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# Optimization of Time Restriction in Construction Project Management Using Lingo and M.S.Excel

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**ABSTRACT**
This study is an attempt to identify the minimum time of a construction project using the critical path method and linear programming model. A systematic analysis is attempted by developing a work breakdown structure for entire project to establish work elements for quantifying various resources against time and cost. A network is established taking into consideration all the predecessor and successor activities. The network is then optimized through crashing of activities so as to obtain optimal solution and serves as a base for optimizing total project cost. Finally, linear programming model is used to formulate the system of crashing network for minimum time by LINGO model and Microsoft Excel. These models consider many considerations of project thus reducing the duration of project. Ultimately, comparison of both the software outputs and the manual calculations is done and the best verifier is determined.

***Keywords -*** Linear Programming, Crashing, CPM, LINGO, Microsoft Excel etc.

## I. INTRODUCTION

Construction industry is one of the largest industries and encompasses projects of all scales like highway, bridge, sky-scrappers, dams, canals, flyovers etc. These projects are complex and consist of large number of activities. Each activity requires certain amount of resources such as time, labor, material, machineries and money. Basically it is a combination of multiple activities, which are inter-related with one another and must be executed in some particular order and specified time limit to complete the entire task.

Construction planning is a fundamental and challenging activity in the management and execution of construction project involving the choice of technology, the definition of work tasks, the estimation of the required resources and durations for individual tasks, and ultimately the identification of any inter-relations among the different work tasks.

Linear Programming deals with the optimization of a function of variables known as the 'objective function', subject to a set of linear equations with inequalities known as 'constraints'.

## II. OBJECTIVES OF PRESENT STUDY

1 A construction project is to be selected to demonstrate the applicability.
2 Linear Programming optimization objective function, constraints and variables are to be determined so as to minimize the construction project duration.
3 This manual linear programming approach is then verified by suitable software.

## III. BASIC PROBLEMS IN PROJECT FACED BY THE MANAGEMENT

With the scarce resources and limited time it is the prime duty of the management to look after the following areas for the better management

1. Completion time of the project.
2. Activities that is critical for the successful completion of the project.
3. Whether enough resources are available?
4. Whether a project is running on schedule?
5. Whether the money spent is within the budgeted cost at any particular time?
6. If the project is to be finished in shorter time what activities should be crashed?
7. What is the best way to accomplish the least cost?

## IV. NETWORK TECHNIQUES IN CONSTRUCTION INDUSTRY

Bar chart/ Gantt chart: A bar chart has two coordinates, one representing the time (x-axis) and the other representing the work or activity to be performed. Each activity is depicted in the form of a horizontal line or a bar. The length of these bars indicates the duration of the activity. In a project some activities should be taken concurrently and some are required to be completed before or after the other works. That's the reason in this bar chart some bars run parallel and some serially, with one bar beginning after the other ends.

Critical Path Method (CPM): Critical Path Method is the process of applying a logical order to the activities as defined in the work orders. Basically it is a management technique, which has to plan, schedule and control the complex projects. It includes:

¬ Project planning
¬ Project scheduling
¬ Project controlling

The crashing is done to optimize the project. The various assumptions are:

¬ $N_T$ = Normal Time
¬ $C_T$ = Crash Time
¬ $N_C$ = Normal Cost
¬ $C_C$ = Crash Cost
¬ Crash time = Maximum time
¬ Crash time < Normal time

Crash cost is formulated using the formula

Crash Cost = (Normal time/ Crash time) X Normal Cost

$C_C = (N_T/C_T) \times N_C$ ……..Eq (4.1)

The maximum compression is obtained by the difference of normal time and cost time

Max compression = $N_T$-$C_T$ ……Eq (4.2)

Cost slope is obtaining by dividing the difference of crash cost and normal cost by the difference of normal time and crash time

Cost slope = $(C_C-N_C)/(N_T-C_T)$ ……Eq (4.3)

Once all the above calculations are made, crashing is started.

Here total cost also changes due to crashing, as the number of days decreases, the cost increases.

Therefore total cost after crashing = direct cost + increase in cost + indirect cost …....Eq (4.4)

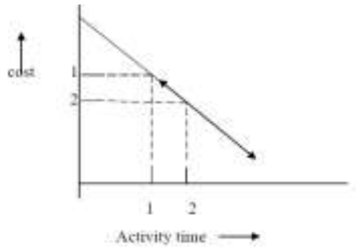
Direct cost = The summation of all the normal costs .……Eq (4.5)

Increase in cost = Maximum compression X Cost slope ……Eq (4.6)

Indirect cost = Number of days X Increase in cost per day ……Eq (4.7)

## V. CRASHING OF PROJECT USING LINEAR PROGRAMMING

A mathematical optimization model consists of an objective function and a set of constraints in the form of system of equalities or inequalities. These optimization models are used in almost every field of decision making, such ad engineering, design and financial portfolio selection.




The basic formulation in a time-cost trade-off problem is a U shaped curve. Once the cost details of the activities are available, the formulation of linear programming is possible. At this point it must be recognized that shortening the critical path leads to the reduced floats of their activities. Mathematical programming is becoming increasingly important, to have a sound system of time and cost control construction projects. In this work, Linear programming is used as a mathematical model to describe the problem with the optimization of linear objective function subjected to a set of constraints in the form of equalities or inequalities.

Linear programming analysis can be used to maximize or minimize a linear function subjected to a finite number of linear constraints.

## VI. SOFTWARE'S REQUIREMENTS

The software's used in this project are

1. Lingo model.
2. Microsoft Excel.

## VII. METHODOLOGY

In order to achieve the objectives of the research the following tasks are performed:

**1. Manual calculations :**

Manual calculations are done using the formulae given below

Crash cost is obtained by dividing the normal time by crash time and multiplying with normal cost

**Crash Cost = (Normal time/ Crash time) X Normal Cost**

**$C_C = (N_T/C_T) \times N_C$** ……..Eq (7.1)

The maximum compression is obtained by the difference of normal time and cost time

**Max compression = $N_T$-$C_T$** ……Eq (7.2)

Cost slope is obtaining by dividing the difference of crash cost and normal cost by the difference of normal time and crash time

**Cost slope = $(C_C-N_C)/(N_T-C_T)$** ……Eq (7.3)

Once all the above calculations are made, crashing is started.

Here total cost also changes due to crashing, as the number of days decreases, the cost increases.

Therefore **total cost after crashing = direct cost + increase in cost + indirect cost** …....Eq (7.4)

**Direct cost = the summation of all the normal Cost** ....Eq (7.5) ……Eq (7.1.5)

**Increase in cost = Maximum compression X Cost slope**
…Eq (7.6)
**Indirect cost = Number of days X Increase in cost per day** ……Eq (7.7)

**2. LINGO Calculations :**
LINGO calculation is done based on the formulae given below
**Defining the earliest start by:**
LTASK = @SIZE( TASKS);
@FOR( TASKS( J)| J #GT# 1:
ES( J) = @MAX( PRED( I, J):
ES(I) + TIME(I)-CRASH(I)));
**Defining the earliest finish by:**
@FOR( PRED( I, J):
EF( J) >= EF( I) + TIME( J) - CRASH( J));
**Defining the latest start by:**
@FOR( TASKS( I)| I #LT# LTASK:
LS( I) = @MIN( PRED( I, J):
LS( J) - TIME( I)+CRASH(I)););
**Defining the latest finish by:**
@FOR( TASKS( I): CDUR( I) = TIME( I) - CRASH( I));
@FOR( TASKS( I): LF( I) = LS( I) + CDUR( I));
**Calculation of slack time by:**
@FOR( TASKS( I): SLACK( I) = LS( I) - ES( I));

**Calculation of max crash time by:**
@FOR( TASKS( J):
CRASH( J) <= TIME( J) - TMIN( J));
**Calculation of due date by:**
DUEDATE= ES (LTASK);
ES( 1) = 0;
EF( @SIZE( TASKS)) <= DUEDATE;
ES( LTASK) - ES(1) = DUEDATE;
**Calculation of crash cost by:**
MIN=@SUM (TASKS: CCOST * CRASH);
END

**3. Excel calculations :**
Excel calculations has three steps as explained below:
**Step 1:** Determination of crash cost
**Step 2:** Defining variables
**Step 3:** Drawing the network diagram according to the preceding and succeeding activities
**Step 4:** Inputting the Linear Programming model by giving the preceding activity '-1' co-efficient and the corresponding succeeding activity '1' and its difference in cost ($N_C$-$C_C$) as '1'
**Step5**: Inputting the solver formula "**SUMPRODUCT (array1, array2)**"
Where array 1 is the solution row and array 2 is the row showing the difference in cost ($N_C$-$C_C$). Also input the crash and normal time
**Step 6**: Inputting all the values in solver window Target cell is the cell for total increase in cost changing cells is the solution row Constraints is the column with the formula SUMPRODUCT, their relations and the corresponding times

## VIII. PRESENT INVESTIGATION

The construction of HLB at Km 6/2 of Chirrakunta-Venkatapuram is considered for Road (Via) Lemur in Adilabad District. Authority and funding agency is A.P.Rural Development Fund (APRDF). The road is rural and connecting to Mandamarri town (S.H.1) and N.H-16. There is an bridge crossing at 6/2.six villages and will be connected to N.H-16. By construction of a bridge un-interrupted transportation facility is provided for agricultural goods and forest.

**8.1 Data:**
**Bridge location feature @ km 6/2**

- Alignment: The HLB is proposed in normal crossing.
- Traffic particulars: Intensity of traffic 100 CVPD.

**Hydraulic and sub soil particulars**
a) vagu crosses the alignment in normal crossing (skew &deg).
Complete hydraulic data including scour depth details:
Flood discharge (A.V/C.A method): 156.00cumees
MFL : +97.45
Velocity: 2.331m/sec
Effective L.W.W : 23.65M
Span arrangement: 6 vents of 6.0M effective span as per SD
Sill level: +94.62M
RCL : +99.175M
Max scour depth for piers: 7.87M
Abutments: 5.00M
Sub-soil particulars: As per the bore charts Sand/ Silt and/ clay exists up to a depth of 9.0m and soft Clay stone is available as follows:
Raft foundations are recommended:

- raft top level +93.62M
- raft bottom level +92.62M
- wing walls +94.51M

**Bridge details and specifications:**
Foundations: Raft foundations
Wing walls: Box type wings are proposed with open foundations with skin reinforcement of 8mm dia at 150mm c/con both ways as per drawing.
Abutment: Wall type abutments are proposed in VCC M15 grade concrete with skin reinforcement of 8mm dia at 150mm c/c on both ways as per drawing.
Piers: Wall type abutments are proposed in VCC M15 grade concrete with skin reinforcement of 8mm dia at 150mm c/con both ways as per drawing
Bed blocks over abutments & piers: bed blocks are proposed in VRCC M20 grade as per drawing

Backing walls over abutments: Bed blocks and backing walls are proposed in VRCC M20 grade as per drawing
Wing walls: Box type wing walls are proposed for a length of 4.0M with skin reinforcement in VCC M15 grade concrete as per drawing.
Super structure
Deck slabs of 6.00m (effective) span without foot paths are provided as per SD drawing No's SD/110.
10.90M carriage way, 12.0M outer to outer width
Hand rails and hand posts: RCC m20 grade as per SD Drawing No SD/105
Wearing coat: As per drg.no.BD/1-9A in VRCC M30. Thickness=75mm.
Drainage Spouts: Drainage spouts are proposed as per SD/103
Approach slab: As per SD drg.no.in VRCC M30.
Length=3.90M, width10.90M, Thickness=(435+300)/2mm
Leveling course below approach slab: PCC M15 grade with 150mm thick is proposed
Concrete quantity and steel quantities:
Concrete quantity per span = 42.433 cum (as per dwg no SD/110)
Steel quantity per span =2.825MT (as per dwg no SD/ 110)
Bridge length the total bridge length from back to back of backing walls is 38.54m
Protection works:
Stone revetment: 330mm thick grouted revetment 150mm thick granular filling is proposed under grouted revetment for quadrants
Estimate quantities and specifications adopted
The estimate quantities are calculated based on approved drawings and the specifications for the bridge work are framed based on specifications for the road and bridge works. (Fourth revision, 2001) of MORTH

L.S. provisions provided for bridge proper
1 confirmatory bores
2. Mastic pads, paper bearing, copper strip
3. Pylon construction
4. Painting to railing

**Bridge approaches:**

The bridge approaches are proposed with reference to R.C.L of the bridge (RCL +99.175m) and it is proposed to provide 15.00m level portion on either side of bridge.

Nature of existing soils: BC soils
Chainages:
Towards Chain ages length gradient starting ending
Chirrankunta side 219.0M
Venkatapur side: 138.0M
Formation width: 10.00M
Side slopes: 1H: 2.0V
Carting lead: borrowed earth with 3.0 km lead
Type of compaction: OMC
Carriageway: 7.00M
Crust provided: granular sub base- 150mm thick
Wet Mix Macadam 225mm thick
S.D.B.C 25mm thick
Estimate quantities and specifications: The estimate quantities are calculated based on approved drawings and the specifications for the bridge work are framed based on specifications for road and bridge works (fourth revision, 2001) of MORTH.
L.S. provisions provided for bridge approaches
Caution boards, name boards & guide posts
The work shall be carried out as per approved, designs and drawing and MORTH specifications, relevant IRC and IS codes, relevant circulars issued by the department from time to time.

### 8.2 ANALYSIS

**Major Work breakdown structure**

A) Earth work in excavation of foundation of structure.
B) P.C.C nominal mix in foundation.
C) HYSD bar reinforcement in foundations.
D) Vibrated cement concrete in open foundation M-15G for footings of abutments piers and wings and aprons @ cut off walls.
E) HYSD bar reinforcement in substructure.
F) Vibrated cement concrete in substructure on M-15G for abutments, piers & wings,
G) HYSD bar reinforcement in super-structure.
H) Furnishing and placing reinforced cement concrete M-20G in super –structure.
I) P.C.C. M-15G leveling course below approach slab
J) Reinforced cement concrete M-30G for approach slab.
K) Providing and laying cement concrete wearing coat M-30G.
L) Construction of precast VRCC-railing M-20G.
M) Vibrated Reinforced cement concrete in substructure VRCC M-20 substructure bed blocks & backing walls,
N) Providing and laying filter material with graver underneath pitching in slopes under revetment.
O) Providing grouted revetment.
P) Providing rough stone revetment 300mm thick for toe wall.
Q) Backfilling behind abutments, wing walls, return walls& providing weep holes
R) Providing and laying of filters media, over the entire surface behind abutments, wing wall and return walls and on river beds and drainage spouts.

**Table 8.1: Work Breakdown Structure**

| SI. NO. | DESCRIPTION | MAL TIME (DAY | MAL COST (RS.) |
|---|---|---|---|
| **A** | Earthwork in excavation of | 60 | 169110 |

| | | | |
|---|---|---|---|
| | foundation of structure | | |
| **B** | P.C.C. nominal mix in foundation | 19 | 456280 |
| **C** | HYSD bar reinforcement in foundation | 10 | 352185 |
| **D** | Vibrated cement concrete in open foundation M-15G for footings of abutments, piers, wings and aprons @cut off walls | 42 | 4007260 |
| **E** | HYSD bar reinforcement in sub structure | 13 | 4822080 |
| **F** | Vibrated cement concrete in substructure on M-15G for abutments, piers & wings | 25 | 1515850 |
| **G** | HYSD bar reinforcement in super structure | 16 | 1172105 |
| **H** | Furnishing and placing reinforced cement concrete M-20G in superstructure | 25 | 1286985 |
| **I** | P.C.C. M-15G leveling course below approach slab | 5 | 44810 |
| **J** | Reinforced cement concrete M-30G for approach slab | 12 | 146660 |
| **K** | Providing and laying | 12 | 171305 |
| | cement concrete M-30G for approach slab | | |
| **L** | Construction of precast VRCC-railing M-20G | 14 | 89300 |
| **M** | Vibrated Reinforced cement concrete in substructure bed blocks & backing walls | 12 | 134995 |
| **N** | Providing and laying filter material with gravel underneath pitching in slopes under revetment | 5 | 6880 |
| **O** | Providing grouted revetment | 10 | 42410 |
| **P** | Providing rough stone revetment 300mm thick for toe wall | 5 | 10950 |
| **Q** | Backfilling behind abutment, wing walls, return walls & providing weep holes | 19 | 94045 |
| **R** | Providing and laying of filers media, over the entire surface behind abutments, wing wall and return walls and on river beds and drainage spouts | 26 | 417480 |

**Table 8.2: Work durations**

| Activity | Predecessor |
|---|---|
| **A** | - |
| **B** | A |
| **C** | A,B |
| **D** | C |
| **E** | C,D |
| **F** | E |
| **G** | E,F |
| **H** | G |
| **I** | H |
| **J** | I |
| **K** | J |
| **L** | K |
| **M** | K,L |
| **N** | M |
| **O** | N |
| **P** | O |
| **Q** | P |
| **R** | Q |

**Table 8.3: Activities and their corresponding predecessors**

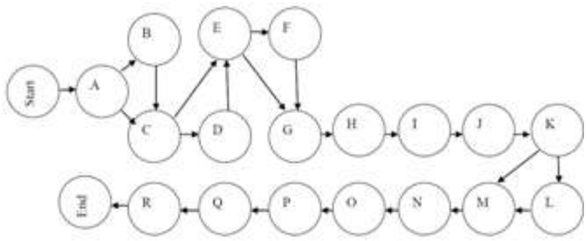



Fig 8.1: Network diagram

**8.3 Crashing of the time**

Calculating the crash time as done in working methodology in the previous chapter we take activity A as an example.
Activity A has 60 days as its normal time.
We crash the 60 days by removing 40% of the 60 days as explained in the chapter
Finally we get the crash time as 24 days.
Hence calibrating in the same manner for all the activities we get the crash time of various activities.
Calculation of crash cost
Activity A has a total cost or normal cost of 169111.
Crash cost = (normal time / crash time) x normal cost
Cc = (Nt / Ct) x Nc
Cc = (60 / 24) x 169110 = 422775
Hence calibrating in the same manner for all the activities we get the crash cost of various activities.

**To calculate Maximum compression:**

After getting the crash cost we proceed by obtaining the maximum compressions

| Activity | Duratio n ($N_T$) | Earliest Start (EST) | Earliest Finish (EFT) | Latest Start (LST) | Latest Finish (LFT) | $T_e^j$ | $T_e^i$ | Float Total | Float Free | Float Input |
|---|---|---|---|---|---|---|---|---|---|---|
| **A** | **60** | **0** | **60** | **0** | **60** | **60** | **0** | **0** | **0** | **0** |
| **B** | **19** | **60** | **79** | **60** | **79** | **79** | **60** | **0** | **0** | **0** |
| **C** | **10** | **79** | **89** | **79** | **89** | **89** | **79** | **0** | **0** | **0** |
| **D** | **42** | **89** | **131** | **89** | **131** | **131** | **89** | **0** | **0** | **0** |
| **E** | **13** | **131** | **144** | **131** | **144** | **144** | **131** | **0** | **0** | **0** |
| **F** | **25** | **144** | **169** | **144** | **169** | **169** | **144** | **0** | **0** | **0** |
| **G** | **16** | **169** | **185** | **169** | **185** | **185** | **169** | **0** | **0** | **0** |
| **H** | **25** | **185** | **210** | **185** | **210** | **210** | **185** | **0** | **0** | **0** |
| **I** | **5** | **210** | **215** | **210** | **215** | **215** | **210** | **0** | **0** | **0** |
| **J** | **12** | **215** | **227** | **215** | **227** | **227** | **215** | **0** | **0** | **0** |
| **K** | **12** | **227** | **239** | **227** | **239** | **239** | **227** | **0** | **0** | **0** |
| **L** | **14** | **239** | **253** | **239** | **253** | **253** | **239** | **0** | **0** | **0** |
| **M** | **12** | **253** | **265:** | **253** | **265** | **265** | **253** | **0** | **0** | **0** |
| **N** | **5** | **265** | **270** | **265** | **270** | **270** | **265** | **0** | **0** | **0** |
| **O** | **10** | **270** | **280** | **270** | **280** | **280** | **270** | **0** | **0** | **0** |
| **P** | **5** | **280** | **285** | **280** | **285** | **285** | **280** | **0** | **0** | **0** |
| **Q** | **19** | **285** | **304** | **285** | **304** | **304** | **285** | **0** | **0** | **0** |
| **R** | **26** | **304** | **330** | **304** | **330** | **330** | **304** | **0** | **0** | **0** |

Maximum compression is obtained taking the difference of normal time and crash time
Max compression = Nt- Ct
Hence for activity A the normal cost is 60 days and crash time is 24 days (60–24=36)
So maximum compression for activity A is 36 days
Calculating in the following manner we get the maximum compression of critical and noncritical activities as in table below.
To calculate the cost slope
For calculation of cost slope by following the procedure written in the working methodology
Cost slope = (Cc-Nc) / (Nt-Ct)
Cost slope = (422775-169110) / (60-24)
Cost slope = 70476.25

The table below shows the calculation of normal time normal cost, crash time and crash cost maximum compression and cost slope of each activity in our project.

**Table 8.4: Slope calculation**

| SI. NO. | Activity | Time (days) | | Cost (Rs.) | | Max Compression | Slope Or Crash Cost Per Week |
|---|---|---|---|---|---|---|---|
| | | Normal | Crash | Normal | Crash | | |
| | | (NT) | (CT) | (NC) | (CC) | NT-CT | (CC-NC)/(NT-CT) |
| 1 | **A** | 60 | 24 | 169110 | 422775 | 36 | 7046.25 |
| 2 | **B** | 19 | 13 | 456280 | 666870 | 6 | 35098.33 |
| 3 | **C** | 10 | 7 | 352185 | 503121 | 3 | 50312 |
| 4 | **D** | 42 | 24 | 4007260 | 7012705 | 18 | 166969.16 |
| 5 | **E** | 13 | 9 | 4822080 | 6965227 | 4 | 535786.75 |
| 6 | **F** | 25 | 15 | 1515850 | 2526417 | 10 | 101056.7 |
| 7 | **G** | 16 | 11 | 1172105 | 1704880 | 5 | 106555 |
| 8 | **H** | 25 | 15 | 1286985 | 2144975 | 10 | 85799 |
| 9 | **I** | 5 | 4 | 44810 | 56013 | 1 | 11203 |
| 10 | **J** | 12 | 8 | 146660 | 219990 | 4 | 18333.5 |
| 11 | **K** | 12 | 8 | 171305 | 256958 | 4 | 21413.25 |
| 12 | **L** | 14 | 10 | 89300 | 125020 | 4 | 8930 |
| 13 | **M** | 12 | 8 | 134995 | 202493 | 4 | 16875.5 |
| 14 | **N** | 5 | 4 | 6880 | 8600 | 1 | 1720 |
| 15 | **O** | 10 | 7 | 42410 | 60586 | 3 | 6058.67 |
| 16 | **P** | 5 | 4 | 10950 | 13688 | 1 | 2738 |
| 17 | **Q** | 19 | 13 | 94045 | 137450 | 6 | 7234.167 |
| 18 | **R** | 26 | 15 | 417480 | 723632 | 11 | 27832 |

**Crashing of activities**

As we have identified the critical path we tabulate the critical activities.
Critical activities are those which lie on the critical path and noncritical activities are those which do not lie on the critical path.
After tabulating them we find the minimum slope of each activity by numbering them with the activity having minimum slope as 1 ,2,3,4,……….accordingly

The activities having the same slope are numbered by the same number

We then start the crashing of activities by first crashing the critical activity which has the minimum value of the cost slope.

**8.3 Calculations:**

**1. Manual Calculations :**

Total cost after crashing = Direct cost + Increase in cost indirect cost
Direct cost = 14940676
Increase in cost = maximum compression x slope
Indirect cost = number of days x increase in cost per day (As by the assumption made in working methodology, increase in cost per day = 0.05% of 14940676 = 7470)
So finally we get

Total cost = 14940690 + (36 X 7046.25 + 6 X 35098.33 + 3 X 50312 + 18 X 535786.75 + 4 X 535786.757 + 10 X 101056.7 + 5 X 106555 + 10 X 857990 + 1 X 11203 + 4 X 18332.5 + 4 X 21413.25 + 4 X 8930 + 4 X 16874.5 + 1 X 1720 + 3 X 6058.67 + 1 X 2738 + 6 X 7234.167 + 11 X 27832) + 7470
=14940690 + **8810710** + 7470
=23758870 Rs.

**The total project cost is increased from 1.49crores to 2.375crores (increase in the project cost = 8810710) and the project duration is reduced from 330 days to 199 days**

**2. LINGO calculations :**

The following input is given into the Lingo software
MODEL:!A CPM Model with crashing;
SETS:
TASKS/START,A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R,LTASK/:
TIME, !Normal time for task;
ES,!Earliest start;
LS,!Latest start;
SLACK,!Slack;

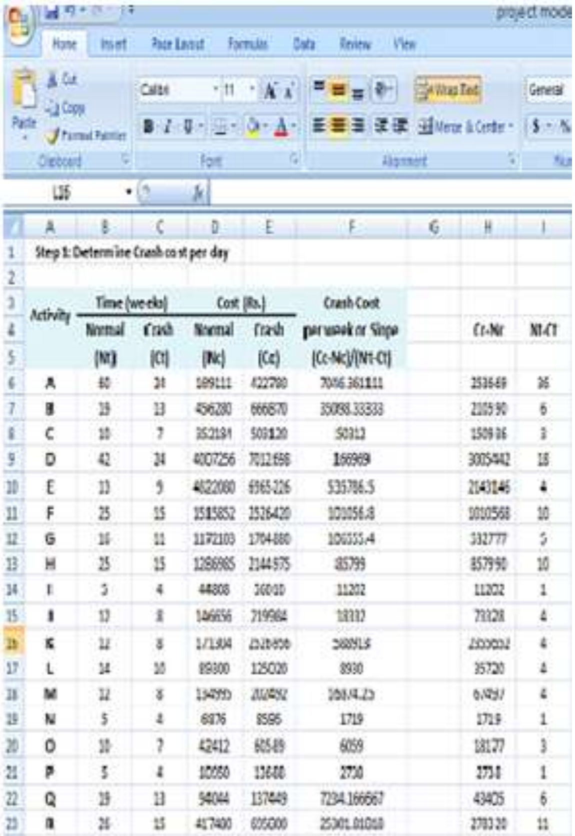
```
EF,!Earliest finish;
LF,!Latest Finish;
CRASH,!Amount of crashing;
CDUR,!Crashed duration time;
TMIN,!Min time at max crash;
CCOST;!Crash cost/unit time;
! Here are the precedence relations;
PRED(TASKS,TASKS)/START,A,A,B,A,C,B,C,C,
D,C,E,D,E,E,F,E,G,F,G,G,H,H,I,I,J,J,K,K,L,K,M,L,
M,M,N,N,O,O,P,P,Q,Q,R,R,
LTASK/;
ENDSETS
DATA:
TIME =
0,60,19,10,42,13,25,16,25,5,12,12,14,12,5,10,5,19,
26,0;  !Normal times;
TMIN = 0,
24,13,7,24,9,15,11,15,4,8,8,10,8,4,7,4,13,15,0;
!Crash times;
CCOST =
0,7046.25,35098.33,50312,166969.16,535786.75,101
056.7,106555,85799,11203,18332.5,21413,8930,168
75.5,1720,6058,2738,
7234,27832,0; !Cost /day to crash;
DUEDATE=199; !Project due date;
ENDDATA
!The crshing LP model;
!Define earliest start, each successor of a task constraints when the earliest time the task can be completed. The earliest the succeeding task can be finished plus the time required for the task minus any time that could be reduced by crashing this task.;
LTASK = @SIZE( TASKS);
@FOR( TASKS( J)| J #GT# 1:
ES( J) = @MAX( PRED( I, J):
ES(I) + TIME(I)-CRASH(I)));
!Define earliest finish, each predecessor of a task constraints when the earliest time the task can be completed. The earliest the preceding task can be finished plus the time required for the task minus any time that could be reduced by crashing this task.;
@FOR( PRED( I, J):
EF( J) >= EF( I) + TIME( J) - CRASH( J));
!Define latest start, each predecessor of a task constraints when the latest time the task can be completed. The latest the preceding task can be finished minus the time required for the task plus any time that could be reduced by crashing this task.;
@FOR( TASKS( I)| I #LT# LTASK:
LS( I) = @MIN( PRED( I, J):
LS( J) - TIME( I)+CRASH(I)););
!Define latest finish, each successor of a task constraints when the latest time the task can be completed. The latest the succeeding task can be finished plus the time required for the task plus any time that could be reduced by crashing this task.;
@FOR( TASKS( I): CDUR( I) = TIME( I) - CRASH( I));
@FOR( TASKS( I): LF( I) = LS( I) + CDUR( I));
!Calculates the slack times of each activity;
@FOR( TASKS( I): SLACK( I) = LS( I) - ES( I));
!For each task, the most it can be crashed is the regular time of that task minus minimum time for that task;
@FOR( TASKS( J):
CRASH( J) <= TIME( J) - TMIN( J));
!Meet the due date;
DUEDATE= ES (LTASK);
!This assumes that there is a single last tasks;
ES( 1) = 0;
EF( @SIZE( TASKS)) <= DUEDATE;
ES( LTASK) - ES(1) = DUEDATE;
!Minimize the sum of crash costs;
MIN=@SUM (TASKS: CCOST * CRASH);
END
```

**The total cost of the project is increased from 1.49crores to 2.375crores (increase in the project cost = 8810710) and the project duration is reduced from 330 days to 199 days**

### 3. Excel calculations :

Excel calculations have three steps as explained below:

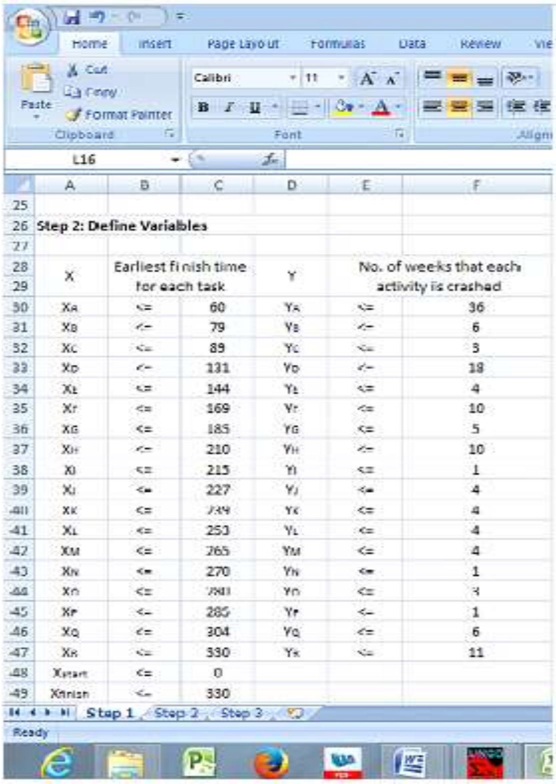
**Step 1:** Determination of crash cost



Step 1: Determine Crash cost per day

| Activity | Time (weeks) Normal (Nt) | Time (weeks) Crash (Ct) | Cost (Rs.) Normal (Nc) | Cost (Rs.) Crash (Cc) | Crash Cost per week or Slope (Cc-Nc)/(Nt-Ct) | Cc-Nc | Nt-Ct |
|---|---|---|---|---|---|---|---|
| A | 60 | 24 | 169111 | 422780 | 7046.361111 | 253669 | 36 |
| B | 19 | 13 | 456280 | 666870 | 35098.33333 | 210590 | 6 |
| C | 10 | 7 | 352184 | 503120 | 50312 | 150936 | 3 |
| D | 42 | 24 | 4007256 | 7012698 | 166969 | 3005442 | 18 |
| E | 13 | 9 | 4822080 | 6965226 | 535786.5 | 2143146 | 4 |
| F | 25 | 15 | 1515852 | 2526420 | 101056.8 | 1010568 | 10 |
| G | 16 | 11 | 1172103 | 1704880 | 106555.4 | 532777 | 5 |
| H | 25 | 15 | 1286985 | 2144975 | 85799 | 857990 | 10 |
| I | 5 | 4 | 44808 | 56010 | 11202 | 11202 | 1 |
| J | 12 | 8 | 146656 | 219984 | 18332 | 73328 | 4 |
| K | 12 | 8 | [illegible] | [illegible] | [illegible] | [illegible] | 4 |
| L | 14 | 10 | 89300 | 125020 | 8930 | 35720 | 4 |
| M | 12 | 8 | 134995 | 202492 | 16874.25 | 67497 | 4 |
| N | 5 | 4 | 6876 | 8595 | 1719 | 1719 | 1 |
| O | 10 | 7 | 42412 | 60589 | 6059 | 18177 | 3 |
| P | 5 | 4 | 10950 | 13688 | 2738 | 2738 | 1 |
| Q | 19 | 13 | 94044 | 137449 | 7234.166667 | 43405 | 6 |
| R | 26 | 15 | 417400 | 695300 | 25301.01010 | 278320 | 11 |

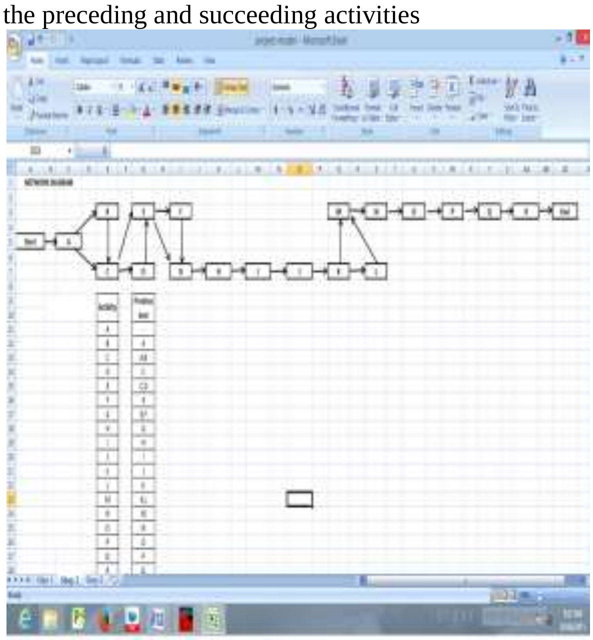
**Step 2:** Defining variables



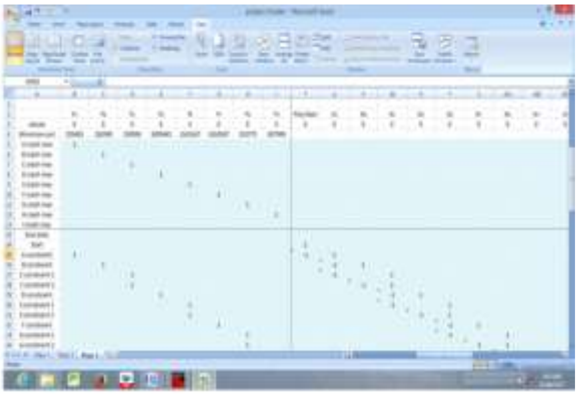
Step 2: Define Variables

| X | | Earliest finish time for each task | Y | | No. of weeks that each activity is crashed |
|---|---|---|---|---|---|
| XA | <= | 60 | YA | <= | 36 |
| XB | <= | 79 | YB | <= | 6 |
| XC | <= | 89 | YC | <= | 3 |
| XD | <= | 131 | YD | <= | 18 |
| XE | <= | 144 | YE | <= | 4 |
| XF | <= | 169 | YF | <= | 10 |
| XG | <= | 185 | YG | <= | 5 |
| XH | <= | 210 | YH | <= | 10 |
| XI | <= | 215 | YI | <= | 1 |
| XJ | <= | 227 | YJ | <= | 4 |
| XK | <= | 239 | YK | <= | 4 |
| XL | <= | 253 | YL | <= | 4 |
| XM | <= | 265 | YM | <= | 4 |
| XN | <= | 270 | YN | <= | 1 |
| XO | <= | 280 | YO | <= | 3 |
| XP | <= | 285 | YP | <= | 1 |
| XQ | <= | 304 | YQ | <= | 6 |
| XR | <= | 330 | YR | <= | 11 |
| Xstart | <= | 0 | | | |
| Xfinish | <= | 330 | | | |

**Step 3:** Drawing the network diagram according to the preceding and succeeding activities

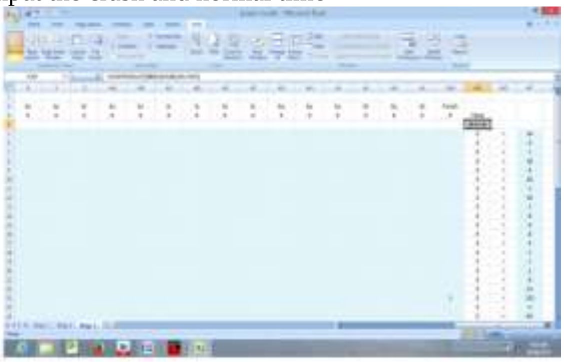


**Step 4:** Inputting the Linear Programming model by giving the preceding activity '-1' co-efficient and the corresponding succeeding activity '1' and its difference in cost ($N_C$-$C_C$) as '1'

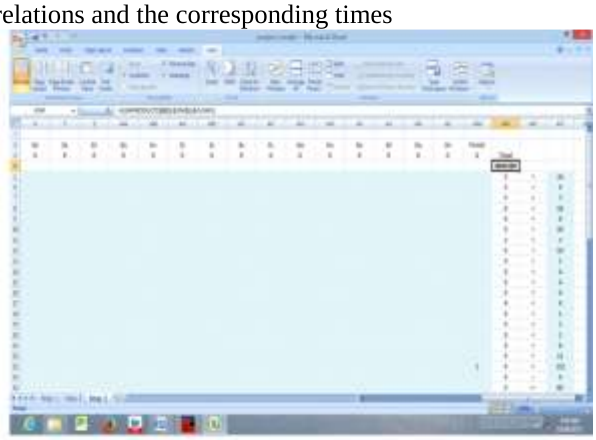


**Step5:** Inputting the solver formula "**SUMPRODUCT(array1,array2)**"

Where array 1 is the solution row and array 2 is the row showing the difference in cost ($N_C$-$C_C$). Also input the crash and normal time



**Step 6:** Inputting all the values in solver window Target cell is the cell for total increase in cost changing cells is the solution row Constraints is the column with the formula SUMPRODUCT, their relations and the corresponding times



**The total cost of the project is increased from1.49 crores to 2.379 crores Project duration is reduced from 330 days to 199 days.**

## IX. RESULTS

**Manual calculations**

1. The project duration is reduced from 330 days to 199 days.
2. Due to the reduction in the time, the total cost of the project is increased from 1.49 crores to 2.375crores (increase in the project cost = 8810710)

**LINGO calculations**

1. The project duration is reduced from 330 days to 199 days
2. Due to the reduction in the time, the total cost of the project is increased from 1.49 crores to 2.375 crores (increase in the project cost = 8810710)

**Excel calculations**

1. The project duration is reduced from 330 days to 199 days
2. Due to the reduction in the time, the total cost of the project is increased from1.49 crores to 2.379 crores (increase in the project cost= 8842580)

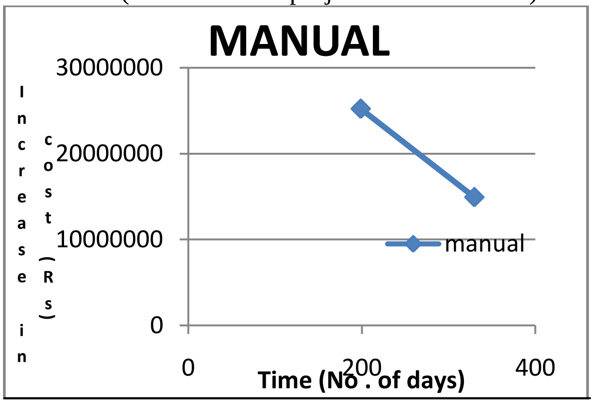



**Fig 9.1: Graphical representation of Manual calculation result**

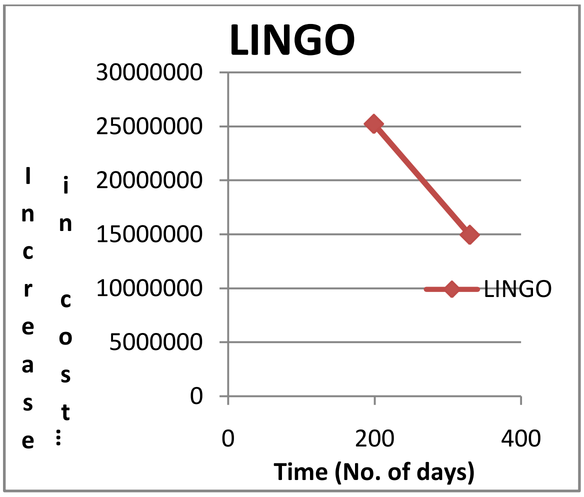



**Fig 9.2: Graphical representation of LINGO result**

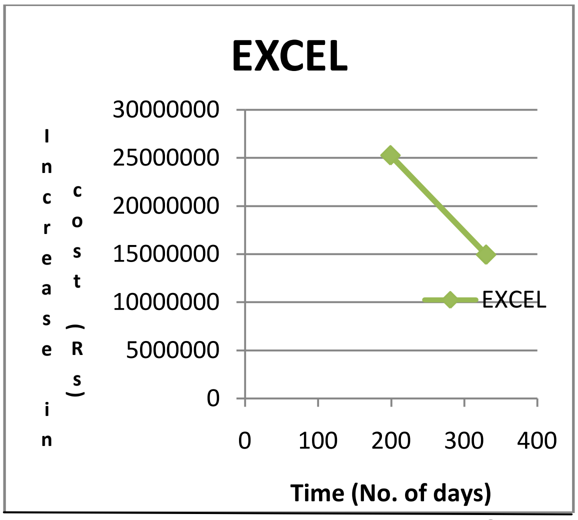



**Fig 9.3: Graphical representation of Excel result**

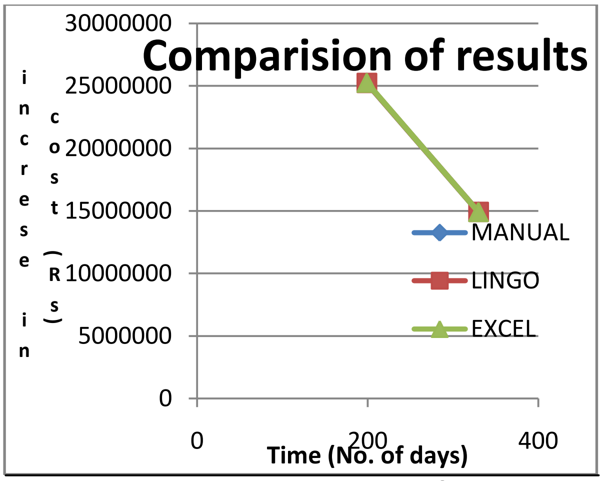



**Fig 9.4: Graphical representation of manual, LINGO and Excel results**

## X. CONCLUSIONS

It is observed that the linear programming model gives some flexibility by providing sensitivity to the mathematical model. The major conclusions drawn from this work are:

- The project duration is 330 days without any flexibility but after the flexibility analysis it is reduced down to 199 days, while increasing the cost from 1.49 crores to 2.37 crores.
- On comparing the results of increase in cost it is found that the LINGO calculation is exactly similar to the manual calculation whereas there is a variation of Rs. 31870 in excel calculation which is equal to 0.36% and hence it can be concluded that both the software's are effective and when compared the LINGO software is more effectively efficient.

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