Analysis of Cross-ply Laminate composite under UD load based on CLPT by Ansys APDL

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Abstract
In current study the strength of composite material configuration is obtained from the properties of constituent laminate by using classical laminate plate theory. For the purpose of analysis various configurations of 2 layer and 4 layered cross ply laminates are used. The material of laminate is supposed to be boron/epoxy having orthotropic properties. The loading in current study is supposed to be of uniformly distributed load type. For the analysis purpose software working on finite element analysis logics i.e. Ansys mechanical APDL is used. By the help of Ansys mechanical APDL the deflection and stress intensity is found out. The effect of variation of laminate layers is also studied in current study along with the effect of variation of stacking patterns. The current study will also help to conclude which stacking pattern is best in 2 layered and 4 layered cross ply laminate.

I. Introduction
The composite materials consists of two components basically the very first is reinforcement and the second is matrix. The reinforcement is surrounded by matrix. It is the reinforcement which give rise to the properties of composite materials like stiffness, strength etc. The orientation, shape, size and aspect ratio is an important criteria in determining the properties of composites. The matrix binds the reinforcement together, holding them aligned in the important stressed directions. The matrix should protect the reinforcement from mechanical damage (e.g. abrasion) and from environmental attack. By comparison with the common reinforcement most matrix materials are weak and flexible and their strengths and modules are often neglected in calculating composite properties.

When certain layers of either same composite material with different orientation or different composite material is stacked in a particular sequence than the laminated structure obtained is known as Laminate composite and is shown below in fig 1. The Laminate composites posses higher stiffness and stress bearing capacity due to the stacking approach of composites together. Any laminate is called cross ply laminate if it has same thickness, same material but is oriented at an angle of 0 and 90.

![Figure 1 Laminate composite](image)

II. Methodology

1. Ansys approach
As we all know, in engineering there are some basic variables with which other parameters vary like in mechanics on varying the displacement, strain produced varies which will vary the magnitude of stress and thus the magnitude of strain energy is also varied. These variables in finite element terminology are called field variables. Practically in analysis of any work piece there are infinite numbers of small elements, finite element method discretize the infinite number of element into a finite number of domains. These domains in Finite Element terminology are called elements or finite elements.

Now each element is governed by certain interpolation functions which are called shape functions. These shape functions are defined in terms of field variables at specified points. These points in finite element method terminology are known as nodes. After all these steps one has to assemble element properties for each element. Once this is done the boundary conditions are being applied on the discritized element. Then by using numerical methods the solution is done. Certain other calculations are also done for some other results.

2. B- matrix
In present work the isoperimetric element is considered to have 3 degree of freedom at each node. The first one is displacement in z-direction, second one is rotation about x-axis and third one is rotation about y-axis. It should also be noted that the displacement along x-axis and y-axis are supposed to be negligible. So the displacement denoted by ‘d’ will be a function of w, \( \theta_x \) and \( \theta_y \).

\[ s = f(w, \theta_x, \theta_y) \]
The general function of displacement at any nodal point in isoperimetric element is given by

\[ w = \sum_{i=1}^{n} N_i w_i \]

where \( \theta_x = \sum_{i=1}^{n} N_i \theta_i \) and \( \theta_y = \sum_{i=1}^{n} N_i \theta_i \).

So in order to define the general displacement function in isoperimetric element the following function is valid.

\[ s = \sum_{i=1}^{n} N_i s_i \]

Here, \( n \) is the no. of nodes in the element. As defined in chapter three the shear strains and bending curvatures can be written in terms of displacement as follows.

\[ \{\varepsilon\} = L_B s \}

\[ \{\varphi\} = L_s s \]

where \( L_B \) and \( L_s \) are matrix whose elements are differential in nature as per equation.

\[ L_B = \begin{bmatrix} 0 & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \]

\[ L_s = \begin{bmatrix} \frac{\partial}{\partial x} & 1 & 0 \\ \frac{\partial}{\partial y} & 0 & 1 \end{bmatrix} \]

Now substituting the value of \( d \) in strain equation, one gets the relation of strains with displacement in the isoperimetric element.

\[ \{\varepsilon\} = L_B \sum_{i=1}^{n} N_i d_i = \sum_{i=1}^{n} B_{ib} s_i \]

\[ \{\varphi\} = L_s \sum_{i=1}^{n} N_i d_i = \sum_{i=1}^{n} B_{is} s_i \]

where,

\[ B_{ib} = \begin{bmatrix} 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \]

\[ B_{is} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 1 & 0 \\ \frac{\partial N_i}{\partial y} & 0 & 1 \end{bmatrix} \]

The total B matrix for the isoperimetric element is given by

\[ B = \begin{bmatrix} B_{ib} \\ B_{is} \end{bmatrix} \]

3. Variation Formulation

As we know the total potential energy of any body is the difference of internal potential energy and external potential energy. For the purpose of minimization this function should be differentiated with respect to the variable in the above function which in this case is displacement denoted by \( \rho \). The total potential energy of structure is given by

\[ \pi = \sum_{e=1}^{E} \pi_e + \sum_{w=1}^{W} W_e \]  \hspace{1cm} (31)

Here,

\[ U_e = \int f \frac{1}{2} \{\varepsilon\}^T \{\sigma\} dv \]

\[ W_e \] is the work done by external forces.

Also,

\[ \{\varepsilon\} = [B]\{\rho\} \]

\[ \sigma = [D]\{\varepsilon\} = [D][B]\{\rho\} \]

Using the above equation the strain energy of body will become

\[ U_e = \frac{1}{2} \int \int \{\rho\}^T [B]^T [D] [B] \{\rho\} dv \]

This equation represents strain energy of body

Let \( F_b \) and \( F_s \) are the resultant bending and shear forces that act on the body, these forces will produce an external work on the body producing the resultant displacement’s. Then the external work done will be given by following.

\[ W_e = - \int \int \{s\}^T F_b dv - \int \int \{s\}^T F_s ds \]

So by above equations the net potential energy becomes

\[ \pi = - \int \int \{\rho\}^T \{N\}^T [F_b] dv + \int \int \{\rho\}^T \{N\}^T [F_s] ds + \frac{1}{2} \int \int \{\rho\}^T [B]^T [D] [B] \{\rho\} dv \]

So by solving the above equation the stiffness matrix is obtained

\[ [K] = \int \begin{bmatrix} 1 & dV \\ B^T D B \end{bmatrix} \]

Here \([J]\) is determinant of jacobian matrix \([J]\).

4. Post computations and validation

In order to compute stress and strains, first one needs to generate assemble equation upon solving which the generalized displacements at the nodes are found out. Then the boundary conditions are imposed on the model. The model is then checked for errors if any. The problem is then solved and the contour plots are being obtained for stresses and strains of various stacking orientation.

The results obtained by above mathematical modeling can be validated by using classical laminate plate theory which suggests

\[ \{N\} = \begin{bmatrix} A & B \end{bmatrix} \{\varepsilon\} \]

\[ \{M\} = \begin{bmatrix} B & D \end{bmatrix} \{K\} \]

Where

A is extensional stiffness matrix
B is extensional bending coupling matrix
D is bending matrix

III. Results and discussion

1. Material Properties

The material which is being used for the purpose of analysis of laminates in this study is Boron epoxy whose properties are as follows:

- Young’s Modulus: 300 GPa
- Poisson’s Ratio: 0.3
- Density: 1500 kg/m³
Table 1 Material properties

2. Geometry and stacking
   The sample is in form of square plate with 1*1 sq meter area. The thickness of each lamina in the stacking is supposed to be 10 mm. In the current study the stacking arrangements for two layered, four layered and six layered cross ply laminates are given in table below.

Table 2 Stacking Pattern

<table>
<thead>
<tr>
<th>Failure Load (kp)</th>
<th>Stacking patterns</th>
<th>stress_{int} (kp)</th>
<th>deflection_{int} (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>76</td>
<td>0/90</td>
<td>63.19</td>
<td>0.1728</td>
</tr>
<tr>
<td>76</td>
<td>90/0</td>
<td>120.83</td>
<td>0.136</td>
</tr>
<tr>
<td>152</td>
<td>90/0/0/0/90</td>
<td>297.88</td>
<td>0.303</td>
</tr>
<tr>
<td>152</td>
<td>0/90/0/90</td>
<td>268.52</td>
<td>0.384</td>
</tr>
<tr>
<td>152</td>
<td>0/90/90/0</td>
<td>140.43</td>
<td>0.397</td>
</tr>
</tbody>
</table>

Table 3 Results

The stacking arrangement of one case from two layered laminate, four layered laminate and one case from six layered laminate is shown if below images on ansys mechanical APDL.

3. Element
   The discretization of entire sample is done. The element chosen for the purpose is 8 noded serendipity element. It is selected for the purpose of sake of minimization of error. The meshed sample is shown below.

4. Results
   After the application of load the above results will be obtained.
The stress variation pattern for four layered case along the central path i.e. along the line which bisects the square into two equal halves. However it is interesting to note down that stress variation pattern is same for all stacking arrangements of four layered and two arrangements of two layered laminates.

Figure 7 Stress v/s stacking pattern plot

Figure 5 and 6 are the plots of resultant, longitudinal and transverse stresses w.r.t. distance. Where as figure 7 are the graphical representation of stress and deflection in various stacking pattern. Figure 7 is stress v/s stacking pattern graph.

IV. Observation

A- 0/90
B- 90/0

The above curve is stress deflection curve pointing out the stress and deflection developed at failure load in two layered laminates. From the above plot it is clear that the stacking pattern {0/90} is the best configuration as it can sustain maximum stress with minimum deflection and thus such stacking pattern can serve better than other for same loading conditions.

C-90/0/0/90
D-0/90/0/90
E-0/90/90/0

The above curve is stress deflection curve pointing out the stress and deflection developed at failure load in four layered laminates. From the above plot it is clear that the stacking pattern {90/0/0/90} is the best configuration as it can sustain maximum stress with minimum deflection and thus such stacking pattern
can serve better than other two for same loading conditions.

![failure load v/s Layer number](image)

**Figure 10 failure load v/s no. of layers**

The above graph makes it clear the increase in layer number will increase the strength of composite material.

### V. Conclusion

In the current study the mathematical modeling of four layered and six layered cross ply composite is done. The element of discretization is 8 node quadratic element. The reason for adopting this element for analysis purpose is that it gives more precise result as compared to other elements. The simulation is run in ansys for different stacking pattern of four layered and six layered cross ply composite laminate. Interesting fact is that with increase in number of laminas the stress and deflection bearing capacity increases. Also the failure load is being increased by about 40 percent on adding two laminas in four layered cross ply laminate. It is also seen that on variation in stacking pattern the stress and deflection patterns are varied despite of same number of laminas. Hence the stacking pattern is an important characteristic while designing the composite plate.

### References

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