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Unsteady MHD Flow Past A Semi-Infinite Vertical Plate With Heat Source/ Sink: A Finite Difference Approach

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ABSTRACT
In the present paper a numerical attempt is made to study the combined effects of heat source and sink on unsteady laminar boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate. A magnetic field of uniform strength is applied normal to the flow. The governing boundary layer equations are solved numerically, using Crank-Nicolson method. Graphical results of velocity and temperature fields, tabular values of Skin-friction and Nusselt are presented and discussed at various parametric conditions. From this study, it is found that the velocity and temperature of the fluid increase in the presence of heat source but they decrease in the presence of heat absorption parameter.
Keywords: Magnetic field, Crank-Nicholson method, heat source/sink.

I. INTRODUCTION

Heat transfer is the energy interaction due to a temperature differences in a medium or between media. Heat is not a storable quantity and is defined as energy in transit due to a temperature difference. In nature, many flows exist which are caused not only by the temperature differences but by concentration differences also. These mass transfer differences show the effect in the rate of heat transfer. In industries, many transport processes exist in which, heat and mass transfer takes place, simultaneously, as a result of combined buoyancy effect of thermal diffusion of chemical species. The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in science and technology. Such phenomenon is observed in buoyancy-induced motions in the atmosphere, in bodies of water, quasi- solid bodies such as earth so on. In the past decades an intensive research effort has been devoted to problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. In addition, the phenomenon of heat and mass transfer frequently exists in chemical processed industries such as food processing and polymer production.

Eckert *et al* [1] have done pioneer work on heat and mass transfer. The equations governing the mass transfer phenomenon are complicated. However, Gebhart [2] simplified these equations by assuming the presence of species concentration at very low levels and made extensive studies on combined heat and mass transfer flow, to highlight the insight of the phenomenon. Due to importance of these flows, several authors [3-11] have studied the problems on free convection and mass transfer flow of a viscous fluid through porous medium. In these studies, the permeability of the porous medium is assumed to be constant. However, a porous material containing the fluid is a non-homogeneous medium and the porosity of the medium may not necessarily be constant. Shreekanth *et al* [13] studied the effect of permeability variation on free convective flow past a vertical porous wall in a porous medium when the permeability varies with time. Singh *et al* [14] studied hydro magnetic free convection and mass transfer flow of a viscous stratified fluid considering variation in permeability with direction. Acharya *et al* [12] discussed magnetic field effects on the free convection flow through porous medium with constant suction and constant heat flux. Singh *et al* [15] studied the effects of permeability variation and oscillatory suction velocity on free convection and mass transfer flow of a viscous fluid past an infinite vertical porous plate to a porous medium when the plate is subjected to a time dependent suction velocity normal the plate in the presence of a uniform magnetic field. Kandasamy *et al* [16] studied the non linear MHD flow with heat and mass transfer characteristics of an incompressible viscous, electrically conducting fluid on a vertical stretching surface with chemical reaction and thermal stratification effects. The effect of thermal radiation, time-dependent suction and chemical reaction on the two-dimensional flow of an incompressible Boussinesq fluid, applying a perturbation technique has been studied by Prakash and Ogulu [17].

In most of the earlier studies there seems to be no significant consideration of the combined effects of heat source and sink, which plays a vital role in maintaining heat transfer at desired level in the applications of Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles. Due to the coupled non-linearity of the problem in most of the earlier investigations, analytical or perturbation methods were applied to obtain the solution of the problem. However, in the present paper a numerical attempt is made to study the combined effects of heat source and sink on unsteady laminar boundary layer flow of viscous incompressible fluid along a semi-infinite vertical plate. A magnetic field of uniform strength is applied normal to the fluid flow. In order to obtain the approximate solution and to describe the physics of the problem, the present non-linear boundary value problem is solved numerically using implicit finite difference formulae known as Crank-Nicholson method, which is more economical from computational view point.

II. MATHEMATICAL ORMULATION

An unsteady laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate, in the presence of thermal and concentration buoyancy effects has been considered. The x'-axis taken along the plate in the vertically upward direction and y'-axis normal to it. A magnetic field of uniform strength applied along y'-axis. Further, due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and t' only. A time dependent suction velocity is assumed normal to the plate. Now, under the usual Boussinesq's approximation, the governing boundary layer equations are:

$$\frac{\partial v'}{\partial y'} = 0 \qquad (1)$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u' \qquad (2)$$

$$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y'^2} + \frac{Q(T - T_\infty)}{\rho c_p} \qquad (3)$$

$$\frac{\partial C}{\partial t'} + v'\frac{\partial C}{\partial y} = \upsilon\frac{\partial^2 C}{\partial y'^2} \qquad (4)$$

The boundary conditions suggested by the physics of the problem are

$$u' = U_0,\ T = T_W + \varepsilon(T_W - T_\infty)e^{n't'},\ C = C_W + \varepsilon(C_W - C_\infty)e^{n't'} \quad at\ \ y' = 0$$

$$u' \to 0, \quad T \to T_\infty, \quad C \to C_\infty \qquad as\ \ y' \to \infty \qquad (5)$$

Integration of continuity equation (1) for variable suction velocity, normal to the plate gives

$$v' = -U_0\left(1 + \varepsilon A e^{n't'}\right) \qquad (6)$$

Where A is the suction parameter and εA is less than unity. U_0 is mean suction velocity, which is a non-zero positive constant and the minus sign indicates that the suction is towards the plate.

Using equation (6) and introducing the following dimensional less quantities

$$u = \frac{u'}{U_0}, \quad t = \frac{U_0^2 t'}{\upsilon}, \quad n = \frac{\upsilon n'}{U_0^2}, \quad y = \frac{y^* U_0}{\upsilon}$$

$$Gr = \frac{g\beta\upsilon(T_w - T_\infty)}{U_0^3},\ Gm = \frac{g\beta^*\upsilon(C_w - C_\infty)}{U_0^3},\ S = \frac{Q\upsilon}{\rho C_p U_0^2},\ M = \frac{\sigma B_0^2\upsilon}{\rho U_0^2}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty},$$

$$\Pr = \frac{\mu C_p}{k},\ Sc = \frac{\upsilon}{D}$$

into equations (1)-(5), we get the equations in dimensional less form as follows

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - Mu \qquad (7)$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + S\theta \qquad (8)$$

$$\frac{\partial \phi}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \qquad (9)$$

with the boundary conditions

$$u = 1,\ \ \theta = 1 + \varepsilon e^{nt},\ \ \phi = 1 + \varepsilon e^{nt} \qquad \text{at } y = 0$$

$$u \to 0,\ \ \theta \to 0,\ \ \phi \to 0 \qquad \text{as } y \to \infty$$

(10)

In order to establish a finite boundary condition $\eta \to 1$ in (10), equations (7)-(9) are transformed to a new system of co-ordinates. So applying the transformation $\eta = 1 - e^{-y}$ on the equations (7) to (10), we get

$$\frac{\partial u}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial u}{\partial \eta}=\left((1-\eta)^2\frac{\partial^2 u}{\partial \eta^2}-(1-\eta)\frac{\partial u}{\partial \eta}\right)+Gr\theta+Gm\phi-Mu \tag{11}$$

$$\frac{\partial \theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial \theta}{\partial \eta}=\frac{1}{Pr}\left((1-\eta)^2\frac{\partial^2 \theta}{\partial \eta^2}-(1-\eta)\frac{\partial \theta}{\partial \eta}\right)+S\theta \tag{12}$$

$$\frac{\partial \phi}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial \phi}{\partial \eta}=\frac{1}{Sc}\left((1-\eta)^2\frac{\partial^2 \phi}{\partial \eta^2}-(1-\eta)\frac{\partial \phi}{\partial \eta}\right) \tag{13}$$

with corresponding boundary conditions

$$\begin{aligned} &u=1: \quad \theta=1+\varepsilon e^{nt}, \quad \phi=1+\varepsilon e^{nt} \quad at \quad \eta=0 \\ &u\to 0: \ \theta\to 0, \qquad \theta\to 1 \qquad as \quad \eta\to 1 \end{aligned} \tag{14}$$

III. METHOD OF SOLUTION

The equations (11)-(13) are coupled, non-linear partial differential equations and obtaining exact solution under the given boundary condition (14) is very difficult. So, using the following finite difference formulae

$$\frac{\partial f}{\partial t}=\frac{f_i^{j+1}-f_i^j}{k}, \qquad \frac{\partial f}{\partial \eta}=\frac{f_{i+1}^j-f_i^j}{h}$$

$$\frac{\partial^2 f}{\partial \eta^2}=\frac{1}{2}\left(\frac{f_{i-1}^j-2f_i^j+f_{i+1}^j}{h^2}+\frac{f_{i-1}^{j+1}-2f_i^{j+1}+f_{i+1}^{j+1}}{h^2}\right)$$, where f stands u, θ and ϕ into the equations (11)-(13) and simplifying implicitly according to the **Crank- Nicholson method**, we get the following system of equations

$$-P_3\, r\, u_{i-1}^{j+1}+\left(1+2P_3\, r\right)u_i^{j+1}-P_3\, r\, u_{i+1}^{j+1}=E_{1i}^j \tag{15}$$

$$-P_3\, r\, \theta_{i-1}^{j+1}+\left(1+2P_3P_4\, r\right)\theta_i^{j+1}-P_3\, r\, \theta_{i+1}^{j+1}=F_{1i}^j \tag{16}$$

$$-\frac{P_3 r}{Sc}\phi_{i-1}^{j+1}+\left(1+\frac{2P_3 r}{Sc}\right)\phi_i^{j+1}-\frac{P_3 r}{Sc}\phi_{i+1}^{j+1}=H_{1i}^j \tag{17}$$

with boundary conditions in finite difference form

$$\begin{aligned} &u\,(0,j)=1, \quad \theta(0,j)=1+\varepsilon\exp(n.\,j.\,\Delta t), \quad \phi=1+\varepsilon\exp(n.\,j.\,\Delta t), \quad \forall\ j \\ &u(10,j)\to 0, \quad \theta(10,j)\to 0, \quad \phi(10,j)\to 1 \quad \forall\ j \end{aligned} \tag{18}$$

where $E_i^j=P_3 r\, u_{i-1}^j-\left(1-P_1P_2\, rh-2P_3 r+P_2 rh-M\,\Delta t\right)u_i^j+\left(P_1\, P_2 rh+P_3 r-P_2 rh\right)u_{i+1}^j+Gr\,\Delta t\theta_i^j+Gm\,\Delta t\phi_i^j$

$$F_i^j=P_3 r\,\theta_{i-1}^j+\left(1-P_1P_2 rh-2P_3 r+P_2 rh+\Delta t\, S\right)\theta_i^j+\left(P_1P_2 rh+P_3 r-P_2 rh\right)\theta_{i+1}^j$$

$$H_i^j=\frac{P_3 r}{Sc}\,\phi_{i-1}^j+\left(1+P_1P_2\, rh-\frac{2P_3 r}{Sc}+\frac{P_2 rh}{Sc}\right)\phi_i^j+\left(\frac{P_3 r}{Sc}-P_1\, P_2 rh-\frac{P_2 rh}{Sc}\right)\phi_{i+1}^j$$

$$P_1=1+\in Ae^{nt},\ P_2=1-i\,h,\ P_3=\frac{(1-i\,h)^2}{2},$$

$r=\Delta t/h^2$, h and Δt are mesh sizes along space and time direction respectively. The finite-difference equations at every internal nodal point on a particular n-level constitute a tri-diagonal system of equations. So, in the equations (15) to (17), taking $i=1(1)n$ and using the boundary conditions (18), the following tri-diagonal system of equations are obtained.

$$DU = A \quad (19)$$
$$E\theta = H \quad (20)$$
$$F\phi = G \quad (21)$$

Where D, E and F are the tri-diagonal matrices of order n whose elements are defined by

$D_{i,i} = B_1; \quad E_{i,i} = B_2; \quad F_{i,i} = B_3;$ at $i = 1(1)n$

$D_{i-1,i} = A_1; \ E_{i-1,i} = A_2; \ F_{i-1,i} = A_3;$ at $i = 2(1)n$

$D_{i,i-1} = A_1; \ E_{i,i-1} = A_2; \ F_{i,i-1} = A_1;$ at $i = 2(1)n$

and U, A, θ, H, ϕ , G are column matrices having n components, namely

u_i^{j+1}, E_{1i}^j, θ_i^{j+1}, F_{1i}^j, ϕ_i^{j+1}, H_{1i}^j, i=1(1)n respectively.

The above equations are solved by using the Thomas algorithm [19], for which simulation is carried out by coding in C-Program. In order to prove the convergence of finite difference scheme, the computation is carried out for slightly changed values of h and Δt, running same program. Negligible change is observed in the values of *u*, θ and ϕ and also after each cycle of iteration the convergence checking is performed, i.e. $\left|u^{n+1} - u^n\right| < 10^{-8}$ is satisfied at all points. Thus, it is concluded that, the finite difference scheme is convergent and stable.

From the technological point of view, after knowing the velocity, temperature and concentration profiles, it is important to know the skin-friction, rate of heat and mass transfer between the plate and the fluid.

3.1 Skin-friction

The Skin friction coefficient τ is given by

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial u}{\partial \eta}\right|_{\eta=0}, \quad (22)$$

3.2 Nusselt number

The rate of heat transfer in terms of Nusselt number is given by

$$Nu = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial \theta}{\partial \eta}\right|_{y=0} \quad (23)$$

IV. RESULTS AND DISCUSSION:

In order to get the physical insight in to the problem, the numerical calculations for the distribution of the velocity, temperature, skin-friction coefficient, rate of heat transfer across the boundary layer for various values of flow parameters such as heat source and sink parameter (S), Magnetic parameter (M), Prandtl number (Pr), have been carried out. During the course of numerical calculations, to be realistic, the values of Prandtl number (Pr) are chosen to be 0.71 and 7.0.

Figures (1) shows the velocity profile for various values of heat source and sink parameter (S) while figures (3) and (4) show the temperature profiles for different values of heat source and sink parameter (S) respectively. It is evident from the figures that the temperature and velocity increase with an increase in the heat source parameter (S). This result qualitatively agrees with expectation, since the effect of heat generation is to increase the rate of heat transport to the fluid there by increasing the temperature of the fluid consequently velocity of the fluid particles also increases. It is also observed that temperature and velocity of the fluid decrease in the presence of heat sink as heat absorption is to decrease the rate of heat transport to the fluid.

Figure (2) show that increasing values of magnetic parameter is to reduce the velocity of the flow due to the magnetic pull of Lorentz force. This type of resistive force tends to slow down the flow field. Figure (5) is drawn for various values of Pr on temperature field in the presence of heat source/sink. A comparative study of the graph reveals that the temperature of the fluid decreases as the value of Prandtl number Pr increases. This is a good agreement with physical fact that an increase in Pr leads to decrease in the thermal boundary layer thickness. The reason underlying such behavior is that the higher Pandtl number fluid has relatively lower thermal conductivity. This results in the reduction of the thermal boundary layer thickness. It is observed from figure (5) that temperature of the fluid increases in the presence of heat source parameter while in the presence of sink it decreases.

Tables (1) and (2) show the numerical values of the Skin-friction and Nusselt number. From the tables, it is concluded that

1. Skin –friction & Nusselt number increase, in the presence of heat source when it is compared to that of its absence of source parameter. But they decrease in the presence of heat absorption parameter.
2. An increase in the M leads to decrease in the Skin –friction.

V. CONCLUSIONS

Effect of heat source/sink on hydromagnetic unsteady laminar boundary layer flow of an incompressible viscous fluid along a semi-infinite vertical plate is studied. From this study the following conclusions are drawn.

(1) Velocity of the fluid decreases in the presence of heat sink and magnetic parameter M. But it increases with the increasing values of source parameter S

(2) The effect of heat source/sink on temperature and velocity is more significant.

(3) Skin –friction and Nusselt number increase, in the presence of heat source. But they decrease in the presence of heat sink.

Nomenclature

ρ Density
C_p Specific heat at constant pressure
ν Kinematic viscosity
k Thermal conductivity
Gr Free convection parameter due to temperature
G Free convection parameter due to concentration
A Suction parameter
n A constant exponential index
D Molar diffusivity
M Magnetic parameter
σ Electrical conductivity
S Heat source/sink parameter
Sc Schmidt number
T Temperature
Pr Prandtl number
ϵ Small reference parameter << 1
β^* Volumetric coefficient of expansion with concentration
U Mean velocity
β Coefficient of volumetric thermal expansion of the fluid

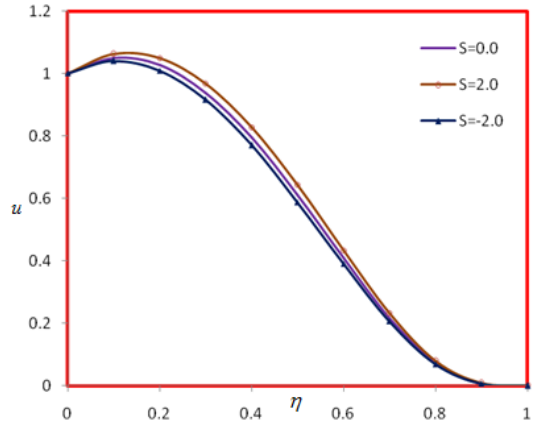

Fig 1: Effect of heat source and sink on velocity field u
(Gr=5.0, Gm=5.0, Pr=0.71, Sc=0.22, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

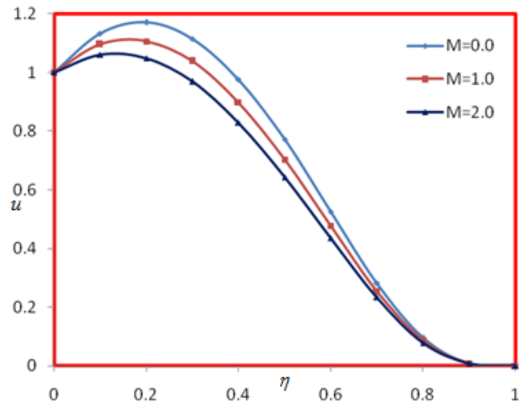

Fig 2: Effect of Magnetic parameter M on velocity field u
(Gr=5.0, Gm=5.0, Pr=0.71, Sc=0.22, S=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

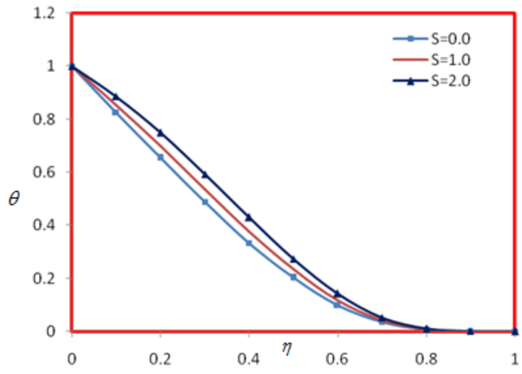

Fig 3: Effect of heat source on temperature field
(Pr=0.71, ε=0.01, n=0.1, A=0.3 and t=1.0)

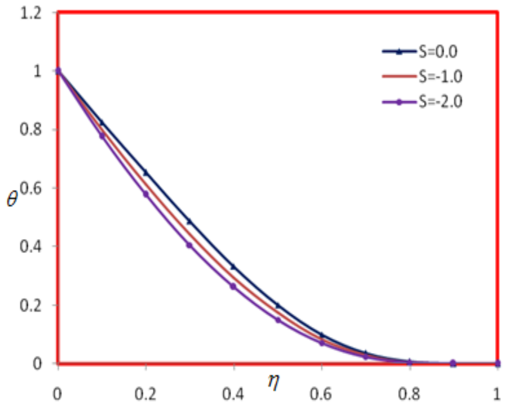

Fig 4: Effect of heat Sink on temperature field
(Pr=0.71, ε=0.01, n=0.1, A=0.3 and t=1.0)

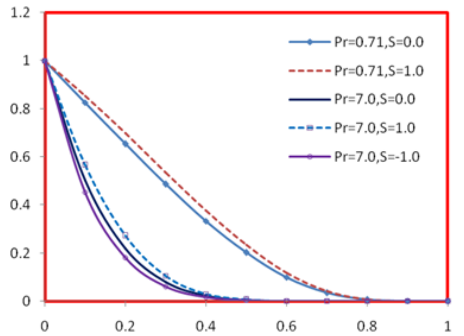

Fig 5: Effect of Prandtl number in the presence of heat source/sink on temperature field
(ε=0.01, n=0.1, A=0.3 and t=1.0)

Table 1 - Effects of M and S on Skin-Friction coefficient
(Gr=5.0, Gm=5.0, Pr=0.71, Sc=0.22, □ =0.01, n=0.1, A=0.3 and t=1.0)

M	S	Sk
2.0	0.0	0.499285
2.0	2.0	0.614797
2.0	-2.0	0.403671
3.0	2.0	0.305407

Table 2 - Effects of Pr and S on Nusselt number
(□ =0.01, n=0.1, A=0.3 and t=1.0)

Pr	S	Nu
0.71	0.0	1.37072
0.71	2.0	0.92109
0.71	-2.0	1.74307

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