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Skolem Mean Labeling Of Five Star Graphs $K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,a_4} \cup K_{1,b}$ where $a_1 + a_2 + a_3 + a_4 - 4 \le b \le a_1 + a_2 + a_3 + a_4 - 3$

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ABSTRACT
A graph G = (V, E) with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to {1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by $f^*(e = uv) = \frac{f(u) + f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u) + f(v) + 1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from the set {2, 3, . . . , p}. In this paper, we prove that five star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,a_4} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1 + a_2 + a_3 + a_4 - 4 \le b \le a_1 + a_2 + a_3 + a_4 - 3$.

Keywords: **S**kolem mean graph, **s**kolem mean labeling, star graphs

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I. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [3]. The symbols V(G) and E(G) will denote the vertex set and edge set of the graph G. A graph with p vertices and q edges is called a (p, q) graph. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1 + a_2 + a_3 + 2 \le b \le a_1 + a_2 + a_3 + 3$.

II. SKOLEM MEAN LABELING

Definition 1.1: A graph G is a non empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) respectively. $|V(G)| = q$ is called the size of G, we say that u and v are adjacent and that u and v are incident with e.

Definition 1.2: A vertex labelling of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y). Similarly, an edge labelling of a graph G is an assignment of labels to the edges of G that induces for each vertex v a label depending on the edge labels incident on it. Total labelling involves a function from the vertices and edges to some set of labels.

Definition 1.3: A graph G with p vertices and q edges is called a mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0, 1, 2, …q in such a way that when each edge e = uv is labeled with $\frac{f(u) + f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u) + f(v) + 1}{2}$ if f(u) + f(v) is odd, then the resulting edge labels are distinct. The labeling f is called a mean labeling of G.

Definition 1.4: A graph G = (V,E) with p vertices and q edges is said to be skolem mean if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 1, 2, …q in such a way that when each edge e = uv is labeled with $\frac{f(u) + f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u) + f(v) + 1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from 2, 3, . . . , p. f is called a skolem mean labeling of G. A graph G = (V, E) with p vertices and q edges is said to be a **skolem mean graph** if there exists a function f from the vertex set of G to {1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by

$$f^*(e=uv)=\begin{cases}\dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even}\\[2mm] \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd}\end{cases}$$

the resulting edges get distinct labels from the set {2, 3, . . . , p}.

Theorem 2.1: The five star $G=K_{1,a_1}\cup K_{1,a_2}\cup K_{1,a_3}\cup K_{1,a_4}\cup K_{1,b}$ where $a_1\le a_2\le a_3\le a_4$ is a skolem mean graph if $a_1+a_2+a_3+a_4-4\le b\le a_1+a_2+a_3+a_4-3$.

Proof: Let $A_i=\sum_{k=1}^{i}a_k$ for $1\le k\le 4$. That is, $A_1=a_1$; $A_2=a_1+a_2$,

$A_3=a_1+a_2+a_3$ and

$A_4=a_1+a_2+a_3+a_4$.

Consider the graph $G=K_{1,a_1}\cup K_{1,a_2}\cup K_{1,a_3}\cup K_{1,a_4}\cup K_{1,b}$.

Let $V=\bigcup_{k=1}^{5}V_k$ be the vertex set of G where $V_k=\{v_{k,i}:0\le i\le a_k\}$ for $1\le k\le 4$ and $V_5=\{v_{5,i}:0\le i\le b\}$. Let $E=\bigcup_{k=1}^{5}E_k$ be the edge set of G where $E_k=\{v_{k,0}v_{k,i}:0\le i\le a_k\}$ for $1\le k\le 4$ and $E_5=\{v_{5,0}v_{5,i}:1\le i\le b\}$.

The condition $a_1+a_2+a_3+a_4-4\le b\le a_1+a_2+a_3+a_4-3$

$\Rightarrow A_3-4\le b\le A_3-3$. Let us prove the graph G is a skolem mean graph when $b=A_4-3$.

Let $b=A_4-3$.

G has $A_4+b+5=2A_4+2$ vertices and $A_4+b=2A_4-3$ edges.

The vertex labeling $f:V\to\{1,2,\dots,A_4+b+5=2A_4+2\}$ is defined as follows:

$f(v_{1,0})=1;\quad f(v_{2,0})=3;\quad f(v_{3,0})=5;$

$f(v_{4,0})=7;\quad f(v_{5,0})=A_4+b+5=2A_4+2$

$f(v_{1,i})=2i \qquad 1\le i\le a_1$

$f(v_{2,i})=2A_1+2i \qquad 1\le i\le a_2$

$f(v_{3,i})=2A_2+2i \qquad 1\le i\le a_3$

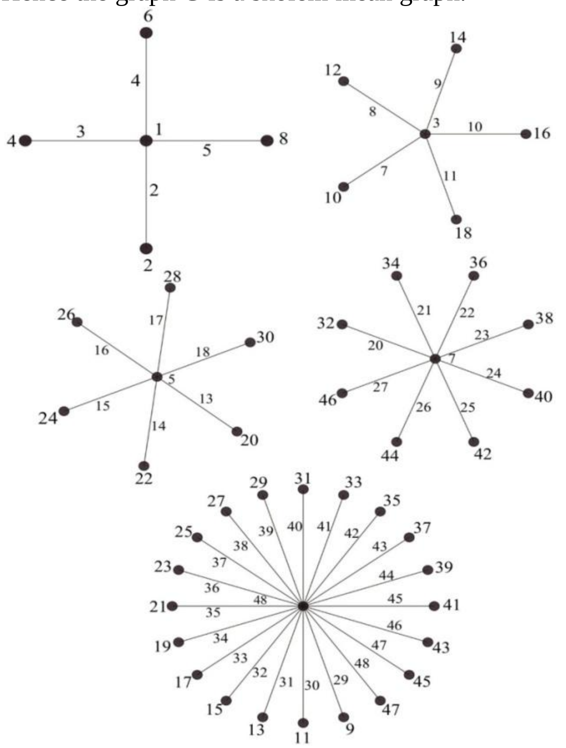
$f(v_{4,i})=2A_3+2i \qquad 1\le i\le a_4$

$f(v_{5,i})=2i+7 \qquad 1\le i\le b=A_4-3$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $1+i$ for $1\le i\le a_1$ (edge labels are $2,3,\dots,a_1+1=A_1+1$), $v_{2,0}v_{2,i}$ is A_1+2+i for $1\le i\le a_2$ (edge labels are $A_1+3,A_1+4,\dots,A_1+a_2+2=A_2+2$), $v_{3,0}v_{3,i}$ is A_2+3+i for $1\le i\le a_3$ (edge labels are $A_2+4,A_2+5,\dots,A_2+a_3+3=A_3+3$), $v_{4,0}v_{4,i}$ is A_3+4+i for $1\le i\le a_4$ (edge labels are $A_3+5,A_3+6,\dots,A_3+a_4+4=A_4+4$) and $v_{5,0}v_{5,i}$ is A_4+5+i for $1\le i\le b=A_4-3$ (edge labels are $A_4+6,\ A_4+7,\dots,2A_4+2$).

Hence the induced edge labels of G are distinct.
Hence the graph G is a skolem mean graph.



$K_{1,4}K_{1,5}\cup K_{1,6}\cup K_{1,8}\cup K_{1,20}$

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