear instability can occur in discrete sections of the Rayleigh number domain with the fluid being linearly stable in a region in between the linear instabilities ones. This is because for certain parameters the neutral curve has a finite isolated oscillatory instability curve lying below the usual unbounded stationary convection one. Straughan and Tracery [10] investigated the effect of an internal heat source on the problem of triply-diffusive convection. Recently, Rionero [11] has study the problem of triple convective diffusive fluid mixture saturating a porous horizontal layer, heated from below and salted from above and below.

The previous studies on triple diffusive convection are dealt with only Newtonian fluid theory. Recently interest in viscoelastic flows through porous media has grown considerably, due largely to the demands of such diverse fields as biorheology, geophysics, chemical, and petroleum industries. Wang and Tan [12] have studied the stability analysis of double diffusive convection in Maxwell fluid in a porous medium. It is worthwhile to point out that the first viscoelastic rate type model, which is still used widely, is due to Maxwell. Keeping in mind the importance in various fields particularly in the soil sciences, ground water-hydrology, geophysics, astrophysics and bio-mechanics, the triply-diffusive convection in Maxwell viscoelastic fluid through porous medium has been considered in the present paper.

II. FORMULATION OF THE PROBLEM

Consider an infinite horizontal Maxwellian viscoelastic fluid layer through porous medium heated and soluted from below and confined between two horizontal planes situated at $z = 0$ and $z = d$, acted upon by a gravity field $\vec{g}(0, 0, -g)$. The temperature T and the solute concentrations $C^{(1)}$ and $C^{(2)}$ at the bottom and top surfaces $z = 0$, $z = d$ are T_0 and T_1; $C_0^{(1)}$, $C_1^{(1)}$ and $C_0^{(2)}$, $C_1^{(2)}$ respectively, and a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and uniform solute gradients $\beta'\left(=\left|\frac{dC^{(1)}}{dz}\right|\right)$ and $\beta''\left(=\left|\frac{dC^{(2)}}{dz}\right|\right)$ are maintained. Now the equations governing the motion of a Maxwellian viscoelatsic fluid through a porous medium and following Boussinesq approximation are

$$\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial \vec{v}}{\partial t}+\frac{1}{\varepsilon}(\vec{v}.\nabla)\vec{v}\right]=$$
$$-\frac{1}{\rho_0}\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla p+\vec{g}\left(1+\lambda\frac{\partial}{\partial t}\right)\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{\nu}{k_1}\vec{v},$$
(1)

$$\nabla.\vec{v}=0, \quad (2)$$

where $\vec{v}$ is the filter velocity, ε is medium porosity, k_1 is the medium permeability and $\nu(=\mu/\rho)$. The fluid velocity $\vec{q}$ and the Darcian (filter) velocity $\vec{v}$ are connected by the relation $\vec{q}=\vec{v}/\varepsilon$. A porous medium of very low permeability allows us to use the Darcy's model. For a medium of very large stable particle suspension, the permeability tends to be small justifying the use of Darcy's model. This is because the viscous drag force is negligibly small in comparison with Darcy's resistance due to the large particle suspension.

When the fluid flows through a porous medium, the equation of heat conduction is

$$(\rho c_f\varepsilon+\rho_s c_s(1-\varepsilon))\frac{\partial T}{\partial t}+\rho c_f(\vec{v}.\nabla)T=\kappa\nabla^2 T \quad (3)$$

and analogous solute concentration equations are

$$(\rho c_f^{'}\varepsilon+\rho_s c_s^{'}(1-\varepsilon))\frac{\partial C^{(1)}}{\partial t}+\rho c_f^{'}(\vec{v}.\nabla)C^{(1)}=\kappa'\nabla^2 C^{(1)}, \quad (4)$$

$$(\rho c_f^{''}\varepsilon+\rho_s c_s^{''}(1-\varepsilon))\frac{\partial C^{(2)}}{\partial t}+\rho c_f^{''}(\vec{v}.\nabla)C^{(2)}=\kappa''\nabla^2 C^{(2)}. \quad (5)$$

Since density variations are mainly due to variations in temperature and solute concentrations, the equation of state for the fluid is given by

$$\rho=\rho_0[1-\alpha(T-T_a)+\alpha'(C-C_a^{(1)})+\alpha''(C-C_a^{(2)})], \quad (6)$$

where $\rho, \rho_0, t, \nu, \kappa, \kappa', \kappa'', \alpha, \alpha'$ and α'' are the fluid density, reference density, time, the kinematic viscosity, the thermal diffusivity, the solute diffusivities, thermal and solvent coefficients of expansion respectively. T_a is the average temperature given by $T_a=\frac{T_0+T_1}{2}$ where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the layer and $C_a^{(1)}$, $C_a^{(2)}$ are the average concentrations given by $C_a^{(1)}=\frac{C_0^{(1)}+C_1^{(1)}}{2}$ and $C_a^{(2)}=\frac{C_0^{(2)}+C_1^{(2)}}{2}$, where $C_0^{(1)}$, $C_1^{(1)}$ and $C_0^{(2)}, C_1^{(2)}$ are constant average concentrations of the lower and upper surfaces of the layer. Here $E = \varepsilon + (1-\varepsilon)\frac{\rho_s c_s}{\rho c_f}$ is a constant, E' and E'' are analogous to E but corresponding to solute rather than heat. ρ, c_f ; ρ_s, c_s stand for density and heat capacity of fluid and solid matrix, respectively.

III. BASIC STATE AND PERTURBATIONEQUATIONS

The basic state was assumed to be quiescent and is given by

$\vec{v} = (0,0,0),\ T = T_b(z),\ p = p_b(z),\ C^{(1)} = C_b^{(1)}(z)$,

$C^{(2)} = C_b^{(2)}(z)$, $\rho = \rho_b(z)$,

with $T_b(z) = T_a - \beta z,\ C_b^{(1)}(z) = C_a^{(1)} - \beta' z$,

$C_b^{(2)}(z) = C_a^{(2)} - \beta'' z$,

$$\rho = \rho_0\,[1 + \alpha\beta z - \alpha'\beta' z - \alpha''\beta'' z]. \qquad (7)$$

To use linearized stability theory and normal mode technique, we assume small perturbations on the basic state solution. Let $\vec{v}(u,v,w) = 0 + \vec{v}(u',v',w')$, $\rho = \rho_b + \rho'$, $p = p_b + p'$, $T = T_b + T'$, $C^{(1)} = C_b^{(1)} + C^{(1)'}$ and $C^{(2)} = C_b^{(2)} + C^{(2)'}$ denote, respectively the perturbations in the fluid velocity, density, pressure, temperature and concentrations. The change in density ρ' caused mainly by the perturbations in temperature and concentrations is given by

$$\rho' = -\rho_0\,[\alpha T' - \alpha' C^{(1)'} - \alpha'' C^{(2)'}]. \qquad (8)$$

Then the linearized perturbation equations are

$$\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \vec{v}'}{\partial t} = -\frac{1}{\rho_0}\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla p' - g\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\alpha T' - \alpha' C^{(1)'} - \alpha'' C^{(2)'}\right) - \frac{\nu}{k_1}\vec{v}', \qquad (9)$$

$$\nabla.\vec{v}' = 0, \qquad (10)$$

$$E\frac{\partial T'}{\partial t} = \beta w + \kappa\nabla^2 T', \qquad (11)$$

$$E'\frac{\partial C^{(1)'}}{\partial t} = \beta' w + \kappa'\nabla^2 C^{(1)'}, \qquad (12)$$

$$E''\frac{\partial C^{(2)'}}{\partial t} = \beta'' w + \kappa''\nabla^2 C^{(2)'}, \qquad (13)$$

Analyzing the perturbations into normal modes, we assume that the perturbation quantities are of the form

$$\left[w, T', C^{(1)'}, C^{(2)'}\right] = [W(z), \Theta(z), \Gamma(z), \Upsilon(z)]\exp\{ik_x x + ik_y y + nt\}, \qquad (14)$$

where k_x and k_y are the wave numbers in x and y directions respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance which is, in general, a complex constant. Using expression (14), equations (9)-(13) in non-dimensional form become

$$\left[\frac{\sigma}{\varepsilon}(1+\sigma F) + \frac{1}{p_l}\right](D^2 - a^2)W + (1+\sigma F)\frac{ga^2d^2}{\nu}(\alpha\Theta - \alpha'\Gamma - \alpha''\Upsilon) = 0, \qquad (15)$$

$$\left(D^2 - a^2 - E\sigma p_1\right)\Theta = -\frac{\beta d^2}{\kappa}W, \qquad (16)$$

$$\left(D^2 - a^2 - E'\sigma q_1\right)\Gamma = -\frac{\beta' d^2}{\kappa'}W, \qquad (17)$$

$$\left(D^2 - a^2 - E''\sigma q_2\right)\Upsilon = -\frac{\beta'' d^2}{\kappa''}W. \qquad (18)$$

Here, we have put $a = kd$, $\sigma = \frac{nd^2}{\nu}$, $F = \frac{\lambda\nu}{d^2}$, $p_1 = \frac{\nu}{\kappa}$, $q_1 = \frac{\nu}{\kappa'}$, $q_2 = \frac{\nu}{\kappa''}$, $p_l = \frac{k_1}{d^2}$ and $D^* = dD$ [(*) is dropped for convenience]. Consider the case of two free boundaries. The case of two free boundaries is slightly artificial, except in stellar atmospheres and in certain geophysical situations where it is most appropriate, but it allows us to find analytical solutions and to make some qualitative conclusions. Both the boundaries are maintained at constant temperatures and constant concentrations. The appropriate boundary conditions w.r.t. which equations (15)-(18) must be solved are

$$W = 0,\ D^2W = 0,\ \Theta = 0,\ \Gamma = 0,\ \Upsilon = 0 \text{ at } z = 0 \text{ and } z = 1. \qquad (19)$$

Eliminating various physical parameters from equations (15)-(18), we obtain the final stability governing equation as

$$\left\{\frac{\sigma}{\varepsilon}(1+\sigma F) + \frac{1}{p_l}\right\}\left(D^2 - a^2 - E\sigma p_1\right)\left(D^2 - a^2 - E'\sigma q_1\right)\left(D^2 - a^2 - E''\sigma q_2\right)\left(D^2 - a^2\right)W$$
$$- Ra^2(1+\sigma F)\left(D^2 - a^2 - E'\sigma q_1\right)\left(D^2 - a^2 - E''\sigma q_2\right)W$$
$$+ Sa^2(1+\sigma F)\left(D^2 - a^2 - E\sigma p_1\right)\left(D^2 - a^2 - E''\sigma q_2\right)W + S_1 a^2(1+\sigma F)\left(D^2 - a^2 - E\sigma p_1\right)\left(D^2 - a^2 - E'\sigma q_1\right)W = 0. \qquad (20)$$

Here, $R = \frac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number, $S = \frac{g\alpha'\beta' d^4}{\nu\kappa'}$ is the analogous solute Rayleigh number and $S_1 = \frac{g\alpha''\beta'' d^4}{\nu\kappa''}$ is another analogous

solute Rayleigh number.
Using the above boundary conditions, it can shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence the proper solution of equation (20) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{21}$$

where W_0 is constant. Substituting the proper solution (21), in equation (20) we get the dispersion relation as

$$R_1 = \left(\frac{1+\omega}{\omega}\right)\left[\left\{\frac{i\sigma_1}{\varepsilon}(1+i\sigma_1\pi^2 F) + \frac{1}{P}\right\}\right.$$
$$\frac{(1+\omega+i\sigma_1 E p_1)}{(1+i\sigma_1\pi^2 F)} + S_2\left(\frac{1+\omega+i\sigma_1 E p_1}{1+\omega+i\sigma_1 E' q_1}\right)$$
$$+ S_3\left(\frac{1+\omega+i\sigma_1 E p_1}{1+\omega+i\sigma_1 E'' q_2}\right). \tag{22}$$

Here, $R_1 = \frac{R}{\pi^4}$, $S_2 = \frac{S}{\pi^4}$, $S_3 = \frac{S_1}{\pi^4}$, $\omega = \frac{a^2}{\pi^2}$,

$i\sigma_1 = \frac{\sigma}{\pi^2}$ and $P = \pi^2 p_l$.

IV. STATIONARY CONVECTION

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (22) reduces to

$$R_1 = \frac{(1+\omega)^2}{\omega P} + S_2 + S_3. \tag{23}$$

Thus, for the case of stationary convection, the relaxation time parameter F vanishes with σ and Maxwellian viscoelastic fluid behaves like an ordinary Newtonian fluid. The above relation expresses the modified Rayleigh number R_1 as a function of the parameters S_2, S_3, P and dimensionless wave number ω. To study the effects of solute gradients and medium permeability, we examine the nature of $\frac{dR_1}{dS_2}$, $\frac{dR_1}{dS_3}$ and $\frac{dR_1}{dP}$ analytically.
From equation (23), we have

$$\frac{dR_1}{dS_2} = 1 \text{ and } \frac{dR_1}{dS_3} = 1, \tag{24}$$

which show that solute gradients have stabilizing effect on the system.
It follows from equation (23) that

$$\frac{dR_1}{dP} = -\frac{(1+\omega)^2}{\omega P^2}, \tag{25}$$

which shows that medium permeability has a destabilizing effect on the system.

V. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Multiplying equation (15) by W^*, the complex conjugate of W, integrating over the range of z and making use of equations (16)-(18) together with the boundary conditions (19), we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{p_l}\left(\frac{1}{1+\sigma F}\right)\right] I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}[I_2 + \sigma^* E p_1 I_3] +$$
$$\frac{g\alpha'\kappa' a^2}{\nu\beta'}[I_4 + \sigma^* E' q_1 I_5]$$
$$+ \frac{g\alpha''\kappa'' a^2}{\nu\beta''}[I_6 + \sigma^* E'' q_2 I_7] = 0, \tag{26}$$

where $I_1 = \int\left(|DW|^2 + a^2|W|^2\right)dz$,
$I_2 = \int\left(|D\Theta|^2 + a^2|\Theta|^2\right)dz,\ I_3 = \int |\Theta|^2\, dz,$
$I_4 = \int\left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz,\ I_5 = \int |\Gamma|^2\, dz,$
$I_6 = \int\left(|D\psi|^2 + a^2|\psi|^2\right)dz,\ I_7 = \int |\psi|^2\, dz,$
$I_8 = \int\left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz,$
$I_9 = \int\left(|DK|^2 + a^2|K|^2\right)dz$

where σ^* is the complex conjugate of σ. The integrals $I_1 - I_9$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ in equation (26), where σ_r and σ_i are real and then equating the real and imaginary parts, we get

$$\left[\frac{\sigma_r}{\varepsilon} + \frac{1}{p_l}\left(\frac{1+\sigma_r F}{(1+\sigma_r F)^2 + \sigma_i^2 F^2}\right)\right] I_1 -$$
$$\frac{g\alpha\kappa a^2}{\nu\beta}\left(I_2 + \sigma_r E p_1 I_3\right) + \frac{g\alpha'\kappa' a^2}{\nu\beta'}[I_4 + \sigma_r E' q_1 I_5]$$
$$+ \frac{g\alpha''\kappa'' a^2}{\nu\beta''}[I_6 + \sigma_r E'' q_2 I_7] = 0 \tag{27}$$

and

$$\left[\left\{\frac{1}{\varepsilon} - \frac{1}{p_l}\left(\frac{F}{\left((1+\sigma_r F)^2 + \sigma_i^2 F^2\right)}\right)\right\} I_1 + \right.$$
$$\frac{g\alpha\kappa a^2}{\nu\beta} E p_1 I_3$$
$$- \frac{g\alpha'\kappa' a^2}{\nu\beta'} E' q_1 I_5$$
$$\left. - \frac{g\alpha''\kappa'' a^2}{\nu\beta''} E'' q_2 I_7 \right]\sigma_i = 0. \tag{28}$$

It is evident from equation (27) that σ_r may be negative or positive i.e. system may be stable or unstable. Also from equation (28), it can be seen that σ_i may be zero or non-zero meaning thereby that the modes may be non-oscillatory or oscillatory. Further, in the absence of solute gradients, equation (28) reduces to

$$\left[\left\{\frac{1}{\varepsilon}-\frac{1}{p_l}\left(\frac{F}{\left((1+\sigma_r F)^2+\sigma_i^2 F^2\right)}\right)\right\} I_1 + \frac{g\alpha\kappa a^2}{\nu\beta} E p_1 I_3\right]\sigma_i = 0 . \quad (29)$$

For the condition $\frac{1}{\varepsilon} > \frac{1}{p_l}\left(\frac{F}{\left((1+\sigma_r F)^2+\sigma_i^2 F^2\right)}\right)$, the coefficient of σ_i in equation (29) is a positive definite and hence implies that $\sigma_i = 0$. Thus, oscillatory modes are not allowed and the principle of exchange of stabilities is valid. So, we can say that oscillatory modes are introduced due to the presence of solute gradients in the system.

VI. THE OVERSTABLE CASE

Here, we discuss the possibility as to whether instability may occur as overstability. Equating the real and imaginary parts of equation (22) and eliminating R_1 between them, we obtain

$$A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \quad (30)$$

where $c_1 = \sigma_1^2$, $b = 1+\omega$

and

$$A_3 = \frac{\pi^4 F^2 E E' E''^2 \; p_1 q_1 q_2^2}{\varepsilon} b(b-1), \quad (31)$$

$$A_0 = \Big[\left(\frac{1}{\varepsilon}-\frac{F\pi^2}{P}\right)(b-1)b^5 + \frac{(Ep_1 - E'q_1)}{P}(b-1)b^4 + S_2(Ep_1 - E'q_1)(b-1)^2 b^3 + S_3(Ep_1 - E''q_2)(b-1)^2 b^3\Big]. \quad (32)$$

The coefficients A_1 and A_2 being quite lengthy and not needed in the discussion of overstability, have not been written here.

Since σ_1 is real for overstability i.e. the three values of c_1 $(=\sigma_1^2)$ should be positive. The product of the roots of equation (30) which is $-\frac{A_0}{A_3}$, should be positive.

From the expressions (31) and (32), It is clear that A_3 is always positive and A_0 is positive if

$$\frac{1}{\varepsilon} > \frac{F\pi^2}{P}, \; Ep_1 > E'q_1 \text{ and } E_1 p_1 > E''q_2 \quad (33)$$

i.e. if $\frac{1}{\varepsilon} > \frac{\lambda\nu}{k_1}$, $\frac{E}{\kappa} > \frac{E'}{\kappa'}$ and $\frac{E}{\kappa} > \frac{E''}{\kappa''}$.

Thus, for the conditions (33), overstability cannot occur and the principle of exchange of stabilities is valid. Hence, these are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily involve the occurrence of overstability.

VII. NUMERICAL RESULTS AND DISCUSSION

For the stationary convection critical thermal Rayleigh number for the onset of instability is determined for critical wave number obtained by the condition $\frac{dR_1}{dx} = 0$ and analyzed numerically using Newton-Raphson method.

In Fig. 1, critical Rayleigh number R_1 is plotted against solute gradient parameter S_2 for fixed values of $S_3 = 20$ and $P = 0.1, 0.5, 0.9$. The critical Rayleigh number R_1 increases with increase in solute gradient parameter S_2 which shows that solute gradient has stabilizing effect on the system.

In Fig. 2, critical Rayleigh number R_1 is plotted against solute gradient parameter S_3 for fixed values of $S_2 = 20$ and $P = 0.01, 0.05, 0.09$. The critical Rayleigh number R_1 increases with increase in solute gradient parameter S_3 which shows that solute gradient has stabilizing effect on the system.

In Fig. 3, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $S_2 = 20$ and $S_3 = 10, 30, 50$. The critical Rayleigh number R_1 decreases with increase in medium permeability P which shows that medium permeability has a destabilizing effect on the system.

In Fig. 4, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $S_3 = 20$ and $S_2 = 10, 50, 90$. The critical Rayleigh number R_1 decreases with increase in medium permeability P which shows that medium permeability has destabilizing effect on the system.

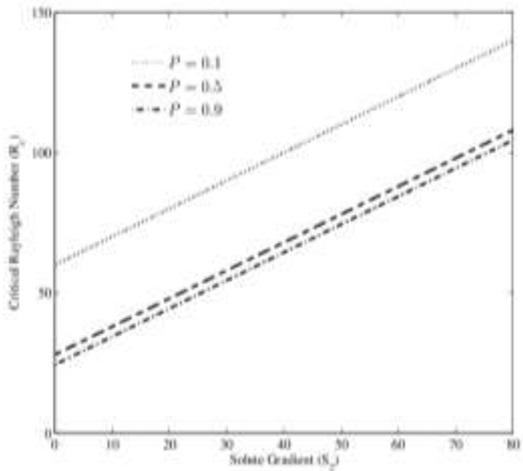

Fig. 1: Variations of critical Rayleigh number R_1 with S_2

for fixed value of $S_3 = 20$ and $P = 0.1, 0.5, 0.9$.

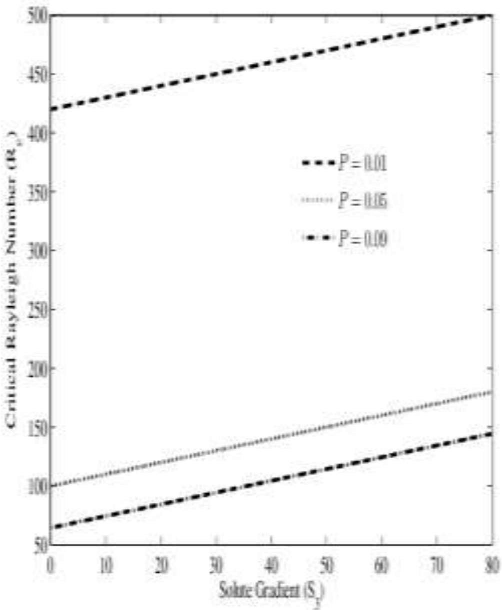

Fig. 2: Variations of critical Rayleigh number R_1 with S_3 for fixed value of $S_2 = 20$ an $P = 0.01, 0.05, 0.09$.

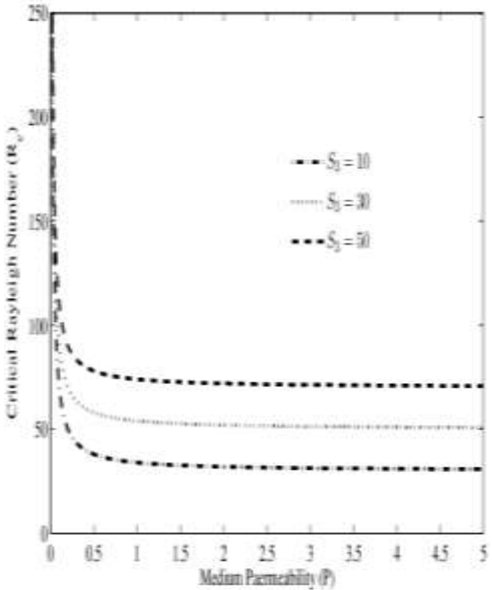

Fig. 3: Variations of critical Rayleigh number R_1 with P for fixed value of $S_2 = 20$ and $S_3 = 10, 30, 50$.

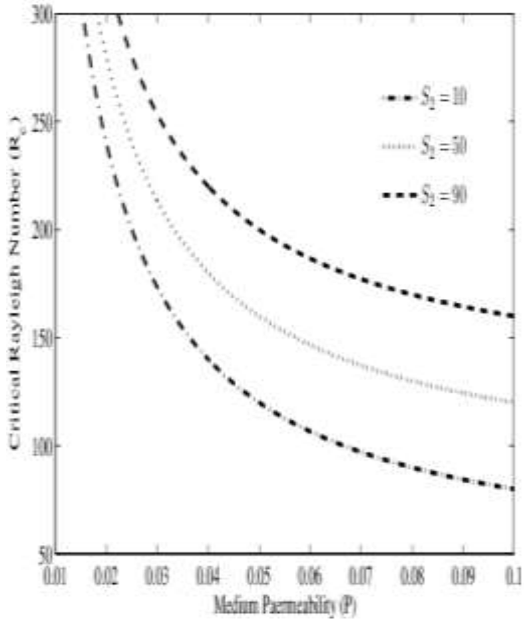

Fig. 4: Variations of critical Rayleigh number R_1 with P for fixed value of $S_3 = 20$ and $S_2 = 10, 50, 90$.

CONCLUSIONS

The subject of double-diffusive convection is still an active research area, however, there are many fluid systems in which more than two components are present. It has been recognized earlier that there are important fluid mechanical systems in which the density depends on three or more stratifying agencies having different diffusivities, which can be called multiply diffusive convection. By analogy with the doubly diffusive case in which the density depends on two independently diffusing stratifying agencies, we refer to the isothermal quaternary and non-isothermal ternary (i.e. three components) case as being triply-diffusive. Keeping this in view, the triply-diffusive convection in Maxwellian viscoelastic fluid through porous medium has been considered in the present paper.

The main conclusions from the analysis of this paper are as follows:

(a) For the case of stationary convection the following observations are made:
- The stable solute gradients have stabilizing effects on the system.
- The medium permeability has a destabilizing effect on the system.

(b) It is also observed from the Figs. 1-4 that stable solute gradients have stabilizing effects whereas the medium permeability has a destabilizing effect on the system.

(c) It is observed that stable solute gradients introduce oscillatory modes in the system.

(d) The conditions $\frac{1}{\varepsilon} > \frac{\lambda \nu}{k_1}$, $\frac{E}{\kappa} > \frac{E'}{\kappa'}$ and $\frac{E}{\kappa} > \frac{E''}{\kappa''}$ are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily involve an occurrence of overstability.

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