Expected Time To Recruitment In A Two Grade Manpower System

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Abstract
In this paper a two graded organization is considered in which depletion of manpower occurs due to its policy decisions. Three mathematical models are constructed by assuming the loss of man-hours and the inter-decision times form an order statistics. Mean and variance of time to recruitment are obtained using an univariate recruitment policy based on shock model approach and the analytical results are numerically illustrated by assuming different distributions for the thresholds. The influence of the nodal parameters on the system characteristics is studied and relevant conclusions are presented.

Key words: Man power planning, Univariate recruitment policy, Mean and variance of the time for recruitment, Order statistics, Shock model.

I. Introduction
Exits of personnel which is in other words known as wastage, is an important aspect in the study of manpower planning. Many models have been discussed using different kinds of wastages and also different types of distributions for the loss of man-hours, the threshold and the inter-decision times. Such models could be seen in [1] and [2]. Expected time to recruitment in a two graded system is obtained under different conditions for several models in [3],[4],[5],[6],[7],[8] and [9] according as the inter-decision times are independent and identically distributed exponential random variables or exchangeable and constantly correlated exponential random variables. Recently in [10] the author has obtained system characteristic for a single grade man-power system when the inter-decision times form an order statistics. The present paper extend the results of [10] for a two grade manpower system when the loss of man-hours and the inter decision times form an order statistics. The mean and variance of the time to recruitment of the system characteristic are obtained by taking the distribution of loss of man-hours as first order (minimum) and kth order (maximum) statistics respectively. This paper is organized as follows: In sections 2, 3 and 4 models I, II and III are described and analytical expressions for the thresholds and the inter-decision times are independent and identically distributed exponential random variables with density function g(.) and mean 1/c_i(c>0). Let X(1),X(2),...,X(k) be the order statistics selected from the sample X_1,X_2,...,X_k with respective density functions g_1(.), g_2(.),..., g_k(.). Let U_i, i=1,2,3,...k are independent and identically distributed exponential random variables with density function f(.). Let U(1),U(2),...,U(k) be the order statistics selected from the sample U_1,U_2,...,U_k with respective density functions f_1(.), f_2(.),..., f_k(.). Let T be a continuous random variable denoting the time for recruitment in the organization with probability density function (distribution function) I(.)(U(.)). Let


\[ l'(\cdot) f^*(\cdot), f_{u(\cdot)}^*(\cdot) \text{ and } f_{u(\cdot)}^*(\cdot) \text{ be the Laplace transform of } l(\cdot), f(\cdot), f_{u(\cdot)}(\cdot) \text{ and } f_{u(\cdot)}(\cdot) \text{ respectively. Let } Y_A \text{ and } Y_B \text{ be independent random variables denoting the threshold levels for the loss of man-hours in grades } A \text{ and } B \text{ with parameters } \alpha_A \text{ and } \alpha_B \text{ respectively. In this model the threshold } Y \text{ for the loss of man-hours in the organization is taken as max } (Y_A, Y_B). \text{ The loss of manpower process and the inter-decision time process are statistically independent. The univariate recruitment policy employed in this paper is as follows: Recruitment is done as and when the cumulative loss of man-hours in the organization exceeds } Y. \text{ Let } V_i(t) \text{ be the probability that there are exactly } k \text{ decision epochs in } (0, t]. \text{ Since the number of decisions made in } (0, t] \text{ form a renewal process we note that } V_i(t) = F_i(t) - F_{i+1}(t), \text{ where } F_0(t) = 1. \text{ Let } E(T) \text{ and } V(T) \text{ be the mean and variance of time for recruitment respectively.} \]

### III. Main results

The survival function of \( T \) is given by

\[
P(T > t) = \sum_{k=0}^{\infty} V_k(t) P(\sum_{i=1}^{k} X_i \leq Y)
\]

\[
= \sum_{k=0}^{\infty} V_k(t) \int_0^\infty p(y > x) g_k(x) dx
\]

**Case 1:**

\( Y_A \text{ and } Y_B \text{ follow exponential distribution with parameters } \alpha_A \text{ and } \alpha_B \text{ respectively. In this case it is shown that} \)

\[
p(Y > x) = \sum_{k=0}^{\infty} V_k(t) \int_0^\infty \left(e^{-\alpha_A x} + e^{-\alpha_B x} - e^{-(\alpha_A + \alpha_B)x}\right) g_k(x) dx
\]

From (1) and (2) we get

\[
P(T > t) = \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] \left[ g_k^*(\alpha_A) + g_k^*(\alpha_B) - g_k^*(\alpha_A + \alpha_B) \right]
\]

**Case 2:**

Since \( L(t) = 1 - P(T > t) \text{ and } l(t) = \frac{d}{dt} \{L(t)\} \)

from (3) and (4) it is found that

\[
l(t) = [1 - g^*(\alpha_A) \sum_{k=1}^{\infty} f_k(t)(g^*(\alpha_A))^{k-1}] + [1 - g^*(\alpha_B) \sum_{k=1}^{\infty} f_k(t)(g^*(\alpha_B))^{k-1} -
\]

\[
[1 - g^*(\alpha_A + \alpha_B) \sum_{k=1}^{\infty} f_k(t)(g^*(\alpha_A + \alpha_B))^{k-1}]
\]

Taking Laplace transform on both sides of (5) it is found that

\[
l'(s) = \frac{1 - g^*(\alpha_A) f^*(s)}{1 - f^*(s) g^*(\alpha_A)} + \frac{1 - g^*(\alpha_B) f^*(s)}{1 - f^*(s) g^*(\alpha_B)} - \frac{1 - g^*(\alpha_A + \alpha_B) f^*(s)}{1 - f^*(s) g^*(\alpha_A + \alpha_B)}
\]

The probability density function of \( \tau^\text{th} \text{ order statistics is given by} \)

\[
f_{\tau(\cdot)}(t) = r k c_{\tau} [F(t)]^{r-1} f(t)[1 - F(t)]^{k-r}, r = 1, 2, 3, k
\]

If \( f(t) = f_{u(t)}(t) \) then \( f^*(s) = f_{u(t)}^*(s) \)

From (7) it is found that

\[
f_{u(t)}(t) = k f(t)(1 - f(t))^{k-1}
\]

Since by hypothesis \( f(t) = \lambda e^{-\lambda t} \)

from (9) and (10) we get
\[ f_{u(t)}(s) = \frac{k \lambda}{k \lambda + s} \]  

It is known that

\[ E(T) = -\frac{d(l^*(s))}{ds} \bigg|_{s=0}, \quad E(T^2) = \frac{d^2(l^*(s))}{ds^2} \bigg|_{s=0} \quad \text{and} \quad V(T) = E(T^2) - (E(T))^2 \]  

Therefore from (6), (11) and (12) we get

\[ E(T) = \frac{1}{\lambda} \left[ V_1 + V_2 - V_3 \right] \]  

\[ E(T^2) = \frac{2}{\lambda^2} \left[ V_1^2 + V_2^2 - V_3^2 \right] \]  

Where \( V_1 = \frac{1}{1 - g^*(\alpha_A)}, V_2 = \frac{1}{1 - g^*(\alpha_B)} \) and \( V_3 = \frac{1}{1 - g^*(\alpha_A + \alpha_B)} \)  

If \( f(t) = f_{u(k)}(t) \)  

In this case \( f^*(s) = f_{u(k)}^*(s) \)  

From (7) it is found that

\[ f_{u(k)}(t) = (F(t))^{k-1} f(t) \]  

From (10), (16) and on simplification we get

\[ f_{u(k)}^*(s) = \frac{k! \lambda^k}{(s + \lambda)(s + 2\lambda) \ldots (s + k\lambda)} \]  

Therefore from (6), (17) and (12) we get

\[ E(T) = \frac{1}{\lambda} \sum_{n=1}^{k} \frac{1}{n} \left[ V_1 + V_2 - V_3 \right] \]  

\[ E(T^2) = \frac{2}{\lambda^2} \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 \left[ V_1^2 + V_2^2 - V_3^2 \right] + \frac{\sum_{n=1}^{k} 1/n}{\lambda^2} \left[ V_1 + V_2 - V_3 \right] \]  

In (18) & (19) \( V_1, V_2 \) and \( V_3 \) are given by (15).

The probability density function of \( n^{th} \) order statistics is given by

\[ g_{(n)}(x) = n k c_n [G(x)]^{n-1} g(x) [1 - G(x)]^{k-n}, n = 1, 2, 3, k \]  

If \( g(x) = g_{(n)}(x) \)

then in (13), (14), (18) and (19) \( g^*(\tau) = g_{(n)}^*(\tau) \) for \( \tau = \alpha_A, \alpha_B \) and \( \alpha_A + \alpha_B \)

From (20) it is found that

\[ g_{(n)}(x) = k g(x) (1 - g(x))^{k-1} \]  

Since by hypothesis \( g(x) = ce^{-cx} \)

from (21) and (22) we get

\[ g_{(n)}^*(\tau) = \frac{k c}{k c + \tau} \]  

In (13), (14), (18) and (19) \( g^*(\alpha_A), g^*(\alpha_B) \& g^*(\alpha_A + \alpha_B) \) are given by (23) when \( s = 1 \).

and \( V(T) = E(T^2) - (E(T))^2 \)

If \( g(x) = g_{(n)}(x) \)

then \( g^*(\tau) = g_{(n)}^*(\tau) \) for \( \tau = \alpha_A, \alpha_B \) and \( \alpha_A + \alpha_B \)
From (20) it is found that
\[ g_{(k)}(x) = (G(x))^{k-1} g(x) \]  
(24)

From (22), (24) and on simplification we get
\[ g_{(k)}^*(\tau) = \frac{k!e^k}{(c+\tau)(2c+\tau)(3c+\tau)\ldots(kc+\tau)} \text{ for } \tau = \alpha_A, \alpha_B \text{ and } \alpha_A + \alpha_B \]  
(25)

In (13), (14), (18) and (19) \( g^*(\alpha_A), g^*(\alpha_B) \& g^*(\alpha_A + \alpha_B) \) are given by (25) when \( s = k \) and \( V(T) = E(T^2) - (E(T))^2 \)

Case 2:

\( Y_A \) and \( Y_B \) follow extended exponential distribution with scale parameters \( \alpha_A \) and \( \alpha_B \) respectively and shape parameter 2. In this case it can be shown that

If \( f(t) = f_{ua1}(t) \)

\[ E(T) = \frac{1}{\lambda} \left[ 2V_1 + 2V_2 - 4V_3 + 2V_4 + 2V_5 - V_6 - V_7 - V_8 \right] \]  
(26)

\[ E(T^2) = \frac{2}{\lambda^2} \left[ 2V_1^2 + 2V_2^2 - 4V_3^2 + 2V_4^2 + 2V_5^2 - V_6^2 - V_7^2 - V_8^2 \right] \]  
(27)

where \( V_4 = \frac{1}{1 - g^*(2\alpha_A + \alpha_B)}, V_5 = \frac{2}{1 - g^*(\alpha_A + 2\alpha_B)}, V_6 = \frac{1}{1 - g^*(2\alpha_A + 2\alpha_B)}, V_7 = \frac{1}{1 - g^*(2\alpha_A)} \) and \( V_8 = \frac{1}{1 - g^*(2\alpha_B)} \)  
(28)

when \( n = 1 \) in (26) & (27) \( V_1, V_2, V_3, V_4, V_5, V_6, V_7 \) and \( V_8 \) are given by (15), (28) and (23).

when \( n = k \) in (26) & (27) \( V_1, V_2, V_3, V_4, V_5, V_6, V_7 \) and \( V_8 \) are given by (15), (28) and (23).

If \( f(t) = f_{uk1}(t) \)

Proceeding as in case (i) it can be found that

\[ E(T) = \sum_{n=1}^{k} \frac{1}{n^2} \left[ 2V_1 + 2V_2 - 4V_3 + 2V_4 + 2V_5 - V_6 - V_7 - V_8 \right] \]  
(29)

\[ E(T^2) = \frac{2}{\lambda^2} \left[ 2V_1^2 + 2V_2^2 - 4V_3^2 + 2V_4^2 + 2V_5^2 - V_6^2 - V_7^2 - V_8^2 \right] \left( \sum_{n=1}^{k} \frac{1}{n^2} \right) - \frac{1}{\lambda^2} \]  
\[ \left[ 2V_1 + 2V_2 - 4V_3 + 2V_4 + 2V_5 - V_6 - V_7 - V_8 \left( \sum_{n=1}^{k} \frac{1}{n^2} \right) - \left( \sum_{n=1}^{k} \frac{1}{n^2} \right) \right] \]  
(30)

when \( n = 1 \) in (26) & (27) \( V_1, V_2, V_3, V_4, V_5, V_6, V_7 \) and \( V_8 \) are given by (15), (28) and (23).

when \( n = k \) in (26) (27) \( V_1, V_2, V_3, V_4, V_5, V_6, V_7 \) and \( V_8 \) are given by (15), (28) and (25).

Case 3:

\( Y_A \) follows extended exponential distribution with scale parameters \( \alpha_A \) and shape parameter 2 and \( Y_B \) follows exponential distribution with parameter \( \alpha_B \).
Proceeding as in case 1 it can be shown that
\[
E(T) = \frac{1}{\lambda} \left[ 2V_1 + V_2 + V_4 - 2V_3 - V_7 \right] \tag{31}
\]
\[
E(T^2) = \frac{2}{\lambda^2} \left[ 2V_1^2 + V_2^2 + V_4^2 - 2V_3^2 - V_7^2 \right] \tag{32}
\]

when \( n = 1 \), in (31) & (32) \( V_1, V_2, V_3, V_4 \) and \( V_7 \) are given by (15), (28) and (23).

when \( n = k \), in (31) & (32) \( V_1, V_2, V_3, V_4 \) and \( V_7 \) are given by (15), (28) and (25).

If \( f(t) = f_{\text{univ}}(t) \)

Proceeding as in case (i) it can be shown that
\[
E(T) = \frac{1}{\lambda} \left[ \sum_{i=1}^{k} \frac{1}{n} \left( 2V_i + V_{i+1} - 2V_3 - V_7 \right) \right] \tag{33}
\]
\[
E(T^2) = \frac{2}{\lambda^2} \left[ \sum_{i=1}^{k} \frac{1}{n} \left( 2V_i^2 + V_{i+1}^2 - 2V_3^2 - V_7^2 \right) \right]
- \frac{1}{\lambda} \left[ \sum_{i=1}^{k} \frac{1}{n} \left( 2V_i + V_{i+1} - 2V_3 - V_7 \right) \right]^2 \tag{34}
\]

when \( n = 1 \), in (33) & (34) \( V_1, V_2, V_3, V_4 \) and \( V_7 \) are given by (15), (28) and (23).

when \( n = k \), in (33) & (34) \( V_1, V_2, V_3, V_4 \) and \( V_7 \) are given by (15), (28) and (25).

Case 4:

The distributions of \( Y_A \) has SCBZ property with parameters \( \alpha_A, \mu_1 \) & \( \mu_2 \), and the distribution of \( Y_B \) has SCBZ property with parameters \( \alpha_B, \mu_3 \) & \( \mu_4 \). In this case it can be shown that

If \( f(t) = f_{\text{univ}}(t) \)

\[
E(T) = \frac{1}{\lambda} \left[ p_1 V_9 + p_2 V_{10} - p_1 p_2 V_{13} - p_1 q_2 V_{14} - p_2 q_1 V_{15} - q_1 q_2 V_{16} + q_1 V_{11} + q_2 V_{12} \right] \tag{35}
\]
\[
E(T^2) = \frac{2}{\lambda^2} \left[ p_1 V_9^2 + p_2 V_{10}^2 - p_1 p_2 V_{13}^2 - p_1 q_2 V_{14}^2 - p_2 q_1 V_{15}^2 - q_1 q_2 V_{16}^2 + q_1 V_{11}^2 + q_2 V_{12}^2 \right] \tag{36}
\]

where
\[
V_9 = \frac{1}{1 - g^* (\alpha_A + \mu_1)}, \quad V_{10} = \frac{2}{1 - g^* (\alpha_B + \mu_2)}, \quad V_{11} = \frac{1}{1 - g^* (\mu_2)}, \quad V_{12} = \frac{1}{1 - g^* (\mu_4)}
\]
\[
V_{13} = \frac{1}{1 - g^* (\alpha_A + \alpha_B + \mu_1 + \mu_2)}, \quad V_{14} = \frac{1}{1 - g^* (\alpha_A + \alpha_B + \mu_1 + \mu_2)}, \quad V_{15} = \frac{1}{1 - g^* (\alpha_B + \mu_3 + \mu_4)}
\]
\[
V_{16} = \frac{1}{1 - g^* (\mu_2 + \mu_4)} \tag{37}
\]

when \( n = 1 \), in (35) & (36) \( V_9, V_{10}, V_{11}, V_{12}, V_{13}, V_{14}, V_{15} \) and \( V_{16} \) are given by (37) and (23).

when \( n = k \), in (35) & (36) \( V_9, V_{10}, V_{11}, V_{12}, V_{13}, V_{14}, V_{15} \) and \( V_{16} \) are given by (37) and (25).

If \( f(t) = f_{\text{univ}}(t) \)

Proceeding as in case (i) it can be shown that
\[
E(T) = \frac{1}{\lambda} \left[ \sum_{i=1}^{k} \frac{1}{n} \left( p_1 V_9 + p_2 V_{10} - p_1 p_2 V_{13} - p_1 q_2 V_{14} - p_2 q_1 V_{15} - q_1 q_2 V_{16} + q_1 V_{11} + q_2 V_{12} \right) \right] \tag{38}
\]
and

\[ E(T^2) = \frac{2}{\lambda^2} \left[ p_1 V_{i9}^2 + p_2 V_{10}^2 - p_1 p_2 V_{13}^2 - p_1 q_2 V_{14}^2 - p_2 q_1 V_{15}^2 \right] \left( \sum_{n=1}^{k} V/n \right)^2 + \frac{2}{\lambda^2} \left( \sum_{n=1}^{k} V^2/n^2 \right) \]

\[ \left[ q_1 V_{i11}^2 + q_2 V_{i12}^2 - q_1 q_2 V_{i16}^2 \right] - \frac{1}{\lambda^2} \left[ q_1 V_{i11} + q_2 V_{i12} - q_1 q_2 V_{i16} \right] \left( \sum_{n=1}^{k} V/n \right)^2 - \sum_{n=1}^{k} V^2/n \]

\[ - \frac{1}{\lambda^2} \left[ p_1 M_{10} + p_2 M_{13} - p_1 p_2 M_{15} - p_1 q_2 M_{14} - p_2 q_1 M_{15} \right] \left( \sum_{n=1}^{k} V/n \right)^2 - \sum_{n=1}^{k} V^2/n \] (39)

when \( n=1 \), in (35) & (36) \( V_{i0}, V_{i11}, V_{i12}, V_{i13}, V_{i14}, V_{i15} \) and \( V_{i16} \) are given by (37) and (23).

when \( n=k \), in (35) & (36) \( V_{i10}, V_{i11}, V_{i12}, V_{i13}, V_{i14}, V_{i15} \) and \( V_{i16} \) are given by (37) and (25).

IV. Model description and analysis for Model-II

For this model \( Y = \min(Y_A, Y_B) \). All the other assumptions and notations are as in model-I. Then the values of \( E(T) \) & \( E(T^2) \) when \( r=1 \) and \( r=k \) are given by:

**case 1:**

If \( f(t)=f_{\text{ext}}(t) \)

Proceeding as in case 1 it can be shown that

\[ E(T) = \frac{1}{\lambda} \left[ V_3 \right] \]

\[ E(T^2) = \frac{2}{\lambda^2} \left[ V_3^2 \right] \] (40)

when \( n=1 \), in (40) & (41) \( V_3 \) is given by (15) and (23).

when \( n=k \), in (40) & (41) \( V_3 \) is given by (15) and (25).

If \( f(t)=f_{\text{sin}}(t) \)

Proceeding as in case 1 it can be shown that

\[ E(T) = \sum_{n=1}^{k} \frac{1}{\lambda} \left[ V_3 \right] \]

\[ E(T^2) = \frac{2}{\lambda^2} \left[ \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 \right] - \frac{1}{\lambda^2} \left[ \sum_{n=1}^{k} V_3^2/n \right] \] (42)

when \( n=1 \), in (42) & (43) \( V_3 \) is given by (15) and (23).

when \( n=k \), in (42) & (43) \( V_3 \) is given by (15) and (25).

and \( V(T) = E(T^2) - (E(T))^2 \)

**Case 2:**

If \( f(t)=f_{\text{ext}}(t) \)

Proceeding as in case 1 it can be shown that

\[ E(T) = \frac{1}{k \lambda} \left[ 4V_3 + V_6 - 2V_4 - 2V_5 \right] \]

\[ E(T^2) = \frac{2}{k \lambda^2} \left[ 4V_3^2 + V_6^2 - 2V_4^2 - 2V_5^2 \right] \] (44)

when \( n=1 \), in (44) & (45) \( V_3, V_4, V_5 \) and \( V_6 \) are given by (15), (28) and (23).
when \( n=k \) in (44) & (45) \( V_3, V_4, V_5 \) and \( V_6 \) are given by (15),(28) and (25).

If \( f(t)=f_{u(t)} \)

Proceeding as in case 1 it can be shown that

\[
E(T) = \frac{1}{k\lambda} \left[ 4V_3 + V_6 - 2V_4 - 2V_5 \right]
\]

(46)

\[
E(T^2) = \frac{2}{k^2\lambda^2} \left[ 4V_3^2 + V_6^2 - 2V_4^2 - 2V_5^2 \right] - \frac{1}{\lambda^2} \left[ \sum_{n=1}^{k} \frac{1}{n^2} \right] \left[ \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 - \sum_{n=1}^{k} \frac{1}{n^2} \right]
\]

(47)

when \( n=1 \) in (46) & (47) \( V_3, V_4, V_5 \) and \( V_6 \) are given by (15),(28) and (23).

when \( n=k \) in (46) & (47) \( V_3, V_4, V_5 \) and \( V_6 \) are given by (15),(28) and (25).

Case 3:

If \( f(t)=f_{u(t)} \)

Proceeding as in case 1 it can be shown that

\[
E(T) = \frac{1}{k\lambda} \left[ 2V_3 - V_4 \right]
\]

(48)

\[
E(T^2) = \frac{2}{k^2\lambda^2} \left[ 2V_3^2 - V_4^2 \right]
\]

(49)

when \( n=1 \) in (48) & (49) \( V_3 \) and \( V_4 \) are given by (15),(28) and (23).

when \( n=k \) in (48) & (49) \( V_3 \) and \( V_4 \) are given by (15),(28) and (25).

If \( f(t)=f_{u(t)} \)

Proceeding as in case 1 it can be shown that

\[
E(T) = \sum_{n=1}^{k} \frac{1}{n} \left[ 2V_3 - V_4 \right]
\]

(50)

\[
E(T^2) = \frac{2}{k^2\lambda^2} \left[ \left( 2V_3^2 - V_4^2 \right) \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 - \frac{1}{\lambda^2} \left[ \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 - \sum_{n=1}^{k} \frac{1}{n^2} \right] \right]
\]

(51)

when \( n=1 \) in (50) & (51) \( V_3 \) and \( V_4 \) are given by (15),(28) and (23).

when \( n=k \) in (50) & (51) \( V_3 \) and \( V_4 \) are given by (15),(28) and (25).

and \( V(T) = E(T^2) - (E(T))^2 \)

Case 4:

If \( f(t)=f_{u(t)} \)

Proceeding as in case 1 it can be shown that

\[
E(T) = \frac{1}{k\lambda} \left[ p_1 p_2 V_{13} + p_1 q_2 V_{14} + p_2 q_1 V_{15} + q_1 q_2 V_{16} \right]
\]

(52)

\[
E(T^2) = \frac{2}{k^2\lambda^2} \left[ p_1 p_2 M_{13}^2 + p_1 q_2 M_{14}^2 + p_2 q_1 M_{15}^2 + q_1 q_2 M_{16}^2 \right]
\]

(53)

when \( n=1 \) in (52) & (53) \( V_{13}, V_{14}, V_{15} \) and \( V_{16} \) are given by (37) and (23).

when \( n=k \) in (52) & (53) \( V_{13}, V_{14}, V_{15} \) and \( V_{16} \) are given by (37) and (25).

If \( f(t)=f_{u(t)} \)
Proceeding as in case 1 it can be shown that

\[
E(T) = \frac{1}{\lambda} \left[ p_1, p_2 V_{13} + p_1 q_1 V_{14} + p_2 q_1 V_{15} + q_1 q_2 V_{16} \right] (54)
\]

\[
E(T^2) = \frac{2}{\lambda^2} \left[ p_1, p_2 V_{13} + p_1 q_1 V_{14} + p_2 q_1 V_{15} + q_1 q_2 V_{16} \right] \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 - \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 - \sum_{n=1}^{k} \frac{1}{n^2} (55)
\]

when \( n=1 \), in (54) & (55) \( V_{13}, V_{14}, V_{15} \) and \( V_{16} \) are given by (37) and (23).

when \( n=k \), in (54) & (55) \( V_{13}, V_{14}, V_{15} \) and \( V_{16} \) are given by (37) and (25).

V. Model description and analysis for Model-III

For this model \( Y = Y_A + Y_B \). All the other assumptions and notations are as in model-I. Then the values of \( E(T) \) & \( E(T^2) \) when \( n=1 \) and \( n=k \) are given by

case 1: If \( f(t)=f_{\text{res}}(t) \)

Proceeding as in case 1 it can be shown that

\[
E(T) = \frac{1}{\lambda} \left[ \frac{\alpha_A}{\alpha_A - \alpha_B} \right] V_2 - \left[ \frac{\alpha_B}{\alpha_A - \alpha_B} \right] V_1 \] (56)

\[
E(T^2) = \frac{2}{\lambda^2} \left[ \frac{\alpha_A}{\alpha_A - \alpha_B} \right] V_2^2 - \left[ \frac{\alpha_B}{\alpha_A - \alpha_B} \right] V_1^2 \] (57)

when \( n=1 \), in (56) & (57) \( V_1 \) and \( V_2 \) are given by (15) and (23).

when \( n=k \), in (56) & (57) \( V_1 \) and \( V_2 \) are given by (15) and (25).

If \( f(t)=f_{\text{res}}(t) \)

Proceeding as in case 1 it can be shown that

\[
E(T) = \frac{1}{\lambda} \left[ \sum_{n=1}^{k} \frac{1}{n} \right] \left[ \frac{\alpha_A}{\alpha_A - \alpha_B} \right] V_2 - \left[ \frac{\alpha_B}{\alpha_A - \alpha_B} \right] V_1 \] (58)

\[
E(T^2) = \frac{2}{\lambda^2} \left[ \sum_{n=1}^{k} \frac{1}{n} \right]^2 \left[ \frac{\alpha_A}{\alpha_A - \alpha_B} \right] V_2^2 - \left[ \frac{\alpha_B}{\alpha_A - \alpha_B} \right] V_1^2 \right] - \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 - \sum_{n=1}^{k} \frac{1}{n^2} \left( \sum_{n=1}^{k} \frac{1}{n^2} \right) \lambda^2 \] (59)

when \( n=1 \), in (58) & (59) \( V_1 \) and \( V_2 \) are given by (15) and (23).

when \( n=k \), in (58) & (59) \( V_1 \) and \( V_2 \) are given by (15) and (25).

Case 2:

If \( f(t)=f_{\text{tra}}(t) \)

Proceeding as in case 1 it can be shown that

\[
E(T) = \frac{1}{\lambda} \left[ \sum_{n=1}^{k} \frac{1}{n} \right] \left[ \frac{\alpha_A}{\alpha_A - \alpha_B} \right] V_2 - \left[ \frac{\alpha_B}{\alpha_A - \alpha_B} \right] V_1 \] (60)

\[
E(T^2) = \frac{2}{\lambda^2} \left[ \sum_{n=1}^{k} \frac{1}{n} \right]^2 \left[ \frac{\alpha_A}{\alpha_A - \alpha_B} \right] V_2^2 - \left[ \frac{\alpha_B}{\alpha_A - \alpha_B} \right] V_1^2 \right] - \left( \sum_{n=1}^{k} \frac{1}{n} \right)^2 - \sum_{n=1}^{k} \frac{1}{n^2} \left( \sum_{n=1}^{k} \frac{1}{n^2} \right) \lambda^2 \] (61)

when \( n=1 \), in (60) & (61) \( V_1 \) and \( V_2 \) are given by (15) and (23).

when \( n=k \), in (60) & (61) \( V_1 \) and \( V_2 \) are given by (15) and (25).
\[ E(T) = \frac{1}{k\lambda} \left[ \left( \frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) V_1 + \left( \frac{4\alpha_A}{\alpha_A - \alpha_B} \right) \right] + \frac{1}{k\lambda} \left[ \left( \frac{2\alpha_B}{2\alpha_A - \alpha_B} \right) V_7 + \left( \frac{\alpha_A}{\alpha_A - \alpha_B} \right) \right] \]  
\[ E(T^2) = \frac{2}{k^2\lambda^2} \left[ \left( \frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) V_1^2 + \left( \frac{4\alpha_A}{\alpha_A - \alpha_B} \right) \right] + \frac{2}{k^2\lambda^2} \left[ \left( \frac{2\alpha_B}{2\alpha_A - \alpha_B} \right) V_7^2 + \left( \frac{\alpha_A}{\alpha_A - \alpha_B} \right) \right] \]  
\[ \text{when } n=1 \text{ in (60) & (61) } V_1, V_2, V_7 \text{ and } V_8 \text{ are given by (15), (28) and (23).} \]
\[ \text{when } n=k \text{ in (60) & (61) } V_1, V_2, V_7 \text{ and } V_8 \text{ are given by (15), (28) and (25).} \]

If \( f(t) = f_{sk}(t) \)

Proceeding as in case 1 it can be shown that

\[ E(T) = \sum_{n=1}^{k} \frac{1}{n} \frac{1}{\lambda} \left[ \left( \frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) V_1 + \left( \frac{4\alpha_A}{\alpha_A - \alpha_B} \right) \right] + \sum_{n=1}^{k} \frac{1}{n} \frac{1}{\lambda} \left[ \left( \frac{2\alpha_B}{2\alpha_A - \alpha_B} \right) V_7 + \left( \frac{\alpha_A}{\alpha_A - \alpha_B} \right) \right] \]  
\[ E(T^2) = \frac{2}{\lambda^2} \left[ \sum_{n=1}^{k} \frac{1}{n} \left( \frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) V_1^2 + \left( \frac{4\alpha_A}{\alpha_A - \alpha_B} \right) \right] + \frac{2}{\lambda^2} \left[ \sum_{n=1}^{k} \frac{1}{n} \left( \frac{2\alpha_B}{2\alpha_A - \alpha_B} \right) V_7^2 + \left( \frac{\alpha_A}{\alpha_A - \alpha_B} \right) \right] \]  
\[ \text{when } n=1 \text{ in (62) & (63) } V_1, V_2, V_7 \text{ and } V_8 \text{ are given by (15), (28) and (23).} \]
\[ \text{when } n=k \text{ in (62) & (63) } V_1, V_2, V_7 \text{ and } V_8 \text{ are given by (15), (28) and (25).} \]

Case 3:
If \( f(t) = f_{sk}(t) \)

Proceeding as in case 1 it can be shown that
\[ E(T) = \frac{1}{k\lambda} \left[ \left( \frac{\alpha_B}{2\alpha_A - \alpha_B} \right) V_7 - \left( \frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_1 + \left( \frac{2\alpha_A}{\alpha_A - \alpha_B} \right) \left( \frac{2\alpha_B}{\alpha_A - \alpha_B} \right) \right] (64) \]

\[ E(T^2) = \frac{2}{k^2\lambda^2} \left[ \left( \frac{2\alpha_A}{\alpha_A - \alpha_B} \right) V_7^2 + \left( \frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_7 - \left( \frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_1^2 \right] (65) \]

when \( n=1 \) in (64) & (65) \( V_1, V_2, V_7 \) and \( V_8 \) are given by (15), (28) and (23).

when \( n=k \) in (64) & (65) \( V_1, V_2, V_7 \) and \( V_8 \) are given by (15), (28) and (25).

If \( f(t)=f_{wk}(t) \)

Proceeding as in case 1 it can be shown that

\[ E(T) = \frac{1}{\lambda} \left[ \left( \frac{2\alpha_A}{\alpha_A - \alpha_B} \right) V_2 + \left( \frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_7 - \left( \frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_1 \right] (66) \]

and

\[ E(T^2) = \frac{2}{\lambda^2} \left[ \left( \frac{2\alpha_A}{\alpha_A - \alpha_B} \right) V_2^2 + \left( \frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_7 - \left( \frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_1^2 \right] (67) \]

when \( n=1 \) in (66) & (67) \( V_1, V_2, V_7 \) and \( V_8 \) are given by (15), (28) and (23).

when \( n=k \) in (66) & (67) \( V_1, V_2, V_7 \) and \( V_8 \) are given by (15), (28) and (25).

Case 4:

If \( f(t)=f_{wk1}(t) \)

Proceeding as in case 1 it can be shown that

\[ E(T) = \frac{1}{k\lambda} \left[ \left( \frac{p_1 p_2 (\alpha_A + \mu_1)}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{q_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right) \right] V_9 + \left( \frac{p_1 q_2 (\alpha_A + \mu_1)}{\alpha_A + \mu_1 - \mu_4} \right) \left( \frac{q_1 q_2 \mu_3}{\mu_2 - \mu_4} \right) \] (68)

and

\[ E(T^2) = \frac{2}{k^2\lambda^2} \left[ \left( \frac{p_1 p_2 (\alpha_A + \mu_1)}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{q_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right) \right] V_9^2 + \left( \frac{p_1 q_2 (\alpha_A + \mu_1)}{\alpha_A + \mu_1 - \mu_4} \right) \left( \frac{q_1 q_2 \mu_3}{\mu_2 - \mu_4} \right) \] (69)

when \( n=1 \) in (68) & (69) \( V_9, V_{10}, V_{11} \) and \( V_{12} \) are given by (28) and (23).

when \( n=k \) in (68) & (69) \( V_9, V_{10}, V_{11} \) and \( V_{12} \) are given by (28) and (25).

If \( f(t)=f_{wk1}(t) \)

Proceeding as in case 1 it can be shown that
\[ E(T) = \frac{1}{\lambda} \sum_{n=1}^{k} \frac{1}{n} \left[ \left( \frac{p_1 p_2 (\alpha_A + \mu_1) + q_1 p_2 \mu_2}{\alpha_A - \alpha_B + \mu_1 - \mu_3} \right) \frac{1}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{q_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right] V_{10} + \left( \frac{p_2 q_1 (\alpha_A + \mu_1) + q_1 q_2 \mu_2}{\alpha_A + \mu_1 - \mu_4} + \frac{q_1 q_2 \mu_2}{\mu_2 - \mu_4} \right) V_{12} \]

\[ \frac{1}{\lambda} \sum_{n=1}^{k} \frac{1}{n} \left[ \left( \frac{p_1 p_2 (\alpha_A + \mu_1) + q_1 p_2 \mu_2}{\alpha_A - \alpha_B + \mu_1 - \mu_3} \right) \frac{1}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{q_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right] V_{9} + \left( \frac{p_2 q_1 (\alpha_A + \mu_1) + q_1 q_2 \mu_2}{\alpha_A + \mu_1 - \mu_4} + \frac{q_1 q_2 \mu_2}{\mu_2 - \mu_4} \right) V_{11} \]  

(70)

and

\[ E(T^2) = \frac{2}{\lambda^2} \sum_{n=1}^{k} \frac{1}{n^2} \left[ \left( \frac{p_1 p_2 (\alpha_A + \mu_1) + q_1 p_2 \mu_2}{\alpha_A - \alpha_B + \mu_1 - \mu_3} \right) \frac{1}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{q_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right] V_{10}^2 + \left( \frac{p_2 q_1 (\alpha_A + \mu_1) + q_1 q_2 \mu_2}{\alpha_A + \mu_1 - \mu_4} + \frac{q_1 q_2 \mu_2}{\mu_2 - \mu_4} \right) V_{12}^2 \]

\[ - 2 \left( \frac{1}{n} \sum_{n=1}^{k} \frac{1}{n} \right)^2 \left( \frac{p_1 p_2 (\alpha_A + \mu_1) + q_1 p_2 \mu_2}{\alpha_A - \alpha_B + \mu_1 - \mu_3} \right) \frac{1}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{q_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right] V_{9}^2 + \left( \frac{p_2 q_1 (\alpha_A + \mu_1) + q_1 q_2 \mu_2}{\alpha_A + \mu_1 - \mu_4} + \frac{q_1 q_2 \mu_2}{\mu_2 - \mu_4} \right) V_{11}^2 \]

\[ \left( \frac{1}{n} \sum_{n=1}^{k} \frac{1}{n} \right)^2 \left( \frac{p_1 p_2 (\alpha_A + \mu_1)}{\alpha_A - \alpha_B + \mu_1 - \mu_3} \right) \frac{1}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{p_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right] V_{10} + \left( \frac{p_2 q_1 (\alpha_A + \mu_1) + q_1 q_2 \mu_2}{\alpha_A + \mu_1 - \mu_4} + \frac{q_1 q_2 \mu_2}{\mu_2 - \mu_4} \right) V_{12} \]

\[ + \left( \frac{1}{n} \sum_{n=1}^{k} \frac{1}{n} \right)^2 \left( \frac{p_1 p_2 (\alpha_A + \mu_1)}{\alpha_A - \alpha_B + \mu_1 - \mu_3} \right) \frac{1}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{p_1 q_2 \mu_4}{\mu_2 - \alpha_B - \mu_3} \right] V_{9} + \left( \frac{p_2 q_1 (\alpha_A + \mu_1) + q_1 q_2 \mu_2}{\alpha_A + \mu_1 - \mu_4} + \frac{q_1 q_2 \mu_2}{\mu_2 - \mu_4} \right) V_{11} \]  

(71)

when \( n=1 \) in (70) & (71) \( V_9, V_{10}, V_{11} \) and \( V_{12} \) are given by (28) and (23).

when \( n=k \) in (70) & (71) \( V_9, V_{10}, V_{11} \) and \( V_{12} \) are given by (28) and (25).

and \( V(T) = E(T^2) - (E(T))^2 \)

VI. Numerical illustration

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment is studied numerically. In the following tables these performance measures are calculated by varying the parameter ‘\( p \)’ at a time and keeping the other parameters fixed as \( \alpha_A=0.1, \alpha_B=0.3, \lambda=0.5, \mu_1=0.4, \mu_2=0.8, \mu_3=0.6, \mu_4=0.7 \).
### Table 1: Effect of ‘c’ and ‘k’ on E(T) for Model-I

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<th>1</th>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
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<tr>
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<td>33.1667</td>
<td>44</td>
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<td>22.0667</td>
<td>22</td>
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Table 2: Effect of ‘c’ and ‘k’ on E(T) for Model-II

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<td>4</td>
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<td>n=1</td>
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Table 3: Effect of ‘c’ and ‘k’ on $E(T)$ for Model-III

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Findings
From the above tables it is found that
1. When the probability density function of inter decision time is same as the probability density function of first order statistics, as ‘k’ increases the mean time to recruitment decreases for the first and kth order statistics for the loss of manhours but it is increases when the probability density function of inter decision time is same as the kth order statistics.
2. When the probability density function of inter decision time is same as the probability density function of first order statistics or the kth order statistics, as ‘c’ increases the mean time to recruitment increases for the first and kth order statistics for the loss of manhours.

Conclusion
Since the time to recruitment is more elongated in model-III than the first two models, model-III is preferable from the organization point of view.

References:


