Experimental Study, Simulation and Model Predictions of Recycled PET Strip-Reinforced Concrete Flexion Members

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ABSTRACT
This study presents results from a theoretical-experimental program of beams partially pre-stressed made with continuous recycled PET strip-reinforced concrete (plain concrete strength of 20 MPa). These studies mainly attempted to determine the strip influence in altering the flexural strength at first and final crack. Also the load-deflection, ductility, energy absorption capacity of the beams are observed and the studies can be used in predicting the flexural behavior of longitudinally reinforced concrete. The model theory assumes that concrete has a tensile load capacity different from zero, characterized by a uniaxial tensile stress-strain diagram. The need for non-linear geometric and the material models imply the use of numerical methods such as the finite element method; so that, a finite element analysis of reinforced concrete beam with strips-reinforced plastic is performed. The obtained results were compared with computer analysis and experimental data to corroborate the validity of the suggested method, showing that the theory also predicts correctly the post-cracking creep deformation.

Keywords- Reinforced concrete, Finite element method, Mechanical properties, Prediction model, Recycled PET strips

I. INTRODUCTION
Concrete is the most widely used construction material in the world due to its high compressive strength, durability and low cost. However, it has inherent disadvantages such as low tensile strength and cracking. Several studies about polymeric reinforced concrete have been made with the aim of improving such deficiencies of the material. The reported results show that it is possible to improve the performance of concrete by reinforcing it with different plastic elements [1]. Reinforced concrete (RC) is a composite material that results from the addition of diverse elements to the brittle matrix of ordinary concrete. Several investigations have been done on the reinforcement of structural materials using plastic elements [2-7]. In the case of concrete, the main reinforcing materials are steel, glass and polymeric strips. Some of these reinforcements are used in structures such as columns, beams and walls [8-10].

Recently the use of recycled materials as reinforcement in concrete elements has received increasing attention worldwide, being one of them polyethylene terephthalate (PET) obtained from recycled plastic bottles. PET has higher resistance to degradation, lower cost and density, and is also non-conductive of electricity and magnetism. Therefore, polymer reinforced concrete can be an excellent alternative for structures built in or near marine environments or similar corrosive environments (deicing salts), where electromagnetic neutrality and / or electrical insulation are required. Unfortunately, the use of polymers as reinforcements is not free of obstacles that must be solved before application. The main obstacles are the high initial cost, low Young modulus, low tensile strength and the absence of design parameters. The high initial cost is greatly influenced by manufacturing processes; however, different methods are currently under development to reduce the cost. The low Young modulus and tensile strength are considered the main engineering disadvantages of PET. This is because PET shows a lower safety margin when compared to its counterpart, i.e., reinforced concrete with steel bars. As for the lack of design parameters, more experimental and theoretical data are needed.

It is worth mentioning that the currently available design formulas have been originally developed for steel reinforcement, and nowadays they are used by many design codes. However, these formulas are not applicable to the type of reinforcing proposed in this study. Thus, it is necessary to modify the currently used formulas to ensure a better prediction of the deflections in beams that are manufactured with these materials.

The flexural rigidity of RC beams under service loads is considerably lower than the stiffness
calculated on the basis of the uncracked cross-section. This is because the beam contains numerous cracks that become active under tensile stress. However, at the same time, the stiffness is significantly higher than that calculated when the tensile strength of concrete is neglected. This phenomenon, often called tension stiffening, is attributed to the fact that the concrete does not break suddenly and completely, but undergoes progressive crack nucleation, which triggers later the effect of crack growth and fracture.

Based on numerous tests [11-17], Branson [18] derived an empirical formula that adequately describes the results of the tests and has been approved by an ACI committee [11]. While this formula is used for practical purposes, it is not derived from the intrinsic properties of the composite material reinforced with strips or bands. In this article, an approach that includes the characteristics and properties that such reinforcements add to the concrete was developed. The model is able to predict the curvatures and deformations beyond the service stress range and the whole development of the ultimate load.

The finite element method (FEM) is a general method of structural analysis in which the solution of a problem in continuum mechanics is approximated by the analysis of an assembly of finite element, which are interconnected at a finite number of nodal points and that represents the solution domain of the problem. The FEM is a general numerical method for the solution of partial differential equations subjected to known boundary and initial conditions, making it the most powerful general technique for the numerical solution of a variety of engineering problems.

The behavior of reinforced concrete structures is distinctly nonlinear, because of several factors: (1) nonlinear material behavior of concrete and reinforcement and their interaction; (2) cracking of concrete; and (3) time dependent effects such as creep, shrinkage, temperature and load history. Nevertheless in this study only the elastic part of the composite has been carried out.

II. THEORETICAL MODELING

Figure 1 shows the arrangement of the reinforcement as well as the cross section of the beam (width \(H\), height \(h\)), the assumed deformation profile distribution through the cross section and the corresponding stress distribution. Each row of reinforcement is referenced from the top of the cross section, and its distance is denoted by \(d_i\). For the strain profile, the hypothesis to consider is that there is a linear distribution of the concrete compressive strain \(\varepsilon_{cc}\), concrete tensile strain \(\varepsilon_{ct}\) and strain in a PET layer \(\varepsilon_{pi}\) \(i=1,2,3\). The neutral axis (N.A.) position in the beam cross section corresponds to the distance “\(x\)” measured from the upper end of the beam cross section (Fig. 1c). In the stress distribution diagram of the beam cross section (Fig. 1d), the force \(F_{ci}\) acting in the zone under compression stress (above the N.A.) was obtained by integrating the non-linear distribution of the stress acting in this area. In the case of tension (below N.A.), the acting forces are divided into two. First the forces provided exclusively by concrete i.e., tension on the higher part \(T_{ci}\) and tension on the lower part \(T_{ci}\); and second, those of the reinforcement i.e., tension in each PET layer \(T_{pi}\) \(i=1,2,3\). The \(T_{ci}\) force is obtained by considering a linear variation of tensile stress as provided by concrete, with a maximum breaking strength value, denoted by the critical stress \(f_{cc}\) in figure 1c. In the case of the \(T_{ci}\) force, a constant stress distribution is assumed. \(f_{cc}\) is the maximum fracture strength.

An important contribution of this research is to propose a value for the coefficient \(T_c\). The magnitude and position of the resulting compressive strength of concrete are obtained by using the stress-strain diagram proposed by Hoggestad [19] (Figure 2). In each of the points to be determined, the procedure starts by assuming the strain value \(\varepsilon_{cc}\) for the conditions under investigation, and then obtaining the depth of the neutral axis that meets the equilibrium conditions of the forces by a trial-and-error approximation. The stress-strain diagram is assumed to fit a parabolic curve as given by Eq. 1 and it is used to calculate the compressive strength of concrete in the plastic behavior region.

![Figure 1. Plot of the internal actuating stresses and strains: (a) cross-section with reinforcement, (b) schematic model of cross-section, (c) strain and (d) stress distribution profile.](image)

![Figure 2. Stress-strain curve for concrete type Hoggestad.](image)
\[ f_i = f_r \left[ 2 \left( \frac{\varepsilon_{cr}}{\varepsilon_0} \right) - \left( \frac{\varepsilon_{cr}}{\varepsilon_0} \right)^2 \right] \]  

(1)

The behavior of the Figure 2 corresponds to the equation (1), which was given by Hognestad for concrete (\(\varepsilon_0\), strain for \(f_c\) of the concrete in the stress-strain curve and \(f_c''\), compressive strength of concrete).

Deformations at each row of reinforcement were calculated by using Eq. 2. In this manner, the tensile stresses corresponding to the experimental stress-strain curve of PET were obtained (Fig. 3). The experimental data were previously obtained by tension tests on the PET strips along with the longitudinal direction. An Instron 4469 universal testing machine with a load cell of 50 kN was used with displacement rate of 0.333 mm/s. This procedure results in the internal forces diagram shown in Figure 1 i.e., \(F_{cr}\) (Eq. 3), \(T_{zi}\) (Eq. 4) where \(\varepsilon_{cr}\) is the critical deformation, \(T_{zi}\) (Eq. 5), where \(T_{pl}\) and \(T_{pl}^2\) and \(T_{pl}^3\) (Eq. 6), where \(A_r\) is the reinforcement area and \(f_r\) is the stress in a PET bars reinforcement layer. Thereby the resisting moment \(M_r\) to the deformation \(\varepsilon_{cr}\) can be calculated with Eq. 7 (Fig. 4), where \(Y_{cr}\) is the distance between \(N.A.\) and \(F_{cr}\), \(Y_{cr}\) is the distance between \(N.A.\) and \(T_{zi}\), \(Y_{cr2}\) is the distance between \(N.A.\) and \(T_{zi}\).

\[ \varepsilon_r = \frac{\varepsilon_{cr}}{x} (d_i - x) \]  

(2)

\[ F_{cr} = f_r \ast h \ast \left[ \frac{x \varepsilon_{cr}}{\varepsilon_0}\left(1 - \frac{\varepsilon_{cr}}{3\varepsilon_0}\right) \right] \]  

(3)

\[ T_{zi} = \frac{1}{2} f_{cr} x \varepsilon_{cr} B \frac{\varepsilon_{cr}}{\varepsilon_0} \]  

(4)

\[ T_{zi} = T_{zi} f_{zi} \left[ H - \left(1 + \frac{\varepsilon_{cr}}{\varepsilon_0}\right) x \right] \]  

(5)

\[ T_{pi} = A_r f_{zi}, \quad i = 1, 2, 3, \ldots \]  

(6)

\[ M_r = F_{cr} Y_{cr} + T_{zi} Y_{zi} + T_{zi} Y_{cr2} + \sum T_{pi} (d_i - x) \]  

(7)

The generation of points of the load-deflection plot depends on the load conditions and support of the beam to be studied. In this work a simply supported beam with point center-point loading is considered (Fig. 5). Thus the parameters to be followed are the critical load \(P_r\) (Eq. 8) where \(L\) is the length between supports of the beam; a calculated curvature factor \(\text{curv}\) (Eq. 9), rigidity of the section \(E I_r\) (concrete Young modulus \(E\) and moment of inertia \(I_r\) (Eq. 10) and the deflection of the beam at the center of the load \(\delta r\) (eq. 11). Figure 6 shows the algorithm of the calculation method of \(P_r\) and \(\text{curv}\) used in this investigation.

\[ P_r = \frac{4 M_r}{L} \]  

(8)

\[ \text{curv} = \frac{\varepsilon_{cr} T_{zi}}{x} \]  

(9)

\[ EI_r = \frac{M_r}{\text{curv}} \]  

(10)

\[ \delta r = \frac{P_r L^3}{48 E I_r} \]  

(11)

Figure 5. Configuration of the supports and load in the beam studied.

III. NUMERICAL SIMULATION: FINITE ELEMENT MODELING

FEM was used to calculate forces acting on reinforced beam (B-0.25, B-0.50 and B-1.00) and on not reinforced beam (B-0.00), considering a load of 5 kN. It was decided to calculate the stress and strains for these conditions, because experimentally unreinforced beams failed under this load. Calculations were made using the theory of small perturbations (Cauchy Green-tensioner), for modeling the concrete solid type deformable elements and isotropic material were used; for bands, an element type “shell” was used. The discretization of the concrete beam was completed using a mesh...
with maximum size of 12.5 mm, the objective was to replicate the dimensions of the concrete aggregate.

IV. RESULTS

According to Fig. 1, the purpose of this work was to determine the value of the coefficient $T_c$ which, when multiplied by the rupture stress ($f_{cr}$), results in the tensile stress distribution in the concrete. This distribution is considered in the equilibrium equations of resisting moments and forces in concrete beams reinforced with recycled PET strips (Eqs. 2-11). Figure 7 shows experimental and numerical results corresponding to the reinforced beam with bands of recycled PET at 0.25% in volume ratio (B-0.25). It is observed that the value of the theoretical and the experimental load to first crack are very similar; the value of the theoretical deflection at this load is 0.80 mm higher in comparison to the experimental, however the difference in the value of the maximum deflection increases in 3 mm. The behavior of the beams containing 0.50% of PET (B-0.50) indicates that the load and deflection to the first crack are very similar, while the calculated maximum deflection is less than the experimental for 0.2 kN (Fig. 8). In the case of the bending behavior of the beams with 1.00% of PET (B-1.00), Fig. 9, the load at the first experimental crack is higher in 0.75kN compared to the theoretically calculated load. On the other hand, in the maximum deflection the experimental crack is greater than that calculated by 9 mm. Experimental and numerical results corresponding to the beams with different ratios ($\rho$) of recycled PET bands reinforcement, 0.0025 (0.25 vol %), 0.0050 (0.50 vol %) and 0.0100 (1.00 vol %), show that generally for all the beams elastic and plastic behavior was predicted accurately. Table 1 presents the maximum loads obtained in each case, finding that at greater value of $\rho$, the percentage error in the approximation is increased, although the model continues precisely predicting the flexural behavior of beams reinforced with PET bands subjected to flexion, for three types of beams to $T_c$ factor is 0.41.

![Algorithm of the proposed method](image)

![Numerical and experimental load-deflection plots for a beam B-0.25.](image)

Table 1. Theoretical and experimental loads.

<table>
<thead>
<tr>
<th>Beam $\rho$</th>
<th>$\rho$</th>
<th>Results</th>
<th>Max load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-0.25</td>
<td>0.0025</td>
<td>Theoretical</td>
<td>7.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Experimental</td>
<td>7.91</td>
</tr>
<tr>
<td>B-0.50</td>
<td>0.0050</td>
<td>Theoretical</td>
<td>9.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Experimental</td>
<td>9.11</td>
</tr>
<tr>
<td>B-1.00</td>
<td>0.0100</td>
<td>Theoretical</td>
<td>14.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Experimental</td>
<td>13.80</td>
</tr>
</tbody>
</table>
It has been shown in the literature and in the present work that the use of recycled PET as reinforcement material, either in the form of short and continuous fibers or strips, substantially improves the strength of concrete beams, as well as their ductility and toughness. However, there is not a model that can predict such improvements. In this paper a methodology to predict these enhancements has been found through the following three steps: 
1. The application of equilibrium equations, 2. The use of experimental stress-strain curves of PET, and 3. The consideration that the concrete-PET beams have a certain tensile strength at their lower part.

A critical stress of 41% of the concrete rupture stress was found to adequately satisfy the theoretical and experimental curves of the specimens tested. The reinforcement percentages used were up to 1% and a condition was to place the PET as continuous strips, in order to have a better control over the percentage of reinforcement and its placement in layers.

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