Closed-Form Performance Analysis of Dual Polarization Based MIMO System in Shadowed-Rician Fading LMS Channels

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Abstract
In this paper, the problem of dual polarization based MIMO Processing in Shadowed-Rician (SR) fading land mobile satellite (LMS) channels is studied. It is shown in literature that polarization is used as interference rejection method; and, most of the existing analytical results are not in closed-form. In this paper, we derive the closed-form expressions of the moment generating function (MGF) of the received signal-to-noise ratio of the MRC based receiver in SR fading LMS channels. Then we provide closed-form expressions of the symbol error rate (SER) by using MGF approach. The analytical diversity order and capacity of the considered scheme is also derived. It is shown by derived closed-form capacity expression that the capacity of the considered dual polarization based scheme is improved; and it is found very useful in practical satellite communication systems.

Index Terms: Land mobile satellite (LMS) channel, M-ary phase shift keying (M-PSK), Polarization, M-ary quadrature amplitude modulation (M-QAM), Shadowed-Rician fading.

I. INTRODUCTION
Satellite systems are very useful in broadcasting, disaster relief, and navigation. These systems are also beneficial in under-populated areas [1]. Multiple antenna based techniques are useful for improving the performance of the satellite based communication system [2]. By using multiple antennas at the earth station and using maximal ratio combining (MRC) the quality of signal reception can be improved. But it is difficult to install multiple antennas at the satellite due to involvement of high cost in designing the satellite system, in practice. Therefore, dual polarization based approach makes this system more useful in terms of available bandwidth. Two independent signals can be transmitted on the same frequency band by means of two orthogonal polarizations. Hence, we can say that the two orthogonal polarizations can be viewed as two data streams transmitting simultaneously and allowing to double the available spectrum. In [3], the Shadowed-Rician (SR) channel model is proposed which describes very accurately the land mobile satellite (LMS) channel and yields significantly less computational burden as compared to other LMS channel models. The MRC scheme for SR fading based LMS channels has been analyzed in [4]–[7]; nevertheless, most of the analytical results are provided in the form of infinite power series, which are not in closed-form. Whereas, closed-form analytical expressions are more suitable for understanding the characteristics and practical implementation of a communication system. Recently, hybrid satellite-terrestrial communication system is discussed in [8]–[13], [21]. Another useful model for the LMS channel is recently discussed in [7] by using the $\kappa - \mu$ shadowed random variables. This model is more accurate for modelling the satellite links than S-R fading but has a complicated form. Because multiple antenna based terrestrial communication systems are widely utilized [25]-[30], therefore it is useful to study the satellite communication system with multiple antennas.

In this paper, we provide closed-form expressions of the moment generating function (MGF) of the received signal-to-noise ratio (SNR) of the MRC scheme based satellite communication system, over SR fading LMS channels. By using these expressions, we get the bit error rate (SER), diversity order, and capacity of the scheme.

II. SYSTEM MODEL
We consider a satellite, with a single antenna, transmitting signals to an earth station containing $N$ receive antennas. The data stream at the satellite is transmitted in two parts; one part uses left-hand circular polarization and other part uses right-hand circular polarization. The signals received at the receiving-end can be written as

$$ y_l = h_l x_l + e_l , \quad (1) $$

and

$$ y_r = h_r x_r + e_r , \quad (2) $$

where $h_l \in \mathbb{C}^{N \times 1}$ and $h_r \in \mathbb{C}^{N \times 1}$ contain the channel gains between left-hand and right-hand feed of the transmit antenna and receive antennas, respectively; $x_l$ and $x_r$, with $E_s$ average power, are the signals transmitted by the satellite; and $e_l \in \mathbb{C}^{N \times 1}$ and $e_r \in \mathbb{C}^{N \times 1}$ denotes the complex-valued additive white Gaussian noise with zero-mean and $\sigma^2$ variance. The channel vectors $h_l$ and $h_r$ with independent and identically distributed (i.i.d.) SR fading entries can be modeled as $h_l = \bar{h} + \bar{h}$, $h_r = \bar{h} - \bar{h}$, where $\bar{h}$ and $\bar{h}$ are the average channel gains.
\[ i = l, r. \text{ Adopting the SR model proposed in [3], the entries of the line-of-sight component } \mathbf{h} \text{ can be modeled as i.i.d. Nakagami-} m \text{ random variables (RVs) with average power } \Omega, \text{ where } m \text{ describes the severity of shadowing varying over the range } m \geq 0. \text{ The entries of the scattered component } \tilde{\mathbf{h}} \text{ are i.i.d. complex Gaussian RVs with zero-mean and unit variance. It is assumed that the receiver contains perfect information of the channel } \mathbf{h}, \text{and } \tilde{\mathbf{h}}. \text{ The received SNR of polarization based scheme at the receiver can be written from (1) and (2) as [14]}

\[ \gamma_l = \frac{\|\mathbf{h}_l\|^2}{\sigma^2} E_s, \]  

(3)

and

\[ \gamma_r = \frac{\|\mathbf{h}_r\|^2}{\sigma^2} E_s, \]  

(4)

where \( \|\cdot\|^2 \) denotes the Euclidean norm.

III. STATISTICS OF THE SNR \( \gamma_l \)

The closed-form expressions of the PDF and CDF of the received SNR \( \gamma_l \) or \( \gamma_r \) are presented in this section. Since the distribution of \( \|\mathbf{h}_l\|^2 \) and \( \|\mathbf{h}_r\|^2 \) is same, therefore, we consider only \( \gamma_l \) for the analysis. Let us re-write \( \gamma_l \) as \( \gamma \), given in (3), as \( \gamma = \|\mathbf{h}_l\|^2 \tilde{\gamma} \), where \( \tilde{\gamma} = E_s/\sigma^2 \).

Lemma 1. The CDF of \( \gamma \) is given by

\[ f_\gamma(x) = a^N \sum_{c=0}^{N} \binom{c}{\gamma} \beta^c \times (F(x, l, d, \tilde{\gamma}) + e^\delta F(x, l, d + 1, \tilde{\gamma})), \]  

(5)

where

\[ F(x, l, d, \tilde{\gamma}) = \frac{(\beta - \delta)^{l-d} x^{\frac{d-l}{2}}}{\tilde{\gamma}^{d-l} \Gamma(d-l)} e^{\frac{\beta-\delta}{2\tilde{\gamma}} x} \times M_{d-l, d-l-1} \left( \frac{\beta-\delta}{\tilde{\gamma}} x \right), \]  

(6)

where \( a = 0.5(2bm/(2bm + \Omega))^m/b, \beta = (0.5/b), \delta = 0.5\Omega/(2b^2m + b\Omega), \) \( 2b \) is the average power of the multipath component, \( c = (d - N)^+, \epsilon = mN - d, \) \( d = \max\{N, \lfloor mN \rfloor \}, \) \( \lfloor z \rfloor \) denotes the largest integer not greater than \( z; \) \( (z)^+ \) indicates that if \( z \leq 0 \), then use \( z = 0; \) \( \max\{\cdot, \cdot\} \) chooses greatest of the two positive integers; \( \Gamma(\cdot) \) denotes the Gamma function, and \( M_{\mu, \nu}(\::) \) represents the Whittaker function.

Proof. Lemma 1 can be proved by using (5), relation \( f_\gamma(x) = \int_0^\infty f_\gamma(y) dy \), and [15, Eq. (2.19.5.3)].

| TABLE I |
|-----------------|-----|-----|
| **LMS CHANNEL PARAMETERS [3]** | b   | m   |
| Frequent heavy shadowing (FHS) | 0.063 | 0.739 | 8.97 \times 10^{-4} |
| Average shadowing (AS)         | 0.126 | 10.1 | 0.835 |
| Infrequent light shadowing (ILS) | 0.158 | 19.4 | 1.29 |

IV. PERFORMANCE ANALYSIS

With the help of the statistics derived in Section III, we now develop a performance analysis of the system.
A. MGF

The MGF of the scheme can be calculated as

\[ M_r(s) = E_{[B_r]}[e^{-sB_r}]. \] (9)

From (5) and (9), the MGF can be expressed as

\[
M_r(s) = a^n \sum_{l=0}^{c} \binom{c}{l} \beta^{-l} \int_0^{\omega} e^{\frac{-sE_{z,1}}{\sigma_z^2}} \frac{z^{d-l-1}}{\Gamma(d-l)}
\times \left[ I_1(d; d - l; - \beta - \delta)z + \frac{\epsilon \delta z^{d-l}}{\Gamma(d - l + 1)} \right]
\times I_1(d + 1; d - l + 1; - \beta - \delta)z)dz.
\] (10)

The integral in (10) can be rewritten by using the relation:

\[ e^{-\frac{sE_{z,1}}{\sigma_z^2}} = G_0^{10}(\frac{sE_{z,1}}{\sigma_z^2}, 1, 0) \] (11)

and

\[ \frac{1}{\Gamma(d - l)} G_{12}^{11}((\beta - \delta)y, 1 - d, 0, 1 - d + l) \] (12)

as

\[
M_r(s) = a^n \sum_{l=0}^{c} \binom{c}{l} \beta^{-l} \left( \int_0^{\omega} \frac{z^{d-l-1}}{\Gamma(d)} G_0^{10}(\frac{sE_{z,1}}{\sigma_z^2}, 1, 0) \right)
\times G_{12}^{11}((\beta - \delta)z, 1 - d, 0, 1 - d + l) + \frac{\epsilon \delta z^{d-l}}{\Gamma(d + 1)} G_0^{10}(\frac{sE_{z,1}}{\sigma_z^2}, 1, 0)
\times G_{12}^{11}((\beta - \delta)z, -d, 0, -d + l)dz.
\] (13)

With the help of [16, Eq. (21)], the integral given in (13) can be solved in closed-form as

\[
M_r(s) = a^n \sum_{l=0}^{c} \binom{c}{l} \beta^{-l} \left( \frac{(s \tilde{y})^{-d+l}}{\Gamma(d)} \right)
\times G_{12}^{12}((\beta - \delta) \tilde{y}, 0, 1 - d + l, 0, -d + l).
\] (14)

where \( \tilde{y} = E_z/\sigma_z^2 \).

B. Average BER

In this subsection, we derive the approximate average BER of the considered MRC based scheme for M-QAM and M-PSK constellations, in closed-form.

1) M-QAM: The instantaneous BER of the M-QAM constellation is given as [17-20]

\[ P_e(\gamma) = \frac{4}{\log M} \left( 1 - \frac{1}{\sqrt{M}} \right) \sum_{k=1}^{\sqrt{M}/2} Q \left( a_k \sqrt{\gamma} \right). \] (15)

where \( a_k = (2k - 1) \sqrt{3/(M - 1)} \). Hence, the average BER of the system will be

\[ P_e(\tilde{y}) = \int_0^{\infty} P_e(\gamma) f_y(\gamma) d\gamma. \] (16)

From (5), (15), and (16), we get

\[ P_e(\tilde{y}) = \frac{4a^n}{\log M} \left( 1 - \frac{1}{\sqrt{M}} \right) \sum_{k=1}^{\sqrt{M}/2} \sum_{l=0}^{c} \binom{c}{l} \beta^{-l}
\times (J(k, l, d, \tilde{y}) + \epsilon \delta J(k, l, d + 1, \tilde{y})). \] (17)

where

\[ J(k, l, d, \tilde{y}) = \frac{(\beta - \delta)^{d-l}}{\tilde{y}^{d-l} \Gamma(d - l + 1)} \int_0^{\omega} \frac{z^{d-l-1}}{\Gamma(d-l+1)} e^{-\frac{\beta - \delta z}{\tilde{y}}}. \]
\[
\times Q\left(a_k \sqrt{\gamma} \right) M_{a_k+\frac{d-1}{2}} \left( \frac{\beta - \delta}{\gamma} x \right) dx.
\]

By using the relation: \( Q(x) = (1/2)\text{erfc}(x/\sqrt{2}) \) in (18), and then using [15, Eq. (2.19.9.4)], it can be shown that

\[
\mathcal{J}(k, l, d, \gamma) = \frac{\Gamma(d - l + 1/2)}{2\sqrt{\pi} \tilde{a}_k^{d-2l} \gamma^{d-1}} \Gamma(d - l + 1) \cdot 3F_2\left(d, d - l, d - l + 1, d - l - 1; \frac{\beta - \delta}{\gamma \tilde{a}_k^2}\right),
\]

where \( \tilde{a}_k = a_k / \sqrt{\gamma} \) and \( 3F_2(c_1, c_2, c_3; d_1, d_2; -z) \) represents the generalized Hypergeometric function. The BER of the considered system can be evaluated by using (17) and (19), in closed-form, for \( M\)-QAM constellation.

\textbf{2. M-PSK}

For \( M\)-PSK constellation, the instantaneous BER is given as [17-20]

\[
P_e(\gamma) \equiv \frac{2}{\max\left(\log M, 2\right)} \sum_{k=1}^{\max(M/4, 1)} Q\left(\sqrt{2} \gamma b_k\right),
\]

where \( b_k = \sin \left(\frac{2k-1}{M}\right) \). By using a similar procedure used in Subsection IV-A.1, the average BER of the MRC scheme for \( M\)-PSK constellation can be obtained from (20), as

\[
P_e(\gamma) = \frac{2\alpha^N}{\max\left(\log M, 2\right)} \sum_{k=1}^{\max(M/4, 1)} \sum_{l=0}^{c} \left(\begin{array}{c}c \\ l\end{array}\right) \beta^{c-l} \\
\times (\mathcal{J}(k, l, d, \gamma) + \epsilon \delta \mathcal{J}(k, l, d + 1, \gamma)),
\]

where

\[
\mathcal{J}(k, l, d, \gamma) = \frac{\Gamma(d - l + 1/2) b_k^{2l-2d}}{2\sqrt{\pi} \gamma^{d-1}} \Gamma(d - l + 1) \cdot 3F_2\left(d, d - l, d - l + 1, d - l - 1; \frac{\beta - \delta}{\gamma b_k^2}\right).
\]

\textbf{C. Coding Gain and Diversity Order}

Let us consider the expression of the average BER of the \( M\)-PSK constellation given in (21) and (22) for diversity order calculation. Let us assume that the value of \( \gamma \) is very high; by observing the fact that \( 3F_2(c_1, c_2, c_3; d_1, d_2; -z) \to 1, z \to 0 \) [22], from (21) and (22), we get the following asymptotic BER of the considered scheme:

\[
P_e(\gamma) = \frac{\alpha^N}{\sqrt{\pi} \max\left(\log M, 2\right)} \sum_{l=0}^{c} \left(\begin{array}{c}c \\ l\end{array}\right) \beta^{c-l} \frac{b_k^{2l-2d}}{\gamma^{d-1}} \\
\times \frac{\Gamma(d - l + 1/2)}{\Gamma(d - l + 1)} \left(1 + \epsilon \delta (d - l + 1/2) / \gamma b_k^2 (d - l + 1)\right).
\]

For very large \( \gamma \), in (1), the term \( A \to 1 \); therefore, we get

\[
P_e(\gamma) = \frac{\alpha^N}{\sqrt{\pi} \max\left(\log M, 2\right)} \sum_{l=0}^{c} \left(\begin{array}{c}c \\ l\end{array}\right) \frac{b_k^{2l-2d}}{\gamma^{d-1}} \\
\times \frac{\Gamma(d - l + 1/2)}{\Gamma(d - l + 1)}.
\]

At very high SNR, the decay of BER is dominated by the highest power of \( \gamma \); therefore, we get the following asymptotic BER from (24):

\[
P_e(\gamma) = \frac{\alpha^N \Gamma(N + 1/2) b_k^{-2N}}{\sqrt{\pi} \max\left(\log M, 2\right) \Gamma(N + 1) \gamma^N}.
\]

At high SNR, the average BER of the communication system is given by \( P_e(\gamma) = (G_c \gamma)^{-G_d} \), where \( G_c \) is coding gain and \( G_d \) denotes diversity order. Therefore, from (25), we have

\[G_d = N\]
\[ G_c = \frac{b^2}{\alpha} \left( \frac{\sqrt{\pi} \max \left( \log M, 2 \right)}{\Gamma(N + 1/2)} \right)^{1/N}. \]  

(26)

**D. Probability of Outage**

The probability of outage is defined as

\[ P_{\text{out}}(\gamma_{\text{th}}) \triangleq \Pr(\gamma \leq \gamma_{\text{th}}), \]  

(27)

where \( \Pr(\cdot) \) denotes the probability. From (7) and (27), we get

\[ P_{\text{out}}(\gamma_{\text{th}}) = \alpha^N \sum_{l=0}^{C} \left( \frac{C}{l} \right) \beta^{l-1} \times \left( G(x, l, d, \gamma_{\text{th}}) + \epsilon \delta G(x, l, d + 1, \gamma_{\text{th}}) \right). \]  

(28)

**E. Capacity**

The average capacity (in bits/second/Hz) of the MRC based scheme is given by [23,24]

\[ C(\gamma) = \int_{0}^{\infty} \log \left( 1 + x \gamma \right) f_{\text{pdf}}(x) dx. \]  

(29)

We can represent \( \ln(1 + x) \) and \( \text{\_F_1}(d; d - l; - (\beta - \delta)x) \), where \( \text{\_F_1}(\cdot; \cdot; \cdot) \) denotes the confluent Hypergeometric function, in the form of Meijer-G function, through [15, Eqs. (8.4.6.5) and (8.4.45.1)]. By using these representations, (29), (5), [16, Eq. (21)]., and after some algebra, the capacity of the MRC scheme can be expressed as

\[ C(\gamma) = \alpha^N \sum_{l=0}^{C} \left( \frac{C}{l} \right) \beta^{l-1} \left( \mathcal{K}(l, d, \gamma) + \epsilon \delta \mathcal{K}(l, d + 1, \gamma) \right). \]  

(30)

In (8), \( \mathcal{K}(l, d, \gamma) \) is given by

\[ \mathcal{K}(l, d, \gamma) = \frac{1}{\gamma^{d-l} \Gamma(d)} \times G_{3,4}^{3,2} \left( \begin{array}{c} \beta - \delta \gamma \\ \gamma \\ \end{array} \right| 0, l - d, l - d, 1 + l - d, l - d, 1 + l - d), \]  

(31)

where \( G_{p,q}^{m,n}(\cdot | \cdot) \) is the Meijer-G function.
V. NUMERICAL RESULTS AND CONCLUSIONS

The analytical capacities (in bits/second/Hz) of the considered polarization based scheme are plotted in Fig. 1 with elevation angles equal to 15°, 30°, 60°, and 90°. The expression of elevation angle is taken from [3]. It can be seen from Fig. 1 that the average capacity of the system is increased with increasing values of elevation angles.

Fig. 2 shows the simulated BER with QPSK constellation for SISO system and proposed scheme for different speed of mobile vehicle. It can be observed from Fig. 2, that proposed scheme outperform the SISO system, especially when the speed of mobile vehicle is low. If speed is high, i.e., 50 to 90 Km/Hr, no improvement is observed.

The simulated and analytical capacity versus SNR plots of the considered MRC scheme are plotted in Fig. 3 for $N = 2, 3, 4$ and under frequent heavy shadowing. The tight matching of the proposed closed-form capacity expression given in (8) and simulated capacity results is evident from the figure. As seen in Fig. 3 that by increasing the receiving antennas from two to four, a capacity gain of approximately 1.2 bits/second/Hz can be achieved at SNR=20 dB under heavy shadowing.
2. Simulated BER of the polarization based scheme with different speed of mobile vehicle
Fig. 3. Analytical and simulated capacity of the MRC scheme with $N = 2, 3, 4$ and FHS.

REFERENCES


