

## Heat and Mass Transfer by MHD Power Law Fluid past a Porous Flat Plate with Heat generation, Chemical reaction and Diffusion-thermo effects.

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### ABSTRACT

In this investigation, the steady laminar boundary layer flow of a non-Newtonian power law fluid past a porous flat plate is considered. The Chemical reaction and diffusion-thermo effects along with heat generation have been studied. A suitable set of dimensionless variables is used and similar equations governing the flow are obtained. The resulting the non-linear ordinary differential equations are first linearised using Quasi-linearization technique and then an adequate implicit finite difference scheme is employed to obtain a numerical solution for the ordinary differential equations. The Representative results for the velocity, temperature and concentration profiles illustrating the influence of various governing parameters are presented and discussed.

**Keywords:** Non-Newtonian fluid, power law index, Heat source/sink parameter, Dufour number, Chemical reaction parameter, Finite Difference method.

Date of Submission: 24-07-2017

Date of acceptance: 05-08-2017

### I. INTRODUCTION

Many models consider only Newtonian fluids, for which strain rate is linearly related to applied shear stress. As non-Newtonian fluids classified as either pseudo-plastic or dilatants have become more common in industry, analysis of the boundary layer flow and heat transfer characteristics of these so called power-law fluids is now a priority. Dilatant fluids are liquids in which viscosity increases as the applied stress increases, whereas pseudo-plastic fluids are characterized by opposite relationship between viscosity and applied stress.

In the existing literature a masterpiece of work has been done on heat and mass transfer of a power-law fluid. Schowalter [1] was the first one, who studied the application of a boundary layer to power-law pseudo-plastic fluids. Laminar boundary layer flow of non-Newtonian power-law fluid past a porous flat plate is examined by Jadhav [2]. Magneto-hydrodynamic flow of a power-law fluid over a stretching sheet investigated by Andersson and Bech [3]. Prasad *et al.* [4] investigated the hydromagnetic flow and heat transfer of a non-Newtonian power-law fluid over a vertical stretching sheet. Recent additions considering magnetohydrodynamic flow were given Ghosh and Pop [5], Mahapatra *et al.* [6] and Shashidar *et al.* [7]. Ali [8, 9] investigated hydromagnetic flow and heat transfer over a non-isothermal permeable surface

stretching with a power-law velocity with heat generation and suction/injection effects and in the presence of a non-uniform transverse magnetic field and boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal and concentration buoyancy effects respectively.

The study of hydro magnetic convective non-Newtonian fluid flows with heat and mass transfer in porous medium attracted many research due to its applications in many fields like, soil physics, geophysics, aerodynamics and aeronautics. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. Many authors like Choudary and Das [10], and Reddy *et al.* [11], investigated an unsteady free convective MHD non-Newtonian flow through a porous medium bounded by an infinite inclined porous plate. MHD effects on non-Newtonian power-law fluid past a continuously moving porous flat plate with heat flux and viscous dissipation was analyzed by Shashidar Reddy and Kishan [12].

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature

gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. Mohammad Mehdi Rashidi et. al [13] examined Heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet with Soret and Dufour effects.

Srinivasa chary and swamy [14] studied the effect of chemical reaction and radiation on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium. Kabeir et.al [15] investigated the effects of radiation, chemical reaction, soret and dufour on heat and mass transfer by MHD stagnation point flow of a power-law fluid towards a stretching surface. Olajuwon [16] studied the effect of Thermo Diffusion and Chemical reaction on heat and mass transfer in a power-law fluid over a flat plate with heat generation. Recently Saritha et. al [17] investigated heat and mass transfer of laminar boundary layer flow of non-Newtonian power-law fluid past a porous flat plate with Soret and Dufour effects.

This study extends earlier works by examining the combined effect of heat generation, diffusion thermo effect and chemical reaction on the heat and mass transfer of a laminar boundary layer flow of a non-Newtonian power law fluid past a porous flat plate in the presence of magnetic field and chemical reaction. Thermal diffusion effect is neglected in this study because it is of a smaller order of magnitude than the magnitude of rate of chemical reaction which exerts a stronger effect on the mass flux.

## II. MATHEMATICAL ANALYSIS:

To construct the model, consider a steady, laminar, incompressible flow of a non-Newtonian power-law fluid past a semi infinite porous flat plate. The coordinate system is chosen such that X-axis is taken along the direction of the flow and y-axis normal to it. It is also assumed that a magnetic field with magnetic field intensity  $B_0$  is applied along the Y-axis and that the magnetic Reynolds number is very small so that the induced magnetic field can be neglected. In addition, Hall Effect and electric field are assumed to be negligible. The fluid properties are assumed to be constant. We assumed that  $T_w(x) = T_\infty + bx$  and  $C_w(x) = C_\infty + cx$ , where  $b$  and  $c$  are constants such that the uniform wall temperature  $T_w$  and concentration  $C_w$  are higher than those of their full stream values  $T_\infty, C_\infty$ . By considering the effects of Chemical reaction, Heat source/sink and Dufour, the governing equations for the flow in this investigation can be written as

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{K}{\rho} \frac{\partial}{\partial y} \left[ -\frac{\partial u}{\partial y} \right]^n - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \frac{Q}{\rho C_p} (T - T_\infty) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - K_1 (C - C_\infty) \quad (4)$$

In the foregoing equations,  $u$  and  $v$  are the velocity components along the X and Y axes,  $n$  is the power-law index,  $K$  is the power-law fluid parameter,  $\rho$  is density,  $\mu$  is the magnetic permeability,  $\sigma$  is the electrical conductivity of the fluid,  $\alpha = k/\rho c_p$  is the thermal diffusivity,  $k$  is the thermal conductivity,  $c_p$  is the specific heat at a constant pressure,  $k_T$  is the

thermal diffusion ratio,  $c_s$  is the concentration susceptibility,  $D_m$  is the coefficient of mass diffusivity,  $T$  is the temperature,  $Q$  is the heat generation constant,  $K_1$  is the reaction rate constant and  $C$  is the fluid concentration.

The boundary conditions associated with the present problem are as follows

$$u = 0, \quad v = V_w, \quad T = T_w(x) \quad \text{and} \quad C = C_w(x) \quad \text{at} \quad y = 0 \quad (5)$$

$$u = U, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as} \quad y \rightarrow \infty$$

Where  $V_w$  is suction velocity. To facilitate the analysis, we follow the previous studies Jadhav [2]

and Mohammad Mehdi Rashidi et al [13] and use the similarity variables

$$\left. \begin{aligned} \Psi(\eta) &= (\gamma x U^{2-n})^{\frac{1}{n+1}} f(\eta) \\ \eta &= y \left[ \frac{U^{2-n}}{\gamma x} \right]^{\frac{1}{n+1}} \\ \theta(\eta) &= \frac{T-T_{\infty}}{T_w-T_{\infty}} \\ \phi(\eta) &= \frac{C-C_{\infty}}{C_w-C_{\infty}} \end{aligned} \right\} \quad (6)$$

Where  $\Psi$  is the stream function defined in the usual way and  $f$  is the reduced stream function for the

flow. Then the velocity components are defined using the similarity variables as

$$\left. \begin{aligned} u &= \frac{\partial \Psi}{\partial x} = U f'(\eta) \\ v &= -\frac{\partial \Psi}{\partial y} = \frac{1}{n+1} \left( \gamma \frac{U^{2-n}}{x^n} \right)^{\frac{1}{n+1}} (\eta f' - f) \end{aligned} \right\} \quad (7)$$

Introducing equation (6) and (7), the continuity equation is satisfied and the momentum, energy and

concentration equations are transformed into ordinary differential equations as follows:

$$n(-f'')^{n-1} f''' + \frac{1}{n+1} f f'' - M f' = 0 \quad (8)$$

$$\theta'' + Pr \left( \frac{1}{n+1} f \theta' - f' \theta \right) + Du \phi'' + S \theta = 0 \quad (9)$$

$$\frac{1}{Le} \phi'' + Pr \left( \frac{1}{n+1} \phi' f - f' \phi \right) - \gamma \phi = 0 \quad (10)$$

Where primes denote indicate differentiation with respect to  $\eta$ .

The transformed boundary conditions in dimensionless form are

$$\left. \begin{aligned} f(0) &= f_w(\text{constant}) \\ f'(0) &= 0, \quad f'(\infty) = 1 \\ \theta(0) &= 1, \quad \theta(\infty) = 0 \\ \phi(0) &= 1, \quad \phi(\infty) = 0 \end{aligned} \right\} \quad (11)$$

Where  $f_w = -\frac{(n+1)}{U} V_w Re^{1/n+1}$  is the suction/injection parameter ( $f_w > 0$  for suction and  $f_w < 0$  for injection),

$M = \frac{\sigma B_0^2 x}{\rho U}$  is the Magnetic parameter,

$Re_x = \frac{U^{2-n} x^n}{\nu}$  is the Reynolds number,

$Pr = \frac{Ux}{\alpha} Re_x^{-\frac{1}{n+1}}$  is the Prandtl number,

$Le = \frac{\alpha}{D_m}$  is the Lewis number,

$\gamma = \frac{K_1 x^2}{\alpha} Re_x^{-\frac{2}{n+1}}$  is the Chemical reaction parameter,

$Du = \frac{D_m k_T}{c_s c_p} \cdot \frac{(C_w - C_{\infty})}{(T_w - T_{\infty}) \alpha}$  is the Dufour number,

$S = \frac{Qx}{U \rho C_p}$  is the Heat Source/Sink.

The physical quantities of engineering interest in this problem are the local Nusselt number and local Sherwood number, which are defined respectively by

$$Nu_x = \frac{q_w x}{k(T_w - T_{\infty})} = -\theta'(0) Re_x^{1/n+1} \quad \text{and} \quad Sh_x = \frac{J_w x}{D_m(C_w - C_{\infty})} = -\phi'(0) Re_x^{1/n+1}$$

Where the rate of heat transfer  $q_w$  and rate of mass transfer  $J_w$  are defined as

$$q_w = -k \left[ \frac{\partial T}{\partial y} \right]_{y=0} \quad \text{and} \quad J_w = -D_m \left[ \frac{\partial C}{\partial y} \right]_{y=0}$$

### III. NUMERICAL PROCEDURE

By using similarity transformation, the governing equations of the problem are reduced to a system of coupled, non linear ordinary differential equations, which are solved numerically. The numerical solutions can be obtained in the following steps:

- Linearize Eq (8) using Quasi Linearization method [18].
- Write the difference equations using implicit finite difference scheme.
- Linearize the algebraic equations by Newton's method, and express them in matrix-vector form and

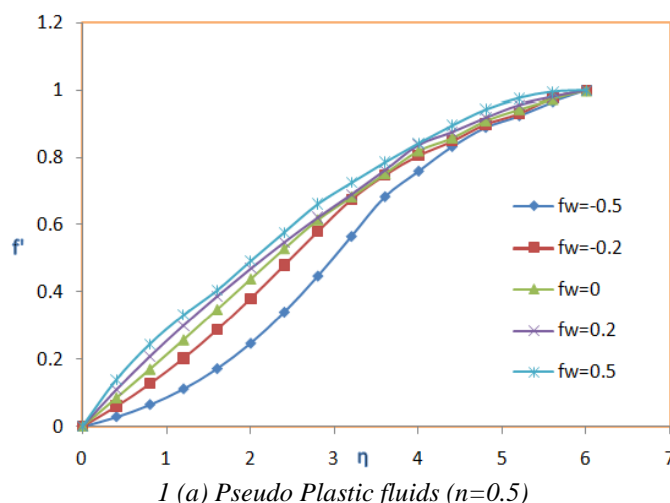
- Solve the linear system by Gauss Seidal Iteration method.

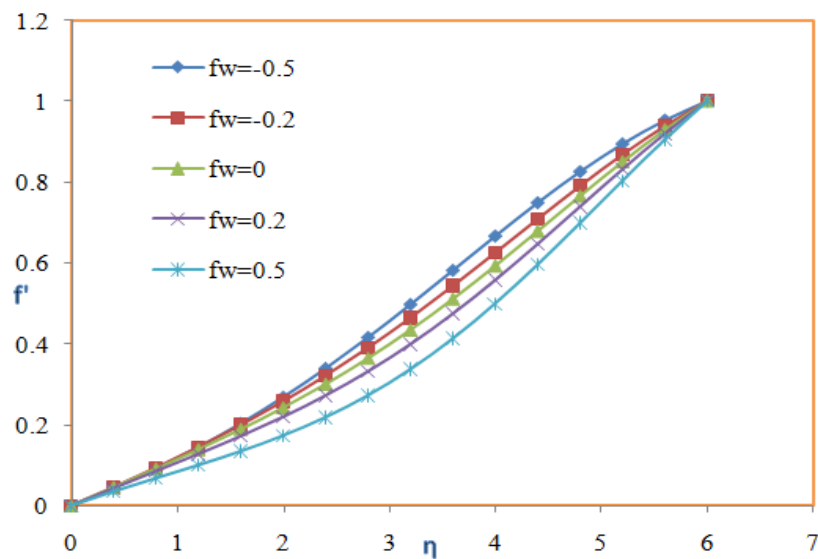
Since the equations governing the flow are nonlinear, iteration procedure is followed. For the sake of brevity, further details of the solution process are not presented here. The numerical computations were carried out C programming. It is also important to note that the computational time for each set of parametric values should be short. The physical domain of the problem is unbounded, whereas the computational domain is finite. The numerical solutions of  $f$  are considered as  $(n+1)^{\text{th}}$  order iterative solutions and  $F$  are the  $n^{\text{th}}$  order iterative solutions. To prove convergence of finite difference scheme, the computation is carried out for slightly changed value of  $h$  by running same program. No significant change was observed in the value. The convergence criterion used in this study is that the maximum change between the current and the previous iteration values in all the dependent variables satisfy  $10^{-5}$ .

#### IV. RESULTS AND DISCUSSION

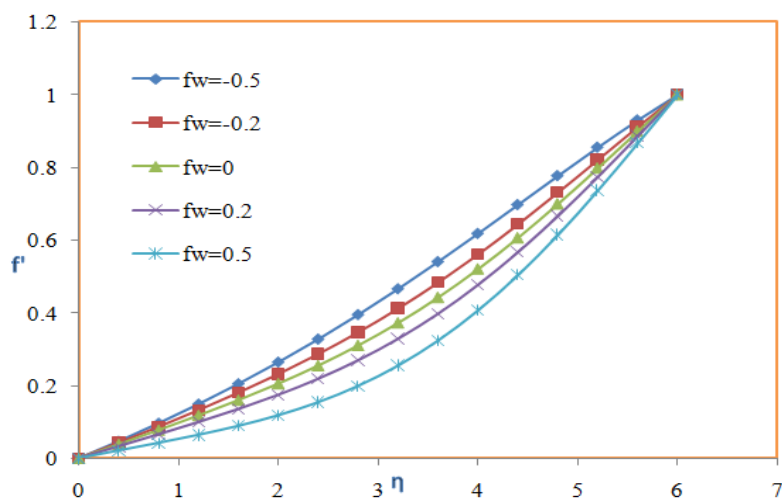
In this section, by applying the numerical values to different flow parameters, the effects on velocity, temperature and concentration fields are discussed. Figure (1) is to show the influence of suction/injection on velocity distribution. It is obvious from the graph that the effect of suction ( $f_w > 0$ ) is to enhance the velocity for Pseudo plastic fluid whereas to decrease the velocity for Newtonian and Dilatant fluids and the effect of injection ( $f_w < 0$ ) is to slow down the velocity for Pseudo plastic fluid but to speed up the velocity for Newtonian and Dilatant fluids. Figure (2) is drawn for temperature profiles for different values of Prandtl number the cases of Newtonian and non-Newtonian fluids. The effect of

Prandtl number is to reduce the temperature for both Newtonian and non-Newtonian fluids. Physically, fluids with smaller Prandtl number have larger thermal diffusivity. Figure (3) & (4) depicts the variation of Velocity profiles and Temperature profiles for Newtonian and non-Newtonian fluids with different values of power-law index. The effect of power-law index is to reduce the velocity and enhance the temperature distributions. The effect of heat generation on temperature is presented in figure 5. It is clear from the figure that temperature rises with heat generation and it falls with heat absorption. Hence the thermal boundary layer thickness reduces with heat absorption but it increases with heat generation which is very much significant for the flow where heat transfer is given prime importance. The heat transfer caused by concentration gradient is called the diffusion thermo or Dufour effect. The effect of diffusion thermo on temperature for Newtonian and non-Newtonian fluids are plotted in Figure 6. Thermal diffusion effect is neglected in this study because it is of a smaller order of magnitude than the magnitude of rate of chemical reaction which exerts a stronger effect on the mass flux. It is evident from the figures that the increase in Dufour number increases the temperature distribution. The effect of Chemical reaction parameter on the concentration field is shown in figure (7). It can be noticed from the graphical presentation that the rise in the chemical reaction parameter will suppress the concentration of the fluid. This is due to the fact that the higher values of  $\gamma$  results in fall of chemical molecular diffusivity and therefore species concentration is suppressed. Hence the concentration distribution decreases at all points of the flow field with the increase in the reaction parameter.



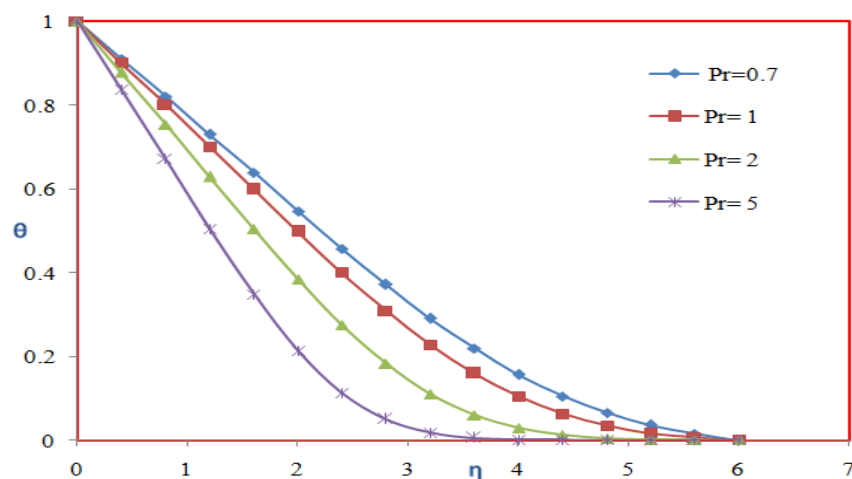


1 (b) Newtonian fluids ( $n=1.0$ )

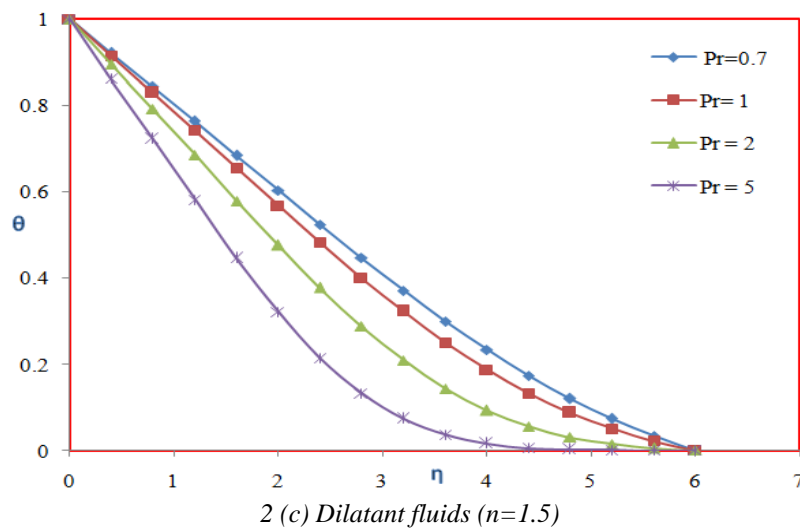
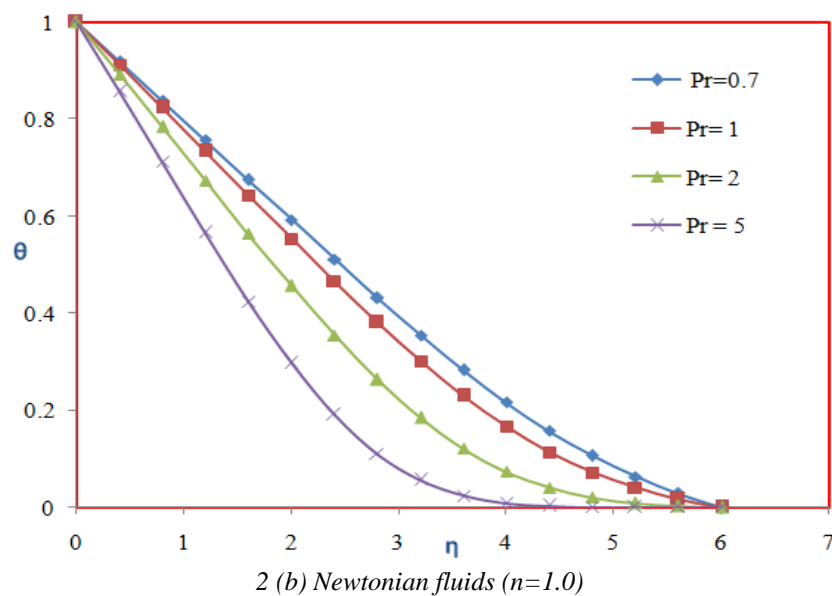


1 (c) Dilatant fluids ( $n=1.5$ )

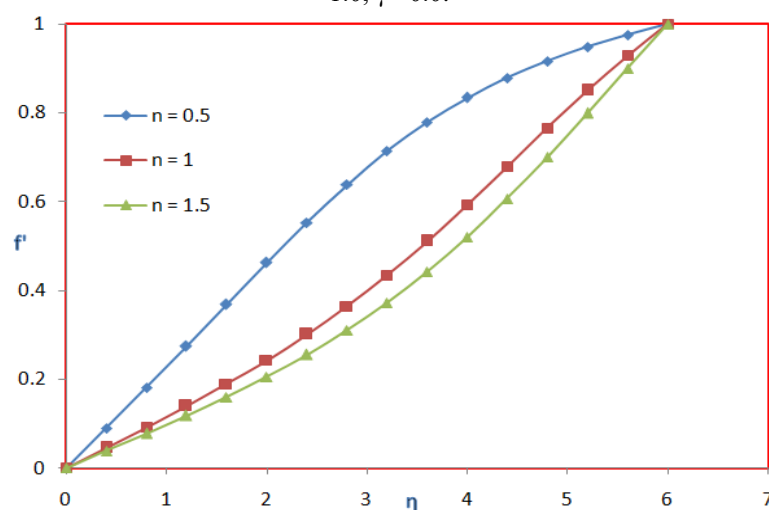
**Figure 2:** Velocity profiles for various values of Suction/injection parameter with  $Pr = 1.0$ ,  $M=0.0$ ,  $S=0$ ,  $Du = 0.08$ ,  $Le = 1.0$ ,  $\gamma = 0.0$ .



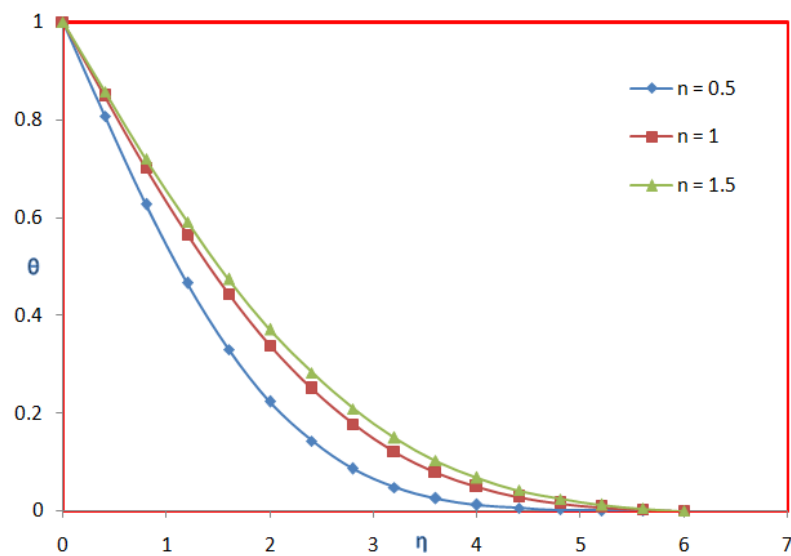
2 (a) Pseudo Plastic fluids ( $n=0.5$ )



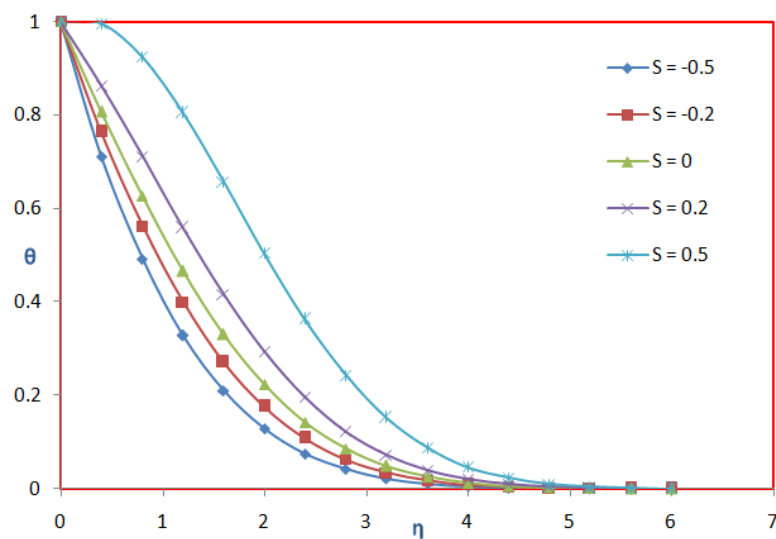
**Figure 2:** Temperature profiles for various values of Prandtl number with  $M=0.1$ ,  $f_w=0.0$ ,  $S=0$ ,  $Du=0.08$ ,  $Le = 1.0$ ,  $\gamma=0.0$ .



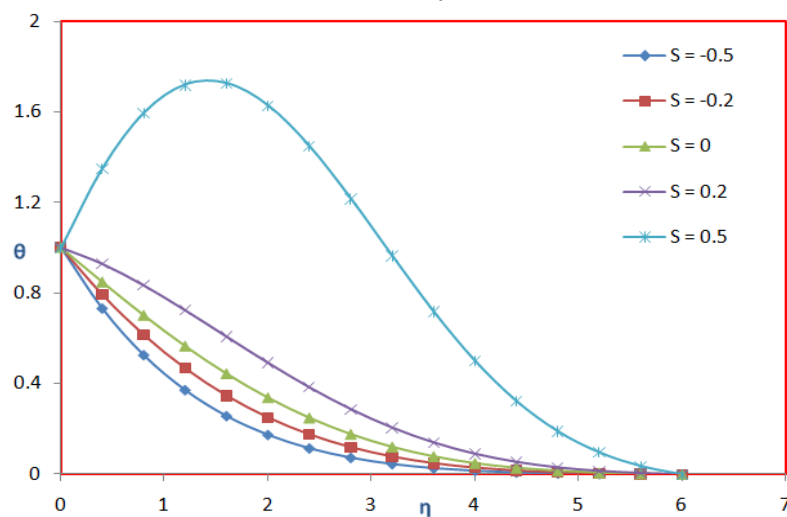
**Figure 3:** Velocity profiles for various values of power law index with  $M = 0.1$ ,  $Pr=1.0$ ,  $f_w=0.0$ ,  $S=0$ ,  $Du = 0.08$ ,  $Le = 1.0$ ,  $\gamma=0.0$ .



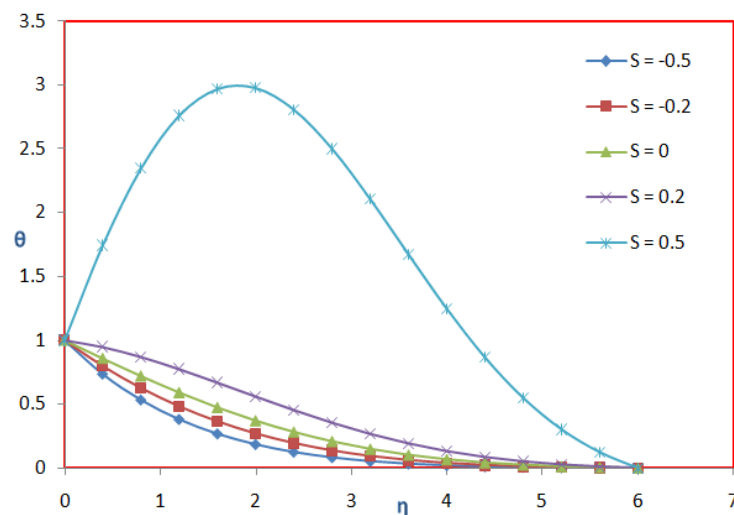
**Figure 4:** Temperature profiles for various values of power law index with  $M = 0.1$ ,  $Pr = 1.0$ ,  $f_w=0.0$ ,  $S=0$ ,  $Du = 0.08$ ,  $\gamma=0.0$ .



5 (a) Pseudo Plastic fluids ( $n=0.5$ )

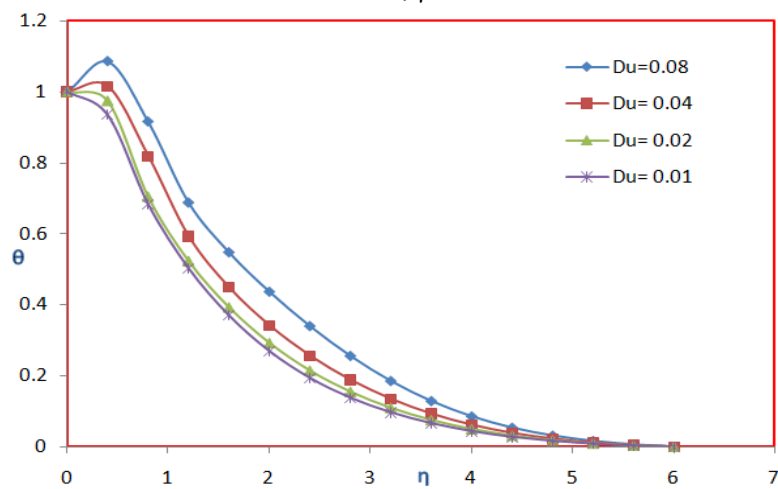


5 (b) Newtonian fluids ( $n=1.0$ )

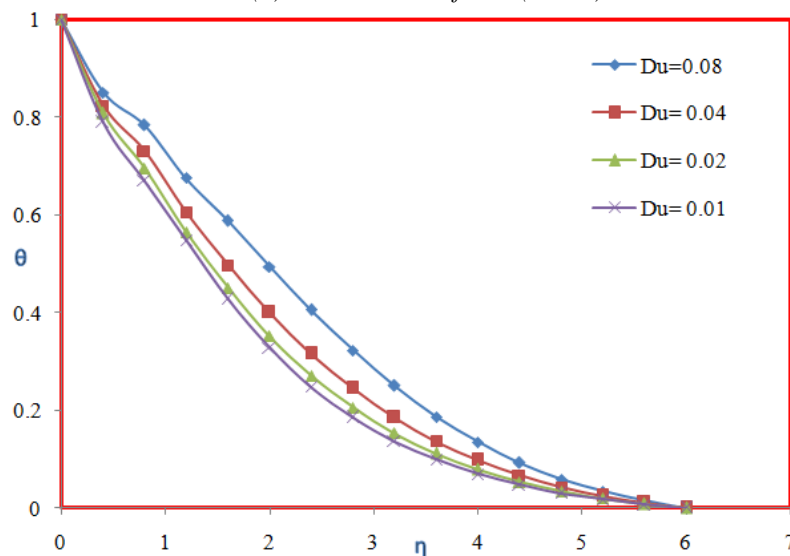


5 (c) Dilatant fluids ( $n=1.5$ )

**Figure 5:** Temperature profiles for various values of Heat Source/Sink parameter with  $M = 0.1$ ,  $Pr=1.0$ ,  $f_w=0.0$ ,  $Du = 0.08$ ,  $\gamma=0.0$ .

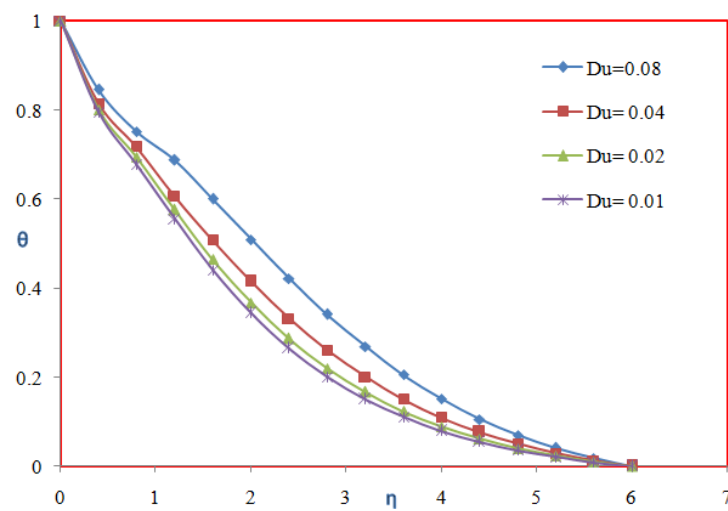


6 (a) Pseudo Plastic fluids ( $n=0.5$ )



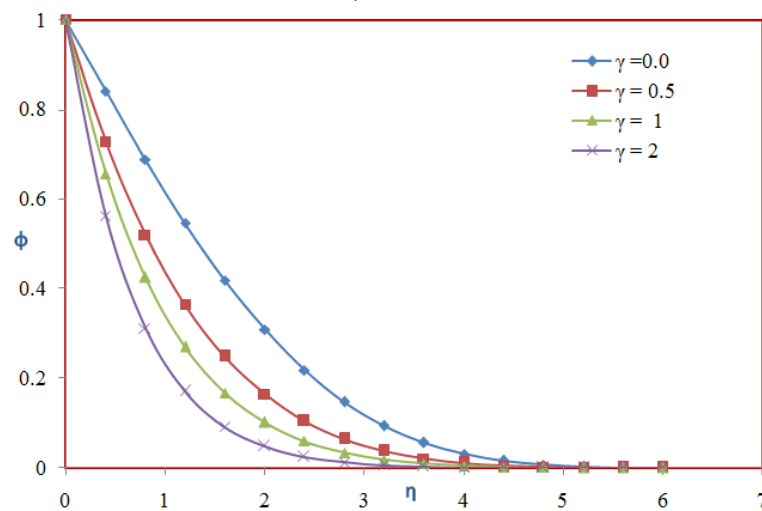
6 (b) Newtonian fluids ( $n=1.0$ )



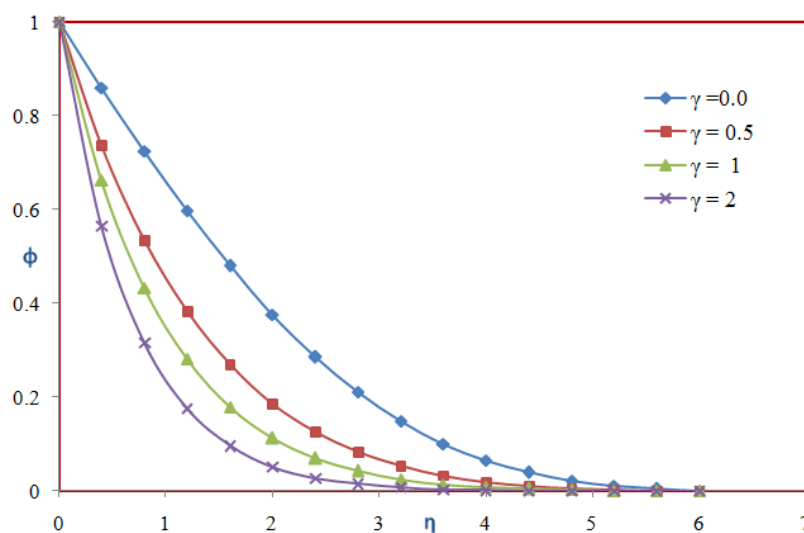


6 (c) Dilatant fluids ( $n=1.5$ )

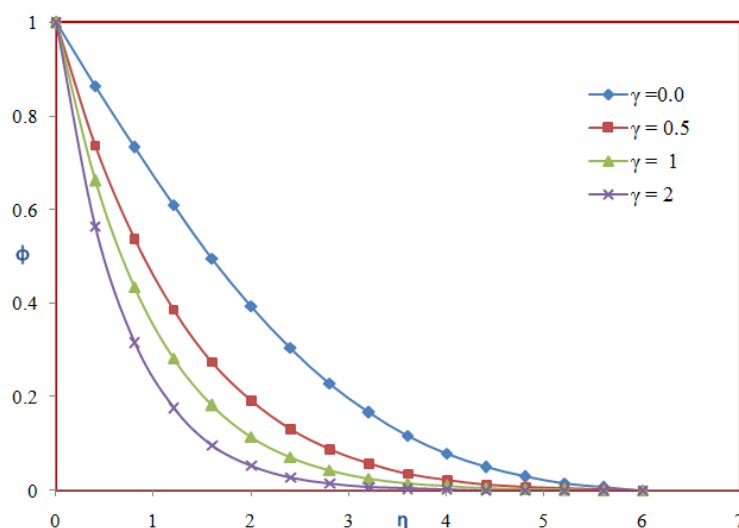
**Figure 6:** Temperature profiles for various values of Dufour numbers with  $M=0.1, Pr=1.0, Le=1.0, f_w=0.0, S=0, \gamma=0.0$ .



7 (a) Pseudo Plastic fluids ( $n=0.5$ )



7. (b) Newtonian fluids ( $n=1.0$ )



7. (c) Dilatant fluids ( $n=1.5$ )

**Figure 7:** Concentration profiles for various values of Chemical reaction parameter with  $M=0.1$ ,  $Pr=1.0$ ,  $Le=1.0$ ,  $f_w=0.0$ ,  $S = 0.0$ ,  $Du = 0.08$ .

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International Journal of Engineering Research and Applications (IJERA) is **UGC approved** Journal with Sl. No. 4525, Journal no. 47088. Indexed in Cross Ref, Index Copernicus (ICV 80.82), NASA, Ads, Researcher Id Thomson Reuters, DOAJ.

Shashidar Reddy B. "Heat and Mass Transfer by MHD Power Law Fluid past a Porous Flat Plate with Heat generation, Chemical reaction and Diffusion-thermo effects." *International Journal of Engineering Research and Applications (IJERA)* 7.8 (2017): 31-41.