RESEARCH ARTICLE OPEN ACCESS

Differential Evolution Algorithm for Gear Ratio Optimization of Vehicles

İlker Küçükoğlu*
*(Department of Industrial Engineering, Uludag University, Turkey)

ABSTRACT
In this study, the gear ratio optimization problem for the vehicles is considered to minimize total fuel consumption amount for a given distance and road gradient. The aim of the study is to find best gear ratio values for a gearbox that provide the best consumption value for the vehicle. In order to analyze the fuel consumption changes for a gearbox, fuel consumption formulations, which have a nonlinear structure, are presented based on the total energy requirement of the vehicle. To solve this nonlinear optimization problem, a differential evolution algorithm (DEA) is proposed which is one of the popular evolutionary optimization algorithms. In computational studies, the DEA is tested on a vehicle set by considering different road conditions. The results of the experiments show that the proposed DEA is capable to find effective solutions for the problem where an improvement is obtained for each experiment according to the real fuel consumption amount of the vehicles given at their technical reports.
Keywords: Differential evolution algorithm, fuel consumption, gear ratio optimization

I. INTRODUCTION

The automobile manufacturers develop various technology to reduce fuel consumption amount of the vehicles. These improvements are carried out in various part of the vehicle, such as engine, transmission, car body, etc. A remarkable amount of reduction on the consumption provides competitive benefits for the firms. Therefore, various researches are focused on how to bring down the vehicle's consumptions.

This study considers the gear ratio optimization problem of the vehicles, where the objective is to find best ratio values for the gearbox of a vehicle. There have been several approaches on gearbox optimization dwelt on different problem type to increase the efficiency of the system. Mohan and Seshaiah [1] proposed a genetic algorithm for spur gear set optimization, where its center distance, weight and tooth deflections are taken into account as objective function. Zhou et al. [2] developed an approach to optimize the economy performance of a two-speed electric vehicle by combining gear shifting schedule design and gear ratio selection. The authors tested their solution approach based on the powertrain model simulation by using different test driving cycles. Kim et al. [3] introduced a fuel economy optimization method for parallel hybrid electric vehicles by using continuously variable transmission, which maximizes the overall system efficiency while meeting the desired performances. On the basis of gear ratio optimization problem, Barr [4] proposed global gear ratio optimization approach for the motorsport applications to make the vehicles faster. Their approach aims to find the best gear ratios that ensures the minimum overlap time for the motorsports cars. Ahangar et al. [5] studied the fuel consumption and gearbox efficiency in fifth gear ratio for a vehicle brand. Robinette and Wehrwein[6] used Monte Carlo Simulation approach for the automatic transmission gear ratio optimization. Goharimanesh et al. [7] selected the best gear ratios by using the Taguchi method where different levels of the gear ratios are tested with a design of experimental analysis.

Distinctly from the existing studies on gearbox, this paper contributes to the literature by introducing a differential evolution algorithm (DEA) to optimize gear ratio for the vehicles. For the considered gear ratio optimization problem, the gearbox performance analyzed for different road conditions. In this context, the remaining of the paper organized as follows. In Section 2, the considered problem for the gearbox optimization is defined with fuel consumption formulations. Section 3 introduces the proposed DEA, and Section 4 shows the computation results formed in order to test the efficiency of the DEA. Finally, conclusion part is given Section 5.

II. PROBLEM DEFINITION

In this study, the gearbox transmission values are analyzed to obtain the best ratios for the gear set that minimizes the fuel consumption amount of the vehicles. Regarding the gearbox transmission rates, the fuel consumption amount for a given distance is determined by using the total energy requirement of the vehicle. This energy is basically equal to total resistance forces (F_T) that the vehicle is exposed while it is going with a constant speed or acceleration/deceleration period. In detail, the F_T includes four main resistance forces: the rolling resistance (F_{Ro}), the aerodynamic resistance (F_{Ae}), the acceleration resistance (F_{Acc}), and the grade resistance (F_G). These resistance forces are determined for a given acceleration and gravity amounts by using the Equations (1-4), where f is the rolling resistance coefficient, m is the vehicle weight, g is the gravitational constant, c_d is the aerodynamic coefficient, A is the frontal surface area of the vehicle, ρ is the air density, v is the speed of vehicle, λ is the transmission variable ($\lambda = 1.04 + 0.0025 \times i^2$, i is the overall gear ratio obtained from the gear ratios), and a is the acceleration value of the vehicle [8].

$$F_{Ro} = f \times m \times g \quad (1)$$

$$F_{Ae} = c_d \times A \times \rho \times \frac{v^2}{2} \quad (2)$$

$$F_{Acc} = \lambda \times m \times a \quad (3)$$

$$F_G = m \times g \times sin\alpha \quad (4)$$

As a result of the calculation of the resistance forces, the fuel consumption amount of a vehicle is calculated by using the Equations (5-7), where P_T is thetotal power at wheels, and u_p is the fuel consumption value per unit power.

$$F_T = F_{Ro} + F_{Ae} + F_{Acc} + F_G \quad (5)$$

$$P_T = F_T \times v \quad (6)$$

$$consumption = u_p \times P_T \quad (7)$$

In order to calculate the total fuel consumption amount of the vehicles for a given distance, the total power and consumption formulations are developed as a function of time which are defined with Equations (8-11), where t_{max} is the finish time of the drive [9].

$$F_T(t) = F_{Ro}(t) + F_{Ae}(t) + F_{Acc}(t) + F_G(t) \quad (8)$$

$$P_T(t) = F_T(t) \times v(t) \quad (9)$$

$$Consumption(t) = u_p \times P_T(t) \quad (10)$$

$$Totalconsumption = \int_0^{t_{max}} u_p \times P_T(t) d_t \quad (11)$$

As it is seen from the equations described above, the gear ratio set of the gearbox is one of the major variables for the fuel consumption calculations, which directly affects the vehicle speed and also acceleration resistance. Therefore, the aim of the study is to find best value set of the gear ratios (i_j, $j = 1, \dots, J$ where J is the number of gears in gearbox) for the gearbox that minimizes the total fuel consumption amount of the vehicles. To analyze the best ratio values for the gearbox, the following three assumptions are considered:

- The physical coefficients, such as air density, rolling resistance coefficient, aerodynamic coefficient, etc., and technical variables for the vehicles except the gear ratios are assumed as constant.
- The road conditions are assumed as stable during the travelling distance.
- For the fuel consumption amount for a given distance, the vehicles first accelerates to the maximum speed limit, then goes with a constant speed and decelerates in order to stop. The Equations (8-11) are practiced with respect to this three stage movement for the vehicles.

III. PROPOSED ALGORITHM

To solve the nonlinear optimization problem described above, in this study DEA is proposed as a solution approach which is one of the evolutionary optimization algorithms introduced by Storn and Price [10]. As with all evolutionary optimization algorithms the DEA works with a population that is formed with continuous variables, where the population is evolved on each generation by using three main operators: mutation, crossover, and selection [11, 12]. At the beginning of the algorithm the population is initialized randomly by using the Equation (12), where NP is the number of population, and i_j^{min} and i_j^{max} are the minimum and maximum value of the gear j, respectively.

$$i_{j,n,0} = i_j^{min} + rnd * \left(i_j^{max} - i_j^{min}\right)$$
$$j = 1 \dots, J; \quad n = 1, \dots, NP \quad (12)$$

In each generation, the mutation, crossover and selection operations are executed for each individual, respectively. For the mutation operation, the offspring vector ($o_{j,n,k}$) for the individual n is formed by using the Equation (13), where r_1, r_2, and r_3 are the random numbers ($r_1, r_2, r_3 \in \{1,2,, \dots, NP\}$; $r_1 \neq r_2 \neq r_3 \neq n$), and MF is the mutation factor [13].

$$o_{j,n,k+1} = i_{j,r_3,k} + MF \times \left(i_{j,r_1,k} - i_{j,r_2,k}\right)$$
$$j = 1, \dots, J; \quad n =, \dots, NP \quad (13)$$

To increase the potential diversity of the perturbed offspring vector, a crossover operation is carried out after the mutation operation, which is shown in Equation (14), where $u_{j,n,k}$ describes the trial vector for selection [13].

$$u_{j,n,k+1} = \begin{cases} o_{j,n,k+1} ifrnd \leq CRorj = j_{rnd} \\ i_{j,n,k} otherwise \end{cases}$$
$$j = 1, \dots, J; \quad n = 1, \dots, NP \quad (14)$$

In selection operation, if the trial vector has an equal or lower objective function value than its target vector, it replaces the target vector in the next generation. Otherwise, the target vector remains in place in the population for at least one more generation. Equation (15) describes the selection condition for the DEA [14].

$$i_{j,n,k+1} = \begin{cases} u_{j,n,k+1} iff\left(u_{j,n,k+1}\right) < f\left(i_{j,n,k}\right) \\ i_{j,n,k} otherwise \end{cases}$$
$$n = 1, \dots, NP \quad (15)$$

Mutation, crossover, and selection operations are repeated for the generations until a stopping criterion is met by the algorithm. For the proposed DEA, a maximum iteration number is used as a stopping criterion, which is commonly used most of the population based algorithms. According to the description above, the pseudo code of the proposed DEA is presented in Fig. 1.

```
Initialize the DEA parameters
Generate initial solution
Do
For n = 1 to NP
        Randomly generate r_1, r_2, r_3, and j_rnd
        Apply mutation operation for individual n and generate offspring vector (o_{j,n,k})
        Apply Crossover operation and generate trial vector u_{j,n,k+1}
If f(u_{j,n,k+1}) < f(i_{j,n,k}) Then
            Select the trial vector (u_{j,n,k+1}) for the next generation
Else
            Select the existing vector (i_{j,n,k}) for the next generation
End If
Next
Until (Stopping Criteria)
```

Figure 1. Pseudo code of the proposed DEA

IV. COMPUTATIONAL STUDIES

In order to test the performance of the proposed DEA a problem set is formed, which includes the technical data of 10 different type vehicles. Some of the major data of the vehicles required for the fuel consumption calculation are given in Table 1. The DEA is performed for each vehicle type in different road conditions by changing the road gradient (0%, 1%, 3%, and 5%) and distance to be travelled (1 km, 10 km, and 100 km). For these experiments the DEA is applied 50 times for each instance of vehicle type and the best solution of the runs is taken into account for computational comparisons. The solutions of the experiments are divided into three groups due to the distance to be travelled by the vehicles, which are shown in Table 2-4. In these tables, also comparisons between the real fuel consumption amount (FCA) of the vehicles given at their technical reports and the new fuel consumption amount (FCA*) obtained by DEA solutions are presented as the percentage fuel consumption reduction (%FCR) for each gradient level. The comparison metric %FCR is calculated as follows:

$$\%FCR = \frac{FCA - FCA^*}{FCA} \times 100\%$$

Table 1. Major Technical Data of the Vehicles

Vehicle Type	Weight (kg)	Frontal Surface Area (m^2)	Wheel Sizes	Engine Speed (r/min)	Gear Ratios 1. Gear	2. Gear	3. Gear	4. Gear	5. Gear	Final Transmission Ratio
1	1380	2.04	195/65/16	2100	4.25	2.24	1.44	1.03	0.80	3.93
2	1900	3.50	225/70/15	2500	5.05	2.60	1.52	1.00	0.78	4.38
3	1400	2.03	195/65/15	2500	4.11	2.12	1.27	0.87	0.66	4.00
4	1950	2.76	215/75/16	2700	4.10	2.22	1.37	1.00	0.77	3.75
5	1800	3.56	245/70/16	2500	4.81	2.54	1.50	1.00	0.76	4.39
6	1680	3.10	205/65/16	2400	3.60	2.19	1.41	1.00	0.83	3.37
7	2320	4.20	215/75/16	2500	4.20	2.24	1.22	1.00	0.79	3.73
8	1920	4.60	215/75/16	2500	4.20	2.24	1.36	1.00	0.76	5.11
9	1080	2.12	165/65/15	2500	3.73	2.05	1.39	1.03	0.82	4.56
10	1460	2.28	235/40/18	2500	3.67	2.14	1.45	1.03	0.81	3.82

Table 2. Computational Results for the Distance Level 1: 1 km

Vehicle Type	Gradient of the Road											
	0%			1%			3%			5%		
	FCA	FCA*	%FCR	FCA	FCA*	%FCR	FCA	FCA*	%FCR	FCA	FCA*	%FCR
1	0.2562	0.2490	2.82	0.2818	0.2746	2.57	0.3330	0.3258	2.17	0.3841	0.3769	1.88
2	0.3732	0.3574	4.24	0.4085	0.3926	3.88	0.4789	0.4631	3.30	0.5493	0.5335	2.88
3	0.2535	0.2463	2.84	0.2794	0.2721	2.58	0.3311	0.3239	2.18	0.3828	0.3756	1.88
4	0.3694	0.3576	3.18	0.4056	0.3938	2.90	0.4779	0.4662	2.46	0.5501	0.5384	2.14
5	0.3563	0.3445	3.32	0.3897	0.3779	3.04	0.4565	0.4446	2.59	0.5231	0.5113	2.26
6	0.3119	0.3049	2.23	0.3430	0.3360	2.03	0.4051	0.3981	1.72	0.4671	0.4601	1.49
7	0.4340	0.4241	2.28	0.4770	0.4670	2.08	0.5627	0.5528	1.76	0.6483	0.6384	1.53
8	0.3813	0.3671	3.72	0.4168	0.4026	3.40	0.4877	0.4736	2.91	0.5586	0.5444	2.54
9	0.2031	0.1970	3.01	0.2231	0.2169	2.74	0.2630	0.2569	2.31	0.3028	0.2967	2.02
10	0.2753	0.2673	2.93	0.3025	0.2944	2.66	0.3567	0.3487	2.26	0.4109	0.4028	1.96
Average	0.3214	0.3115	3.06	0.3527	0.3428	2.79	0.4153	0.4054	2.37	0.4777	0.4678	2.06

Table 3. Computational Results for the Distance Level 2: 10 km

Vehicle Type	Gradient of the Road											
	0%			1%			3%			5%		
	FCA	FCA*	%FCR	FCA	FCA*	%FCR	FCA	FCA*	%FCR	FCA	FCA*	%FCR
1	0.7629	0.7557	0.95	1.0183	1.0110	0.71	1.5286	1.5214	0.47	2.0380	2.0308	0.35
2	1.1757	1.1599	1.34	1.5273	1.5114	1.04	2.2299	2.2141	0.71	2.9312	2.9154	0.54
3	0.7616	0.7543	0.95	1.0205	1.0133	0.71	1.5380	1.5308	0.47	2.0546	2.0474	0.35
4	1.0669	1.0551	1.10	1.4276	1.4159	0.82	2.1488	2.1370	0.55	2.8686	2.8568	0.41
5	1.1536	1.1418	1.02	1.4866	1.4748	0.79	2.1523	2.1405	0.55	2.8167	2.8049	0.42
6	1.0223	1.0154	0.68	1.3330	1.3261	0.52	1.9541	1.9471	0.36	2.5740	2.5670	0.27
7	1.4028	1.3929	0.71	1.8319	1.8220	0.54	2.6895	2.6796	0.37	3.5456	3.5357	0.28
8	1.3535	1.3393	1.05	1.7086	1.6944	0.83	2.4183	2.4042	0.59	3.1268	3.1126	0.45
9	0.6791	0.6730	0.90	0.8788	0.8727	0.70	1.2780	1.2719	0.48	1.6766	1.6704	0.37
10	0.8302	0.8222	0.97	1.1007	1.0927	0.73	1.6414	1.6333	0.49	2.1810	2.1730	0.37
Average	1.0209	1.0110	0.97	1.3333	1.3234	0.74	1.9579	1.9480	0.50	2.5813	2.5714	0.38

Table 4. Computational Results for the Distance Level 3: 100 km

Vehicle Type	Gradient of the Road											
	0%			1%			3%			5%		
	FCA	FCA*	%FCR	FCA	FCA*	%FCR	FCA	FCA*	%FCR	FCA	FCA*	%FCR
1	5.8306	5.8233	0.12	8.3829	8.3757	0.09	13.4845	13.4773	0.05	18.5768	18.5696	0.04
2	9.2012	9.1853	0.17	12.7153	12.6995	0.12	19.7392	19.7234	0.08	26.7504	26.7345	0.06
3	5.8426	5.8354	0.12	8.4319	8.4247	0.09	13.6073	13.6001	0.05	18.7732	18.7660	0.04
4	8.0417	8.0300	0.15	11.6483	11.6366	0.10	18.8571	18.8454	0.06	26.0528	26.0410	0.05
5	9.1270	9.1151	0.13	12.4561	12.4443	0.09	19.1104	19.0986	0.06	25.7525	25.7407	0.05
6	8.1265	8.1195	0.09	11.2336	11.2266	0.06	17.4440	17.4370	0.04	23.6431	23.6362	0.03
7	11.0906	11.0807	0.09	15.3813	15.3714	0.06	23.9577	23.9477	0.04	32.5183	32.5084	0.03
8	11.0755	11.0613	0.13	14.6265	14.6123	0.10	21.7241	21.7099	0.07	28.8088	28.7946	0.05
9	5.4388	5.4327	0.11	7.4362	7.4301	0.08	11.4287	11.4226	0.05	15.4138	15.4077	0.04
10	6.3791	6.3710	0.13	9.0831	9.0751	0.09	14.4879	14.4798	0.06	19.8827	19.8747	0.04
Average	8.0154	8.0054	0.12	11.1395	11.1296	0.09	17.3841	17.3742	0.06	23.6172	23.6073	0.04

For the 1 km travelling distance, where the solutions of this case are given in Table 2, the new gearbox ratios obtained by DEA provide more than 2.00% reduction on fuel consumption amount for most of the experiment. Especially, for the low road gradient ratios, this improvement is higher 2.50%. According to the average %FCR, the reduction on the fuel consumption amounts with respect to the FCA are 3.06%, 2.79%, 2.37%, 2.06 for the gradient rate 0%, 1%, 3%, and 5%, respectively.

Similar to the first distance level, a consumption saving on each vehicle type is provided for the other travelling distance levels by the DEA solutions. However, the reductions on fuel consumption amounts are less than 1% in general. For these experiments, the average %FCR based on the gradient level of road ranges between the 0.38-0.97% for the 10 km travelling distance and 0.04-0.12% for the 100 km travelling distance.

As a result of the computational studies, it can be seen from the results that the proposed DEA is capable to find effective solutions for the gearbox ratios that provides a considerable reduction on fuel consumption amount for the vehicles with respect to their existing values given in their technical reports. Another outcome can be emphasized on the effects of travelling distance to the %FCR. The results show that the rise on distance directly affects the %FCR values, where the %FCR amounts for the long distances are smaller with respect to the solutions of the experiments executed for the short distances. This situation can be explained by the gear changes in a travelling time, where the vehicle uses the last gear for a long time on a long distance travelling while the gears are shifted frequently in a short distance travelling. Therefore, the gearbox optimization studies on real-life applications can be formed by considering the potential values of the physical conditions.

V. CONCLUSION

This paper addresses a DEA based optimization approach for the gear ratio optimization for the vehicle gearbox in order to minimize total fuel consumption amount. In this context, a fuel consumption calculation formulation based on the vehicle technical specifications is taken into account. The proposed DEA optimizes the gear ratios with respect to the fuel consumption amounts for a given distance and road condition. In computational studies, the DEA is tested on different type vehicles for various road conditions. The experimental results show that the proposed DEA is capable to find effective solutions for the gear ratio optimization, where a reduction on fuel consumption is provided for each case. Especially for the short distance cases, the proposed DEA obtained more than 2% savings on fuel consumptions according to the real fuel consumption amount of the vehicles given at their technical reports.

REFERENCES

[1]. Y.M. Mohan, and T. Seshaiah, Spur gear optimization by using genetic algorithm, *International Journal of Engineering Research and Applications, 2(1)*, 2012, 311-318.

[2]. X. Zhou, P. Walker, and N. Zhang, Performance improvement of a two speed EV through combined gear ratio and shift schedule optimization, *SAE Technical Paper*, 2013.

[3]. C. Kim, E.NamGoong, S. Lee, T. Kim, and H. Kim, Fuel economy optimization for parallel hybrid vehicles with CVT, *SAE Technical Report*, 1999.

[4]. T.W. Barr,Global optimization of gear-ratios for motorsport applications, or how to make a fast car faster, Technical Report, 2008.

[5]. R.G. Ahangar, Fuel consumption and gearbox efficiency in the fifth gear ratio of Roa vehicle,*Advances in Theoretical and Applied Mechanics*, *3(7)*, 2010, 299-308.

[6]. D. Robinette, and D.Wehrwein,Automatic transmission gear ratio optimization and Monte Carlo simulation of fuel consumption with parasitic loss uncertainty,*SAE International Journal of Commercial Vehicles, 8*, 2015, 45-62.

[7]. M. Goharimanesh, A. Akbari, and A.A. Tootoonchi, More efficiency in fuel consumption using gearbox optimization based on Taguchi method,*Journal of Industrial Engineering International, 10(2)*, 2014, 54-61.

[8]. H.H. Braess, and U.Seiffert, *Handbook of automotive engineering* (SAE International. Pennsylvania, USA, 2005).

[9]. İ. Küçükoğlu, S.Ene, A.Aksoy, and N. Öztürk,A memory structure adapted simulated annealing algorithm for a green vehicle routing problem,*Environmental Science and Pollution Research, 22(5)*, 2015, 3279-3297.

[10]. R. Storn, and K. Price, Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces,*Journal of Global Optimization, 11(4)*, 1997, 341–359.

[11]. G.C. Onwubolu, Design of hybrid differential evolution and group method of data handling networks for modeling and prediction,*Information Sciences, 178(18)*, 2008, 3616 – 3634.

[12]. G.C. Onwubolu, and D. Davendra, Scheduling flow shops using differential evolution algorithm,*European Journal of Operational Research*, *171(2)*, 2006, 674–692.

[13]. D. Karaboğa, and S.Ökdem, A simple and global optimization algorithm for engineering problems: differential evolution algorithm,*Turkish Journal of Electrical Engineering*, *12(1)*, 2004, 53-60.

[14]. C. Erbao, and L.Mingyong, A hybrid differential evolution algorithm to vehicle routing problem with fuzzy demands,*Journal of Computational and Applied Mathematics*, *231(1)*, 2009, 302–310.