

A Simple Analytical Model for Estimating Long-term Productivity of Multi-fractured Shale Gas Wells

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ABSTRACT

Shale gas production has been successfully developed over the past several years. The horizontal well with multi-stage hydraulic fracturing has proven to be an effective operation of development these unconventional reservoirs. Most of modeling effort has been made for earlier linear flow. A simple analytical model for long-term shale gas production evaluation is still lacking. This work fills this gap. A simple and accurate mathematical model is developed which considers linear flow in both shale matrix and fracture. It can describe the long-term shale gas production with pseudo steady state flow. The model is verified against a Fayetteville field case data. The difference between the production rate given by the model and measured data was found to be less than 10%. The effect of fracture geometry parameter on long-term production rate is also investigated. The results show that fracture spacing and fracture length are the most dominant factors on the long-term shale gas production performance. This model provides reservoir engineers a simple and accurate tool for predicting, evaluating and optimizing the long-term performance of shale gas wells.

I. INTRODUCTION

The combination of horizontal drilling and multi-stage hydraulic fracturing technology has made possible the current gas production from shale gas reservoirs in the United States, as well as the global fast growing investment in shale gas exploration and development. Shale-gas reservoirs are organic-rich formations, varying from one shale formation to another, even within the formation itself, and they serve as both reservoir and source rock. Gas in the shale is mainly composed of free gas in natural fractures and matrix pore structure and adsorbed gas on the surface of shale matrix and in organic materials. The uncertainties of reservoir and fracture parameters have significant effect on shale-gas production.

There have been a significant number of attempts in recent years to evaluate numerically the well performance for earlier-time transient flow in unconventional gas reservoirs (Britt and Smith 2009; Fan et al. 2010; Cipolla et al. 2010; Du et al. 2009; Li et al. 2011; Dahaghi and Mohaghegh. 2011; Novlesky et al. 2011; Yu et al. 2014; Mattar et al. 2008; Freeman et al. 2009). However, to the best of our knowledge, few of analytical or semi-analytical

models consider long-term pseudo steady state flow shale gas production (Fig. 1). Raghavan (1993) developed an analytical model to evaluate the well production of horizontal wells with multiple fractures. But the flow in the fractures was not considered. Guo and Schechter (1997) presented an analytical model coupling reservoir linear flow and fracture linear flow. But that model only considers a single fracture case. Yuan and Zhou (2010) proposed a simple model to predict inflow performance in fractured horizontal wells which is applicable for both fractured wells and non-fractured wells. But it is only valid for steady state flow condition. Vargas et al (2015) extended Guo's model to predict transient pressure behavior with variable flow-rate.

This paper presents a simple model for predicting gas production of multi-fractured horizontal wells at pseudo steady state condition. It provides reservoir engineers a simple and accurate tool for predicting, evaluating and optimizing the long-term performance of shale gas wells. A good agreement was observed between the actual production rate and predicted one for Fayetteville shale well. The effect of fracture geometry parameter on gas production has been analyzed.

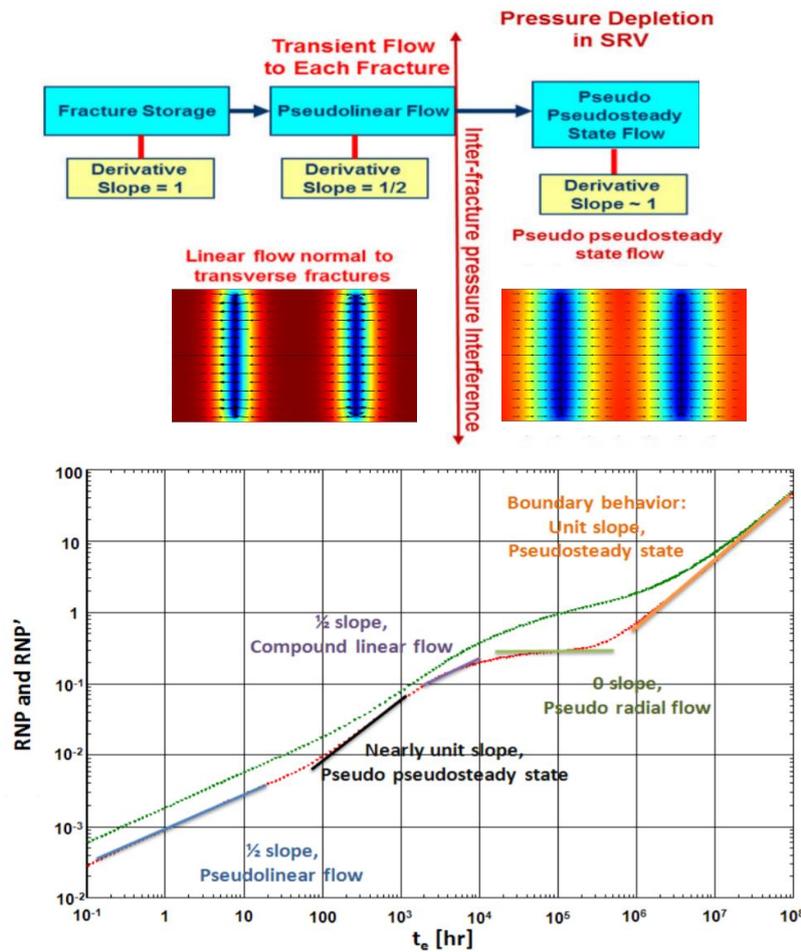


Fig. 1 Potential flow regimes during shale gas production (Song, 2011)

Model Development

The analytical model considering liner flow in the fracture and matrix. Several assumptions are stated as follows:

- 1) The wellbore-flow modeling and fracture skin is not included in this paper
- 2) The shale reservoir is homogeneous and isotropic with uniform thickness and constant porosity.

- 3) Fracture are identical and fully penetrate the reservoir thickness.
- 4) Gravity effect is negligible.
- 5) Single phase gas flow is considered.

The analytical solution of well inflow equation for pseudo steady state flow can be expressed as follows:

$$Q_g = \frac{5.87 \times 10^{-5} n_f k_m h (\bar{p}^2 - p_w^2)}{\mu T S_f \sqrt{c} \left[\frac{1}{1 - e^{-\sqrt{c} x_f}} - \frac{1}{3 x_f \sqrt{c}} \right]} \quad (1)$$

$$c = \frac{96 k_m}{k_f w S_f} \quad (2)$$

Where Q_g is the gas production rate, Mscf/d; n_f is the number of fracture, dimensionless; k_m is the matrix permeability, md; h is the pay zone thickness, ft; \bar{p} is the average formation pressure in psia. p_w is wellbore pressure, psia; μ is gas viscosity. T is the formation temperature, R; S_f is fracture spacing, ft; x_f is the fracture half length.

Methodology Verification

A field cases (Song, 2011) were performed to verify the accuracy and applicability of the proposed analytical model. The basic model parameters for all cases are summarized in **Table 1**. Song (2011) presented a rate-normalized pressure (RNP) method to quantify the flow region, shale permeability, average half-length and effective stimulated reservoir volume (SRV) based on

production data. In the case of Fayetteville shale, the well was stimulated by 72 stages fracturing with a single, perforated interval for each stage. The well

production rate declines form 2500 Mscf/d to 50 Mscf/d over a 1.8 years production period.

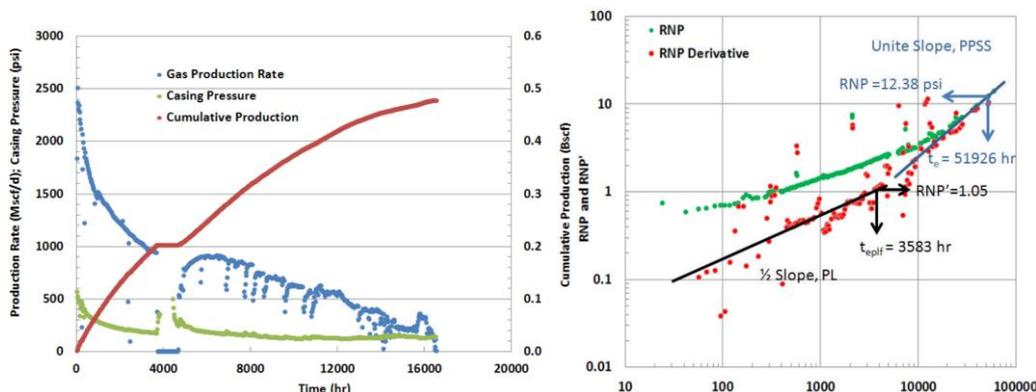


Fig. 2—Fayetteville shale well production and RNP data (Song. 2011)

Based on the well RNP figure (Fig. 2above), a unit slop indicates a pseudo steady state flow period occurs at about 1.7-1.8 years. By using RNP analysis, the fracture half-length is 64ft, Matrix permeability is 2.2e-4 md, effective SRV is 7.6e6 ft³. The final bottom hole pressure is measured about 150 psi. The gas viscosity can be calculated based on average

reservoir pressure (150+1050)/2=600 psia. Based on Guo’sCarr-Kobayashi-Burrows correlation program (2017), the gas viscosity of 0.01159cp can be calculated at 125 °F and 600 psia. Based on mass balanced rule, the average reservoir pressure can be estimated by:

$$\frac{G_p}{G_i} = 1 - \frac{\bar{p} / z}{p_i / z_i} \tag{3}$$

Thus,

$$\bar{p} / z = (1 - G_p / G_i) / (p_i / z_i) = (1 - 0.48 / 1.78)(1050 / 0.86) = 890 \tag{4}$$

Based on Hall-Yarborough correction (Guo. 2017), the average pressure can be calculated in 765 psi. The

theoretical fracture permeability can be estimated using cubic law (Kim and Moridis, 2015):

$$k_f = \alpha_c \frac{h^2}{12} \tag{5}$$

Where, C is the correction factor considering fracture roughness, 0.14 used in this paper. Assuming the fracture width is 0.05 inch. All of parameters are summarized in Table. 1. Substituting these data into Eqs. 1-2 yields the gas production rate of 46.15

Mscf/day. As shown in Fig. 2, thediscrepancy between predicted value (66.01) and measured value (60-70) is within 10%.

Table 1—Reservoir and fluid flow properties (Song. 2011)

Property	Unit	Value
Horizontal well length, L	ft	3528
Initial reservoir pressure, P _i	psia	1050
Average formation pressure, p _{bar}	psia	765
Reservoir thickness, h	ft	322
Reservoir temperature, T	R	580
Gas viscosity, □	cp	0.01159
Bottom hole pressure, p _w	psia	150
Matrix permeability, k _m	md	2.2e-4
Total number of fractures, n _f	-	72
Effective fracture spacing, s _f	ft	49
Fracture half-length, x _f	ft	90
Fracture width, w	in	0.05
Fracture permeability, k _f	md	18817

Sensitivity Study

For hydraulic fracture operation, the fracture permeability is dependent on fracture width (Eq. 5) and the number of fractures is dependent on fracture spacing. Thus, three key operational parameters were selected as design variables in this study: fracture spacing, S_f , fracture width, w , and fracture half-

length, x_f . The RSM (response surface method) optimization method are used in evaluating the effect of operational these parameters on shale gas production. The RSM design variables and test run can be set up as in **Table 2** and **Table 3**(Box and Draper. 1987; Anderson and Whitcomb. 2004).

Table 2 Factors and coded levels for RSM

Factors	S_f -X1 /ft	w-X2 /in	x_f -X3 /ft
+Alpha level (1.682)	250	0.2	400
High level (1)	209.46	0.16	329.46
Levels Zero level (0)	150	0.11	225
Low level (-1)	90.54	0.05	120.94
-Alpha level (-1.682)	50	0.01	50

The response surface diagrams and sensitive analysis are presented in **Figs. 3-5**. The gas production rate increase with increase of fracture width and fracture half length, but increase with decrease of fracture spacing. Based on calculated sensitivities of design variables, the fracture spacing has the most dominant effect on productivity, followed by the fracture half length. The effect of fracture width (as well as fracture permeability in Eq. 5) on gas production is negligible.

Table 3 RSM test scheme

Test No.	Fracture Geometry parameters* (Actual-Coded factors)			Predicted Production Mscf/d
	S_f -X1 /ft	w-X2 /in	x_f -X3 /ft	
1	209.46 (1)	0.16 (1)	329.46 (1)	13.62
2	209.46 (1)	0.16 (1)	120.94 (-1)	5.05
3	209.46 (1)	0.05 (-1)	329.46 (1)	12.77
4	209.46 (1)	0.05 (-1)	120.94 (-1)	4.93
5	90.54 (-1)	0.16 (1)	329.46 (1)	72.39
6	90.54 (-1)	0.16 (1)	120.94 (-1)	26.97
7	90.54 (-1)	0.05 (-1)	329.46 (1)	65.66
8	90.54 (-1)	0.05 (-1)	120.94 (-1)	25.99
9	250 (1.682)	0.11 (0)	225 (0)	6.53
10	50 (-1.682)	0.11 (0)	225 (0)	160.21
11	150 (0)	0.2 (1.682)	225 (0)	18.27
12	150 (0)	0.11 (-1.682)	225 (0)	18.06
13	150 (0)	0.11 (0)	400 (1.682)	31.62
14	150 (0)	0.11 (0)	50 (-1.682)	4.08
15	150 (0)	0.11 (0)	225 (0)	18.01

*Other parameters are the same with Table. 1.

L_1 and L_2 depend on and vary with design parameters α , β and R.

Fig. 6 presents the effect of fracture spacing of gas production. It shows a denser fracture distribution will drastically increase the gas production (nearly 800% in this case). Therefore, increasing the number of fracture as well as the density of fracture is an effective way to maximize the gas production rate. **Fig. 7** shows the effect of fracture half-length on the gas production. A 400% production improvement can be achieved by increasing the fracture length, which implies creating a longer fracture by pumping a large volume of fluid

is also a viable approach to improve the gas production. **Fig. 8** shows the effect of fracture width (as well as fracture permeability) on gas production. Only 50% production improvement can be obtained by increasing the fracture width. Thus, there is no need to increase the investment on increasing the fracture width, like more powerful pump for fast pumping operation. Thus, the same results of single variable analysis are obtained against the RSM results.

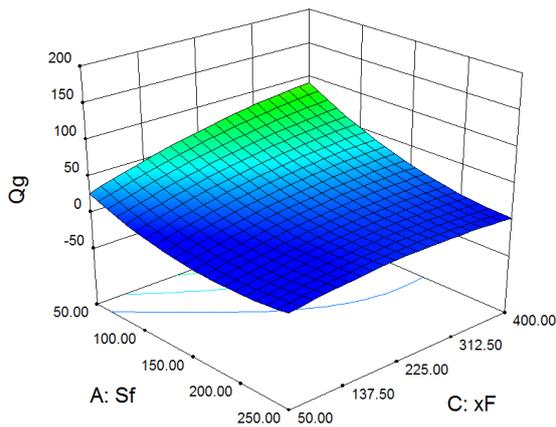


Fig. 3—Response surface for production rate as S_f and x_f vary as S_f and w vary ($w=0.11$ inch)

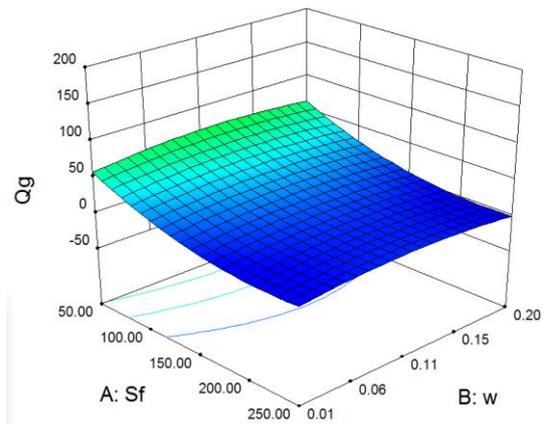


Fig. 4—Response surface for production rate ($x_f=225$ ft)

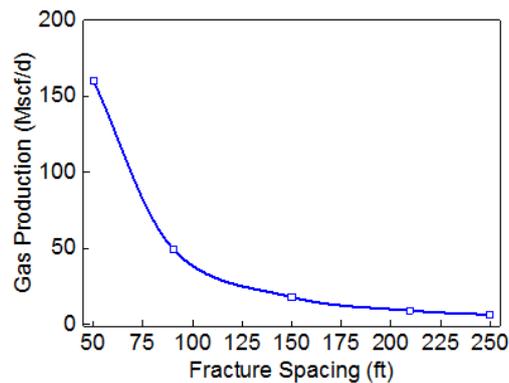
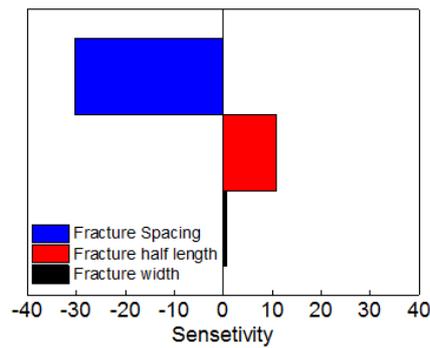
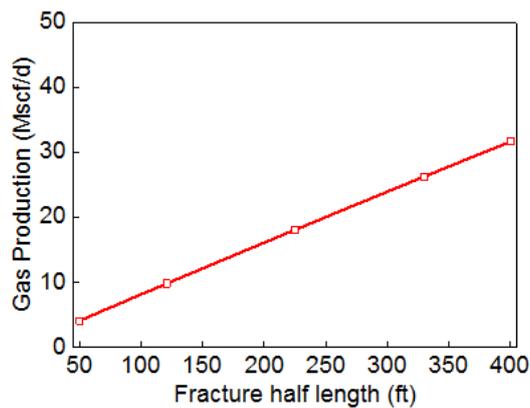


Fig. 5—Sensitivity analysis for fracture geometry parameters

Fig. 6—Effect of fracture spacing on gas production rate



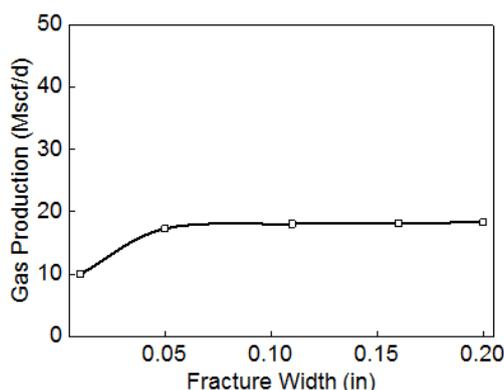


Fig. 7—Effect of fracture half-length on gas production rate Fig. 8—Effect of fracture width on gas production rate

II. CONCLUSIONS

Based on rigorous derivation, a simple and accurate model has been developed for estimating the long-term shale gas production. Several conclusions can be drawn accordingly:

- Case studies on a Fayetteville shale gas well indicates the difference between the production rates given by the model and field data are less than 10%. The proposed is accurate enough for engineering application.
- Based on the sensitivity analysis of fracturing controlling factors, the fracture spacing has the most dominant effect on shale gas production rate, followed by the fracture half length. But the effect of fracture width on gas production is negligible. Thus, increasing the number of fracture and the volume of fracturing fluid are the effective ways in improving the production.
- This model provides reservoir engineers a simple and accurate tool for predicting, evaluating and optimizing the long-term performance of shale gas wells.

III. ACKNOWLEDGMENTS

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Nomenclature

- c_t = total compressibility, ft
 h = pay zone thickness, ft
 k_m = matrix permeability, md
 k_f = fracture permeability, md
 n_f = number of fractures
 p_w = bottom hole pressure, psi
 \bar{p} = reservoir average pressure, psi
 S_f = fracture spacing, ft
 T = formation temperature, R
 x_f = fracture half length, ft
 w = fracture width, inch

ϕ = porosity, inch

μ = fluid viscosity, cp

Q_g = shale gas production rate, Mscf/D

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expressed as follows using the assumptions of constant flow rate and compressibility (Dake, 1978):

$$\frac{\partial p}{\partial t} = -\frac{q}{c_f V} = -\frac{q(x)}{c_f (S_f / 2) h \phi \Delta x} \quad (\text{A-3})$$

Substituting Eq. A-3 into Eq. A-1 gives:

$$\frac{\partial}{\partial y} \left(\frac{\partial p}{\partial y} \right) = -\frac{q(x)}{c_f (S_f / 2) h \phi \Delta x} \frac{\mu \phi c_f}{k_m} = -\frac{2q(x)\mu}{k_m S_f \Delta x h} \quad (\text{A-4})$$

Integrating Eq. A-4 gives:

$$\frac{\partial p}{\partial y} = -\frac{2q(x)\mu}{k_m S_f \Delta x h} y + C_1 \quad (\text{A-5})$$

Applying boundary condition 1 (BC1) on Eq. 5 gives:

$$\frac{\partial p}{\partial y} = \frac{q(x)\mu}{k_m \Delta x h} - \frac{2q(x)\mu}{k_m S_f \Delta x h} y = \frac{q(x)\mu}{k_m \Delta x h} \left(1 - \frac{2}{S_f} y \right) \quad (\text{A-6})$$

Integrating Eq. A-6 gives:

$$p = \frac{q(x)\mu}{k_m \Delta x h} \left(y - \frac{y^2}{S_f} \right) + C_2 \quad (\text{A-7})$$

Applying boundary condition 2 (BC2) on Eq. A-7 gives:

$$p(x, y) = \frac{q(x)\mu}{k_m \Delta x h} \left(y - \frac{y^2}{S_f} \right) + p_f(x) \quad (\text{A-8})$$

The boundary pressure $p_e(x, S_f/2)$ can be calculated as follows:

$$p_e = \frac{q(x)\mu}{k_m \Delta x h} \left(\frac{S_f}{2} - \frac{(S_f / 2)^2}{S_f} \right) + p_f(x) = \frac{q(x)\mu S_f}{4k_m \Delta x h} + p_f(x) \quad (\text{A-9})$$

Thus, the flux (q) from matrix to fracture at a point (x) in the fracture can be obtained by rearranging Eq. A-9:

$$q(x) = \frac{4k_m \Delta x h}{\mu S_f} [p_e - p_f(x)] \quad (\text{A-10})$$

The velocity (v) at a point (x) in the fracture can be calculated as follows (Fig. A-1):

$$v_m(x) = \frac{q(x)}{h \Delta x} = \frac{4k_m}{\mu S_f} [p_e - p_f(x)] \quad (\text{A-11})$$

Therefore, the flow rate for two wings at a point(x) in the fracture can be calculated as follows:

$$Q_m(x) = 2 \int_x^{x_f} \frac{4k_m}{\mu S_f} [p_e - p_f(x)] \cdot h dx \quad (\text{A-12})$$

Also, the pressure at matrix in terms of boundary pressure (p_e) can be expressed by substituting Eq. A-10 into A-8:

$$p(x, y) = \frac{4}{S_f} [p_e - p_f(x)] \left(y - \frac{y^2}{S_f} \right) + p_f(x) \quad (\text{A-13})$$

Fracture linear flow

For linear fracture flow, the fluid velocity at the fracture should be the same with the velocity in the matrix:

$$v_f(x) = \frac{Q_m(x)}{wh} = -\frac{k_f}{\mu} \frac{dp_f(x)}{dx} \quad (\text{A-14})$$

Substituting Eq. A-12 into A-14 gives:

$$\frac{8k_m h}{\mu S_f} \cdot \frac{1}{wh} \int_x^{x_f} [p_e - p_f(x)] dx = -\frac{k_f}{\mu} \frac{dp_f(x)}{dx} \quad (\text{A-15})$$

Taking derivative with respect to x for Eq. A-15 gives:

$$\frac{8k_m}{S_f w k_f} [p_e - p_f(x)] = \frac{d^2 p_f(x)}{dx^2} \quad (A-16)$$

The equation can be further simplified by:

$$c = \frac{8k_m}{S_f w k_f}, \quad p_d(x) = p_e - p_f(x) \quad (A-17)$$

Thus, Eq. A-15 can be expressed as:

$$\frac{d^2 p_d(x)}{dx^2} = c p_d(x) \quad (A-18)$$

$$BC1: \left. \frac{dp_d}{dx} \right|_{p_d=0} = 0 \quad BC2: p_d|_{x=x_f} = p_e - p_w \quad (A-19)$$

Based on Guo and Schechter's derivation (1997), the solution of Eq. A-18 can be expressed as follows:

$$p_f(x) = p_e - (p_e - p_w) e^{\sqrt{c}(x-x_f)} \quad (A-20)$$

Multi-fractured horizontal gas well inflow equation

Substituting Eq. A-20 into Eq. A-13, the pressure distribution can be calculated as follows:

$$p(x, y) = p_e - (p_e - p_w) e^{\sqrt{c}(x-x_f)} + \frac{4}{S_f} (p_e - p_w) e^{\sqrt{c}(x-x_f)} \left(y - \frac{y^2}{S_f} \right) \quad (A-21)$$

Substituting Eq. A-20 into Eq. A-12 and integrating from 0 to x_f , the production rate for this fracture can be calculated as follows:

$$Q = \frac{16k_m h}{\mu S_f \sqrt{c}} (p_e - p_w) (1 - e^{x_f \sqrt{c}}) \quad (A-22)$$

The average pressure can be calculated as follows:

$$\bar{p} = \frac{\int p dv}{\int dv} = \frac{\phi h \int_0^{x_f} \int_0^{S_f/2} p(x, z) dz dx}{\phi h x_f (S_f / 2)} \quad (A-23)$$

Substituting Eq. A-21 into Eq. A-22 gives:

$$\bar{p} = p_e - \frac{p_e - p_w}{3x_f \sqrt{c}} (1 - e^{-x_f \sqrt{c}}) \quad (A-24)$$

Solving $P_e - P_w$ from Eq. A-22 and substituting the results into Eq. A-24 gives the inflow performance in terms of average pressure:

$$Q = \frac{16k_m h (\bar{p} - p_w)}{\mu S_f \sqrt{c} \left[\frac{1}{1 - e^{-\sqrt{c}x_f}} - \frac{1}{3x_f \sqrt{c}} \right]} \quad (A-25)$$

For multi-fractured (n_f) horizontal well, the general inflow performance can be expressed as follows:

$$Q = \frac{16n_f k_m h (\bar{p} - p_w)}{\mu S_f \sqrt{c} \left[\frac{1}{1 - e^{-\sqrt{c}x_f}} - \frac{1}{3x_f \sqrt{c}} \right]}, \quad c = \frac{8k_m}{S_f w k_f} \quad (A-26)$$

For gas well, the gas production rate in standard conditions and field units can be expressed as follows:

$$Q_g = \frac{T_0}{p_0} \frac{p}{T} Q = \frac{T_0}{p_0} \frac{\bar{p} + p_{wf}}{2} \frac{Q}{T} \quad (A-27)$$

Substituting Eq. A-27 into Eq. A-26 gives:

$$\begin{aligned}
 Q_g &= \frac{T_0}{p_0} \frac{p}{T} Q = \frac{T_0}{2p_0} \frac{\bar{p} + p_{wf}}{T} \frac{16n_f k_m h (\bar{p} - p_w)}{\mu S_f \sqrt{c} \left[\frac{1}{1 - e^{-\sqrt{c}x_f}} - \frac{1}{3x_f \sqrt{c}} \right]} \\
 &= \frac{5.87 \times 10^{-5} n_f k_m h (\bar{p}^2 - p_w^2)}{\mu T S_f \sqrt{c} \left[\frac{1}{1 - e^{-\sqrt{c}x_f}} - \frac{1}{3x_f \sqrt{c}} \right]} \quad \text{(A-28)}
 \end{aligned}$$

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