Improving Structural Limitations of Pid Controller For Unstable Processes

School of Post Graduate Studies Enugu State University of Science & Technology
Faculty of Engineering Dept of Electrical & Electronic Engineering Enugu State University of Science & Technology
NASRDA Center for Basic Space Science, Nsukka University of Nigeria, Nsukka

ABSTRACT
PID controllers have structural limitations which make it impossible for a good closed-loop performance to be achieved. A step response with high overshoot and oscillations always results. In controlling processes with resonances, integrators and unstable transfer functions, the PI-PD controller provides a satisfactory closed-loop performance. In this paper, a simple approach to extracting parameters of a PI-PD controller from parameters of the conventional PID controller is presented so that a good closed-loop system performance is achieved. Simulated results from this formation are carried out to show the efficacy of the technique proposed.

Keywords: Disturbance rejection, PID controller, Integrating process, PI-PD, Unstable process.

1. INTRODUCTION
Proportional-Integral-Derivative (PID) controller is a generic control loop feedback mechanism widely used in industrial control system. It attempts to correct the error between a measured process variable and a desired set point by calculating and outputting a corrective action that can adjust the process accordingly. The PID controller calculation (algorithm) involves three separate parameters viz: - k_p – proportional gain value, k_i-Integral gain value and k_d – Differential gain value. These three parameters achieve three things: - k_p – reacts in response to a linear error; k_d – measures the last error difference and will create a larger compensation value i.e. it will accelerate the system quickly if there is a larger error; k_i- the running average smooths out the effect of step responses so it acts as a damping factor to control overshoot and ringing[1]. The weighted sum of these actions is used to adjust the process via a control element such as the position of a control valve or the power supply of heating element. Scaling(tuning) of these three constants in the PID controller algorithm can provide control action designed for specific process requirement. The standard PID controller configuration is shown in figure 1 below.

![Fig 1: Block diagram of PID controller](image)

The extensive use of PID controller in the industry could be attributed to the following reasons:- typical transfer function models used to represent processes can be easily controlled with PID controller; for process without an integral term, an integrator produces zero steady state error to a step input; only a small number of parameters are needed for tuning of PID controller; simple tests such as Ziegler-Nichols[2] and Astrom and Hagglund[3], provide effective controller tuning for processes with typical transfer functions. It is pertinent to note that the use of PID algorithm for control does not guarantee optimal control of a system. Often the PID controller is taken to have error as its input to the close loop system, which produces an unwanted ‘derivative kick’ at its output for step input to the feedback loop even when the D – term has a filter[4]. It is a known fact that it is not easy to get a good
closed-loop step response for processes with resonances and unstable plant transfer functions.

Publications abide addressing the control of unstable processes from different points of view. These can be seen in Park et al(5), Ho and Xu[6].

This paper presents an approach to improving the structural inefficiency of the PID controller in two steps:- moving the PD arm to the feedback loop while retaining the PI arm in the feed forward loop. Next, is using the existing PID parameters in generating the parameters for the formed PI-PD structure. This approach is later shown through examples and simulations to be effective in replacing any existing PID structure and thus improving its performance for processes with resonances, integrators and unstable transfer functions.

II. PI-PD CONTROL STRUCTURE

In conventional PID structure shown in figure 1, it could be seen that the proportional, integral and the derivative terms are all in the feed forward loop thereby acting as the error arising between the set-point and the close-loop response. This leads to a phenomenon called derivative kick which is undesirable. To resolve this matter, the PI-PD structure shown in figure 2 is suggested.

![Fig 2: PI-PD controller structure](image)

Here with the PD arm moved to the feedback loop, unstable or integrating process can now be better stabilized and controlled more effectively by the PI arm in the feed forward loop. In the figure 2, G(S) is the plant transfer function while the Gpd(S) and the Gpi(S) are the PI and PD controller transfer function respectively.

\[ G_{pt}(S) = kp(1 + \frac{1}{T_s S}) \] (1)

\[ G_{pd}(S) = kf(1 + T_d S) \] (2)

It is worthy to mention here that this is not an entirely new concept. Benouaret[7] was the first to mention the PI-PD controller structure but the true potential was not recognized. Kwak et al[8] and Park et al[5] used PID-P PID structure for controlling integrating and unstable processes.

The D term being left in the feed forward path is still prone to derivative kick. Having the PD arm in the feed back loop converts the open-loop unstable or integrating process to open-loop stable process and guarantees more suitable pole location [4]. The general plant transfer function is given by

\[ G(S) = \frac{b_m S^m + b_{m-1} S^{m-1} + \ldots + b_1 S + b_0}{a_n S^n + a_{n-1} S^{n-1} + \ldots + a_1 S + a_0} \] (3)

The closed-loop transfer function for the inner loop \( G_d(S) \) of the form given in equation 3 is

\[ G_d(S) = \frac{b_m S^m + b_{m-1} S^{m-1} + b_1 S + b_0}{a_n S^n + a_{n-1} S^{n-1} + \ldots + a_1 S + a_0} \] (4)

provided \( n > m + 2 \)

A closed look on the modification of the last two terms in equation 4 shows the effect of the insertion of the inner loop. The PI-PD controller gives more flexibility in location of poles of open-loop transfer function \( G_d(S) \) in more desired position with the use of \( K_f \) and \( T_d \) in place of only \( K_f \).

III. EXTRACTING PI-PD PARAMETERS FROM PID PARAMETER

The PID structure shown in figure 2 earlier can through block diagram reduction be reduced to fig 3 below.

![Fig 3: PID equivalent structure of PI-PD](image)

\[ G_{PID} = G_{pi} + G_{pd} \] (5)

Substituting equations (1) and (2) in (5).

\[ G_{PID}(S) = K_f (1 + \frac{1}{T_s S}) + K_f (1 + T_d S) \] (6)

Expanding equation (6) gives

\[ G_{PID}(S) = \left( K_f + \frac{K_f}{(K_f + K_f) T_s} \right) \left( \frac{K_f}{(K_f + K_f) T_s} + \frac{K_f}{(K_f + K_f) T_s} \right) \] (7)

The common structure used for PID controller is
Comparing equation (8) and (7), the following deductions are made:

\[ K^*_c = K_p + K_f \]  
\[ K_p = \frac{\beta K^*_c}{1 + \beta} \]  
\[ K_f = \frac{K^*_c}{1 + \beta} \]  
\[ T_i = \frac{\beta T_i}{1 + \beta} \]  
\[ T_d = (1 + \beta)T_d^* \]

Note that \( \beta = \frac{K_p}{K_f} \)

For different ranges of \( \beta, K_p, K_f, T_i, T_d \) can be calculated and tabulated.

IV. EFFECTS OF \( \beta \) ON ROOT LOCATIONS

From figure(2) above,

\[ \frac{Q(s)}{P(s)} = \frac{G(s)}{1 + G(s)G_{PD}(s)} \]  

Substituting for \( G_{PD}(s) \) from equation (2) and \( T_d \) from equation (13) then,

\[ \frac{Q(s)}{P(s)} = \frac{G(s)}{1 + G(s)k_f(1 + T_d^*s)} \]  
\[ \frac{Q(s)}{P(s)} = \frac{G(s)}{1 + G(s)\frac{k^*_c}{1 + \beta}(1 + T_d^*s)} \]  
\[ \frac{Q(s)}{P(s)} = \frac{G(s)}{1 + G(s)\frac{k^*_c}{1 + \beta}(1 + [1 + \beta]T_d^*s)} \]

From the equation (17) it could be seen that the characteristic equation of the system is

\[ 1 + G(s)\frac{k^*_c}{1 + \beta}(1 + [1 + \beta]T_d^*s) = 0 \]  

V. SIMULATION FUNCTIONS AND PARAMETERS

The following system with the transfer functions given below are used to test the performance of the suggested approach.

1. \( G_1(s) = \frac{1}{S(S + 1)(S + 5)} \)

2. \( G_2(s) = \frac{1}{S(S + 1)(S + 6)(S + 7)} \)

The systems are all controlled by PID controller with the following parameters calculated by Ziegler-Nichols tuning rule [2]. The resulting PID controller parameters are

\( k^*_c = 30, \beta = 0.2, T_d = 0.35124 \) for \( G_1(s) \).

\( k^*_c = 2, \beta = 0.2, T_d = 0.25 \) for \( G_2(s) \)

VI. SIMULATION RESULT

Figure (4) and (5) shows root locations of the systems with and without the PD in the inner feedback loop. It could be easily seen that using the PD in the inner feedback loop produces a better stable system.

Fig4: Root plot (a) without loop (b) with inner loop
VII. CONCLUSION

Proportional-Integral-Derivatives (PID) controller has inherent structural limitations that make it impossible for a good close-loop control performance to be achieved in processes with resonances, integrators and unstable transfer functions. Simulation result above has shown that this approach of extracting parameters of a PI-PD controller from parameters of the conventional PID controller gives a good closed-loop system performance.

REFERENCES


