Intuitionistic Fuzzy Generalized Beta Closed Mappings

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ABSTRACT
In this paper we introduce intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings. We investigate some of their properties. We also introduce intuitionistic fuzzy M-generalized beta closed mappings and intuitionistic fuzzy M-generalized beta open mappings. We provide the relation between intuitionistic fuzzy M-generalized beta closed mappings and intuitionistic fuzzy M-generalized beta closed mappings.

Key words and phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings.

I. Introduction
Zadeh [10] introduced the notion of fuzzy sets. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings and study some of their properties. We also introduce intuitionistic fuzzy M-generalized beta closed mappings as well as intuitionistic fuzzy M-generalized beta open mappings. We provide the relation between intuitionistic fuzzy M-generalized beta closed mappings and intuitionistic fuzzy generalized beta closed mappings.

II. Preliminaries
Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form \( A = \{ (x, \mu_A(x), \nu_A(x)) \} \) where the functions \( \mu_A : X \to [0,1] \) and \( \nu_A : X \to [0,1] \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of non-membership (namely \( \nu_A(x) \)) of each element \( x \in X \) to the set A, respectively, and \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \). Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form \( A = \{ (x, \mu_A(x), \nu_A(x)) \} \) and \( B = \{ (x, \mu_B(x), \nu_B(x)) \} \). Then
(a) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \) for all \( x \in X \)
(b) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \)
(c) \( A' = \{ (x, \nu_A(x), \mu_A(x)) \} \)
(d) \( A \cap B = \{ (x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x)) \} \)
(e) \( A \cup B = \{ (x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x)) \} \)

The intuitionistic fuzzy sets \( 0 = \{ (x, 0, 1) \} \) and \( 1 = \{ (x, 1, 0) \} \) are respectively the empty set and the whole set of \( X \).

We shall use the notation \( A = \{ (x, \mu_A(x), \nu_A(x)) \} \) instead of \( A = \{ (x, \mu_A(x), \nu_A(x)) \} \).

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT for short) on X is a family \( \tau \) of IFSs in X satisfying the following axioms.
(i) \( \emptyset, X \in \tau \)
(ii) \( \bigcap_{i} G_i \in \tau \) for any \( G_i \in \tau \)
(iii) \( \bigcup_{i} G_i \in \tau \) for any family \( \{ G_i : i \in I \} \subseteq \tau \).

In this case the pair \( (X, \tau) \) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement \( A' \) of an IFOS A in IFTS (X, \( \tau \)) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[3] Let \( (X, \tau) \) be an IFTS and \( A = \{ (x, \mu_A, \nu_A) \} \) be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
\( \text{int}(A) = \bigcup \{ G : G \in \tau \text{ and } G \subseteq A \} \)
\( \text{cl}(A) = \bigcap \{ K : K \in \tau \text{ and } K \supseteq A \} \)

Note that for any IFS A in \( (X, \tau) \), we have\( \text{cl}(A') = (\text{int}(A))' \) and \( \text{int}(A') = (\text{cl}(A))' \) [3].

Definition 2.5:[3] An IFS \( A = \{ (x, \mu_A, \nu_A) \} \) in an IFTS \( (X, \tau) \) is said to be an
(i) intuitionistic fuzzy semi closed set (IFSCS in short) if \( \text{cl}(\text{int}(A)) \subseteq A \)
(ii) intuitionistic fuzzy pre closed set (IFPCS in short) if \( \text{cl}(\text{int}(A)) \subseteq A \)
(iii) intuitionistic fuzzy a closed set (IFaCS in short) if \( \text{cl}(\text{int}(A)) \subseteq A \).
The respective complements of the above IFCSs are called their respective IFOSs.

**Definition 2.6:**[3] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS $(X, \tau)$ is said to be
(i) intuitionistic fuzzy beta closed set (IF$\beta$CS for short) if $\text{int}(\text{cl}(A)) \subseteq A$.
(ii) intuitionistic fuzzy beta open set (IF$\beta$OS for short) if $A \subseteq \text{cl}(\text{int}(A))$.

The family of all IF$\beta$CSs (respectively IF$\beta$OSs) of an IFTS $(X, \tau)$ is denoted by IF$\beta$C$(X)$ (respectively IF$\beta$O$(X)$).

**Definition 2.7:**[6] An IF$\alpha$ in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy beta open set (IF$\beta$OS for short) if $f(A)$ is an IF$\alpha$OS in $Y$ for each IF$\alpha$ $A$ in $X$.

**Definition 2.8:**[6] An IFS $A$ in an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized beta closed set (IF$\beta$gCS for short) if $\beta\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $(X, \tau)$.

Every IFCS, IFSCS, IFPCS, IFROCS, IF$\alpha$OS and IF$\beta$CS is an IF$\beta$gCS but the separate converses may not be true in general.[6]

The family of all IF$\beta$gCSs of an IFTS $(X, \tau)$ is denoted by IF$\beta$gC$(X)$.

**Definition 2.9:**[6] The complement $A'$ of an IF$\beta$gCS $A$ in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy generalized beta open set (IF$\beta$gOS for short) in $X$.

Every IFOS, IFPOS, IFROOS, IF$\alpha$OS, IF$\beta$OS and IF$\beta$CS is an IF$\beta$gOS but the separate converses may not be true in general.[6]

The family of all IF$\beta$gOSs of an IFTS $(X, \tau)$ is denoted by IF$\beta$gO$(X)$.

**Definition 2.10:**[6] If every IF$\beta$gCS in $(X, \tau)$ is an IFCS in $(X, \tau)$, then the space can be called as an intuitionistic fuzzy beta $T_{1/2}$ space (IF$\beta$T$_{1/2}$ space for short).

**Definition 2.11:**[4] A map $f : X \rightarrow Y$ is called an intuitionistic fuzzy closed mapping (IFCM for short) if $f(A)$ is an IFCS in $Y$ for each IFCS $A$ in $X$.

**Definition 2.12:**[4] A mapping $f : X \rightarrow Y$ is said to be an intuitionistic fuzzy open mapping (IFOM for short) if $f(A)$ is an IFOS in $Y$ for each IFOS $A$ in $X$.

**Definition 2.13:**[4] A map $f : X \rightarrow Y$ is called an (i) intuitionistic fuzzy semi open mapping (IF$\alpha$OM for short) if $f(A)$ is an IF$\alpha$OS in $Y$ for each IF$\alpha$ $A$ in $X$.

(ii) intuitionistic fuzzy $\alpha$ open mapping (IF$\alpha$OM for short) if $f(A)$ is an IF$\alpha$OS in $Y$ for each IF$\alpha$ $A$ in $X$.

(iii) intuitionistic fuzzy preopen mapping (IF$\alpha$POM for short) if $f(A)$ is an IF$\alpha$OS in $Y$ for each IF$\alpha$ $A$ in $X$.

**Definition 2.14:**[7] The IFS $c(\alpha , \beta ) = \langle x, c_A , c_B \rangle$ where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP for short) in $X$.

Note that an IFP $c(\alpha , \beta )$ is said to belong to an IFS $A = \langle x, \mu_A , \nu_A \rangle$ of $X$ denoted by $c(\alpha , \beta ) \in A$ if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

**Definition 2.15:**[8] Let $c(\alpha , \beta )$ be an IFP of an IFTS $(X, \tau)$. An IFS $A$ of $X$ is called an intuitionistic fuzzy beta neighborhood (IFN for short) of $c(\alpha , \beta )$ if there exists an IF$\beta$OS $B$ in $X$ such that $c(\alpha , \beta ) \subseteq B$.

**Definition 2.16:**[6] Let $c(\alpha , \beta )$ be an IFP in $(X, \tau)$. An IFS $A$ of $X$ is called an intuitionistic fuzzy beta neighborhood (IFN for short) of $c(\alpha , \beta )$ if there is an IF$\beta$OS $B$ in $X$ such that $c(\alpha , \beta ) \subseteq B$.

**Theorem 2.17:** Let $(X, \tau)$ be an IFTS where $X$ is an IF$\beta$T$_{1/2}$ space. An IFS $A$ is an IF$\beta$gOS in $X$ if and only if $A$ is an IF$\beta$gN of $c(\alpha , \beta )$ for each IFP $c(\alpha , \beta ) \in A$.

**Proof:** Necessity: Let $c(\alpha , \beta ) \in A$. Let $A$ be an IF$\beta$gOS in $X$. Since $X$ is an IF$\beta$T$_{1/2}$ space, $A$ is an IF$\beta$OS in $X$. Then clearly $A$ is an IF$\beta$N of $c(\alpha , \beta )$.

Sufficiency: Let $c(\alpha , \beta ) \in A$. Since $A$ is an IF$\beta$N of $c(\alpha , \beta )$, there is an IF$\beta$OS $B$ in $X$ such that $c(\alpha , \beta ) \subseteq B$. Now $A = \cup \{ c(\alpha , \beta ) / c(\alpha , \beta ) \in A \} \subseteq \cup \{ B_{c(\alpha , \beta )} / c(\alpha , \beta ) \in A \}$. This implies $A = \cup \{ B_{c(\alpha , \beta )} / c(\alpha , \beta ) \in A \}$. Since each $B$ is an IF$\beta$OS, $A$ is an IF$\beta$OS and hence an IF$\beta$gOS in $X$.

**Theorem 2.18:** For any IFS $A$ in an IFTS $(X, \tau)$ where $X$ is an IF$\beta$T$_{1/2}$ space, $A \in$ IF$\beta$gO$(X)$ if and only if for every IFP $c(\alpha , \beta ) \in A$, there exists an IF$\beta$gOS $B$ in $X$ such that $c(\alpha , \beta ) \subseteq B$.\[2\]

Proof: Necessity: If $A \in$ IF$\beta$gO$(X)$, then we can take $B = A$ so that $c(\alpha , \beta ) \in B \subseteq A$ for every IFP $c(\alpha , \beta ) \in A$.

Sufficiency: Let $A$ be an IFS in $X$ and assume that there exists $B \in$ IF$\beta$gO$(X)$ such that $c(\alpha , \beta ) \in B \subseteq A$. Since $X$ is an IF$\beta$T$_{1/2}$ space, $B$ is an IF$\beta$OS of $X$. Then $A = \cup_{c(\alpha , \beta ) \in A} c(\alpha , \beta ) \subseteq \cup_{c(\alpha , \beta ) \in A} B \subseteq A$. Therefore $A = \cup_{c(\alpha , \beta ) \in A} B$ is an IF$\beta$OS [6] and...
III. Intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings

In this section we introduce intuitionistic fuzzy generalized beta closed mappings and intuitionistic fuzzy generalized beta open mappings. We study some of their properties.

**Definition 3.1:** A map \( f: X \rightarrow Y \) is called an intuitionistic fuzzy generalized beta closed mapping (IFGβCM for short) if \( f(A) \) is an IFGβCS in \( Y \) for each IFCS \( A \) in \( X \).

The sake of simplicity, we shall use the notation \( A = \langle x, (\mu_A, \nu_A), (\nu_A, \mu_A) \rangle \) instead of \( A = \langle x, (\mu_A, \nu_A), (\nu_A, \mu_A) \rangle \) in the following examples.

Similarly we shall use the notation \( B = \langle y, (\mu_B, \nu_B), (\nu_B, \mu_B) \rangle \) instead of \( B = \langle y, (\mu_B, \nu_B), (\nu_B, \mu_B) \rangle \) in the following examples.

**Example 3.2:** Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = (x, (0.5, 0.6), (0.5, 0.4)), G_2 = (y, (0.3, 0.4), (0.7, 0.6)) \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \).

Then \( f \) is an IFGβCM.

**Theorem 3.3:** Every IFCSM is an IFGβCM but not conversely.

**Proof:** Let \( f: X \rightarrow Y \) be an IFCSM. Let \( A \) be an IFCS in \( X \). Then \( f(A) \) is an IFCS in \( Y \). Since every IFCS is an IFGβCS, \( f(A) \) is an IFGβCS in \( Y \). Hence \( f \) is an IFGβCM.

**Example 3.4:** In Example 3.2 \( f \) is an IFGβCM but not an IFCSM, since \( G_1^c = (x, (0.5, 0.4), (0.5, 0.6)) \) is an IFCS in \( X \), but \( f(G_1^c) = (y, (0.5, 0.4), (0.5, 0.6)) \) is not an IFCS in \( Y \), since \( cl(f(G_1^c)) = G_1^c \neq f(G_1^c) \).

**Theorem 3.5:** Every IFαCM is an IFGβCM but not conversely.

**Proof:** Let \( f: X \rightarrow Y \) be an IFαCM. Let \( A \) be an IFCS in \( X \). Then \( f(A) \) is an IFCS in \( Y \). Since every IFCS is an IFGβCS, \( f(A) \) is an IFGβCS in \( Y \). Hence \( f \) is an IFGβCM.

**Example 3.6:** Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = (x, (0.5, 0.4), (0.5, 0.6), (0.2, 0.3)), G_2 = (y, (0.8, 0.7), (0.2, 0.3)) \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \).

Then \( f \) is an IFGβCM but not an IFαCM. Since \( G_1^c = (x, (0.5, 0.4), (0.5, 0.6), (0.2, 0.3)) \) is an IFCS in \( X \) but \( f(G_1^c) = (y, (0.5, 0.4), (0.5, 0.4)) \) is not an IFCS in \( Y \), since \( cl(int(cl(f(G_1^c)))) = 1 \neq f(G_1^c) \).

**Theorem 3.7:** Every IFSCM is an IFGβCM but not conversely.

**Proof:** Let \( f: X \rightarrow Y \) be an IFSCM. Let \( A \) be an IFCS in \( X \). Then \( f(A) \) is an IFSCS in \( Y \). Since every IFSC is an IFGβCS, \( f(A) \) is an IFGβCS in \( Y \). Hence \( f \) is an IFGβCM.

**Example 3.8:** In Example 3.6, \( f \) is an IFGβCM but not an IFSCM, since \( G_1^c = (x, (0.5, 0.4), (0.5, 0.4)) \) is not an IFSCS in \( Y \), since \( cl(int(cl(f(G_1^c)))) = 1 \neq f(G_1^c) \).

**Theorem 3.9:** Every IFPCM is an IFGβCM but not conversely.

**Proof:** Let \( f: X \rightarrow Y \) be an IFPCM. Let \( A \) be an IFCS in \( X \). Then \( f(A) \) is an IFPCS in \( Y \). Since every IFPCS is an IFGβCS, \( f(A) \) is an IFGβCS in \( Y \). Hence \( f \) is an IFGβCM.

**Example 3.10:** Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = (x, (0.5, 0.6), (0.5, 0.4)), G_2 = (y, (0.2, 0.3), (0.8, 0.7)) \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \).

Then \( f \) is an IFGβCM but not an IFPCM, since \( f(G_1^c) = (x, (0.5, 0.4), (0.5, 0.4)) \) is an IFCS in \( Y \) but not an IFPCS in \( Y \), since \( cl(int(cl(f(X^c)))) \neq G_2^c \).

**Definition 3.11:** A mapping \( f: X \rightarrow Y \) is said to be an intuitionistic fuzzy M-generalized beta closed mapping (IFMGβCM, for short) if \( f(A) \) is an IFGβCS in \( Y \) for every IFCS \( A \) in \( X \).

**Example 3.12:** Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = (x, (0.5, 0.4), (0.5, 0.4)), G_2 = (y, (0.3, 0.2), (0.8, 0.7)) \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \).

Then \( f \) is an IFMGβCM.

**Theorem 3.13:** Every IFMGβCM is an IFGβCM but not conversely.

**Proof:** Let \( f: X \rightarrow Y \) be an IFMGβCM. Let \( A \) be an IFCS in \( X \). Then \( A \) is an IFGβCS in \( X \). By hypothesis \( f(A) \) is an IFGβCS in \( Y \). Therefore \( f \) is an IFGβCM.

**Example 3.14:** Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = (x, (0.5, 0.4), (0.5, 0.6)), G_2 = (y, (0.6, 0.7), (0.4, 0.3)), G_3 = (y, (0.7, 0.8), (0.3, 0.2)) \). Then...
Theorem 3.15: Let \( f: X \rightarrow Y \) be a mapping. Then the following are equivalent if \( Y \) is an IF\( \beta \)T\( _{1/2} \) space:

1. \( f \) is an IFG\( \beta \)CM
2. \( \beta \text{cl}(f(A)) \subseteq f(\text{cl}(A)) \) for each IFS \( A \) of \( X \).

Proof: (i) \( \Rightarrow \) (ii). Let \( A \) be any IFS in \( X \). Then \( \text{cl}(A) \) is an IFCM in \( X \) (i) implies that \( f(\text{cl}(A)) \) is an IFCS in \( Y \). Since \( Y \) is an IF\( \beta \)T\( _{1/2} \) space, \( f(\text{cl}(A)) \) is an IFCS in \( Y \). Therefore, \( \beta \text{cl}(f(\text{cl}(A))) = f(\text{cl}(A)) \).

(ii) \( \Rightarrow \) (i). Let \( A \) be any IFS in \( X \). Then \( \text{cl}(A) = A \). (ii) implies that \( \beta \text{cl}(f(\text{cl}(A))) \subseteq f(\text{cl}(A)) \). But \( f(\text{cl}(A)) \subseteq f(\text{cl}(A)) \). Therefore \( \beta \text{cl}(f(\text{cl}(A))) = f(\text{cl}(A)) \). This implies \( f(\text{cl}(A)) \) is an IFCS in \( Y \). Since every IF\( \beta \)CM is an IFCS, \( f(\text{cl}(A)) \) is an IFCS in \( Y \). Hence \( f \) is an IFG\( \beta \)CM.

Theorem 3.16: Let \( f: X \rightarrow Y \) be a bijection. Then the following are equivalent if \( Y \) is an IF\( \beta \)T\( _{1/2} \) space:

(i) \( f \) is an IFG\( \beta \)CM
(ii) \( \beta \text{cl}(f(A)) \subseteq f(\text{cl}(A)) \) for each IFS \( A \) of \( X \).

Proof: (i) \( \Leftrightarrow \) (ii) is obvious from Theorem 3.15.

Theorem 3.17: A mapping \( f: X \rightarrow Y \) is an IFG\( \beta \)CM if and only if for every IFS \( B \) in \( Y \) and for every IFOS \( U \) containing \( f^{-1}(B) \), there is an IFG\( \beta \)OS \( A \) of \( Y \) such that \( B \subseteq A \) and \( f^{-1}(A) \subseteq U \).

Proof: Necessity: Let \( B \) be any IFS in \( Y \). Let \( U \) be an IFOS in \( X \) such that \( f^{-1}(B) \subseteq U \). Then \( U \) is an IFCS in \( X \). By hypothesis, \( f(U) \) is an IFCS in \( Y \). Let \( A = f(U) \), then \( A \) is an IFG\( \beta \)OS in \( Y \) and \( B \subseteq A \). Now \( f^{-1}(A) = f^{-1}(f(U)) = f(U) \subseteq U \).

Sufficiency: Let \( A \) be any IFCS in \( X \). Then \( \text{cl}(A) \) is an IFCS in \( Y \) and \( f^{-1}(B) \subseteq A \). By hypothesis, there exists an IFG\( \beta \)OS \( A \) in \( Y \) such that \( f(A) \subseteq B \) and \( f^{-1}(B) \subseteq A \). Therefore, \( A \subseteq f^{-1}(B) \). Hence \( A \subseteq f^{-1}(B) \). This implies that \( A = B \).

Theorem 3.18: Let \( f: X \rightarrow Y \) be a bijective map where \( Y \) is an IF\( \beta \)T\( _{1/2} \) space. Then the following are equivalent:

(i) \( f \) is an IFG\( \beta \)CM
(ii) \( f \) is an IFG\( \beta \)OS in \( Y \) for every IFOS \( B \) in \( X \).

Proof: (i) \( \Rightarrow \) (ii) is obvious.

(iii) \( \Rightarrow \) (i). Let \( B \) be any IFS in \( X \). Then \( \text{int}(B) \) is an IFOS in \( X \). By hypothesis, \( f(\text{int}(B)) \) is an IFG\( \beta \)OS in \( Y \). Since \( Y \) is an IF\( \beta \)T\( _{1/2} \) space, \( f(\text{int}(B)) \) is an IFCS in \( Y \). Therefore, \( f(\text{int}(B)) \subseteq \text{int}(f(\text{int}(B))) \subseteq f(\text{int}(f(\text{int}(B)))) \).

Theorem 3.19: Let \( f: X \rightarrow Y \) be a bijective map where \( Y \) is an IF\( \beta \)T\( _{1/2} \) space. Then the following are equivalent:

(i) \( f \) is an IFG\( \beta \)CM
(ii) \( f \) is an IFG\( \beta \)OS in \( Y \) for every IFOS \( B \) in \( X \).

(iii) \( \text{int}(\text{cl}(f(B))) \subseteq f(\text{cl}(B)) \) for every IFOS \( B \) in \( X \).

Proof: (i) \( \Rightarrow \) (ii) is obvious.

(ii) \( \Rightarrow \) (i). Let \( B \) be any IFS in \( X \). Then \( \text{cl}(B) \) is an IFOS in \( X \). By hypothesis, \( f(\text{cl}(B)) \) is an IFCS in \( Y \). Therefore, \( f(\text{cl}(B)) \Rightarrow \text{int}(f(\text{cl}(B))) \subseteq f(\text{cl}(B)) \). This implies \( f(\text{cl}(B)) \) is an IFG\( \beta \)OS in \( Y \) and hence an IFG\( \beta \)CM in \( Y \). Therefore \( f \) is an IFG\( \beta \)CM.
Definition 3.20: A mapping \( f : X \rightarrow Y \) is said to be an intuitionistic fuzzy generalized beta open mapping (IF\( G \)OM for short) if \( f(A) \) is an IF\( G \)OM in \( Y \) for each IFOS in \( X \).

Theorem 3.21: If \( f : X \rightarrow Y \) is a mapping. Then the following are equivalent if \( f \) is an IF\( G \)OM in space:

(i) \( f \) is an IF\( G \)OM.
(ii) \( f(\text{int}(A)) \subseteq \text{int}(f(A)) \) for each IF\( A \) in \( X \).
(iii) \( \text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{int}(B)) \) for every IF\( B \) in \( Y \).

Proof: (i) \( \Rightarrow \) (ii) Let \( f \) be an IF\( G \)OM. Let \( A \) be any IF\( S \) in \( X \). Then \( \text{int}(A) \) is an IF\( A \) in \( X \). (i) implies that \( f(\text{int}(A)) \) is an IF\( G \)OM in \( Y \). Since \( Y \) is an IF\( G \)OM in space, \( f(\text{int}(A)) \) is an IF\( G \)OM in \( Y \). Therefore \( \text{int}(f(\text{int}(A))) = f(\text{int}(A)) \subseteq f(A) \). Now \( \text{int}(f(A)) = \text{int}(f(\text{int}(A))) \subseteq \text{int}(f(A)) \).

(ii) \( \Rightarrow \) (i) Let \( B \) be any IF\( S \) in \( Y \). Then \( f^{-1}(B) \) is an IF\( S \) in \( X \). (ii) implies that \( \text{int}(f^{-1}(B)) \subseteq \text{int}(f^{-1}(B)) \) implies \( \text{int}(f^{-1}(B)) \subseteq \text{int}(f^{-1}(B)) \). Now \( \text{int}(f^{-1}(B)) \subseteq \text{int}(f^{-1}(B)) \) implies \( \text{int}(f^{-1}(B)) \subseteq \text{int}(f^{-1}(B)) \).

Theorem 3.22: A mapping \( f : X \rightarrow Y \) is an IF\( G \)OM if and only if \( \text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{int}(B)) \) for \( B \subseteq Y \) where \( Y \) is an IF\( G \)OM in space.

Proof: Necessity: Let \( B \subseteq Y \) and \( \text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{int}(B)) \) be an IF\( S \) in \( X \). By hypothesis, \( \text{int}(f^{-1}(B)) \) is an IF\( G \)OM in \( Y \). Since \( Y \) is an IF\( G \)OM in space, \( \text{int}(f^{-1}(B)) \) is an IF\( G \)OM in \( Y \). Therefore \( \text{int}(f^{-1}(B)) = \text{int}(f^{-1}(B)) \subseteq \text{int}(f^{-1}(B)) \). This implies \( \text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{int}(B)) \).

Sufficiency: Let \( A \) be an IF\( S \) in \( X \). Then \( \text{int}(A) \subseteq f^{-1}(\text{int}(B)) \). Hence \( f(A) \) is an IF\( G \)OM in \( Y \) and hence an IF\( G \)OM in \( Y \). Thus \( f \) is an IF\( G \)OM.

Theorem 3.23: Let \( f : X \rightarrow Y \) be an onto mapping where \( Y \) is an IF\( G \)OM in space. Then \( f \) is an IF\( G \)OM if and only if for any IF\( S \) in \( \alpha(\beta) \), there is an IF\( N \) of \( \alpha(\beta) \) such that \( \alpha(\beta) \subseteq A \) and \( f^{-1}(A) \subseteq B \).

Proof: Necessity: Let \( \alpha(\beta) \in Y \) and \( \beta \) be an IF\( S \) in \( Y \). Then there is an IF\( G \)OM in \( f^{-1}(\alpha(\beta)) \), there exists an IF\( G \)OM in \( Y \) such that \( f^{-1}(\alpha(\beta)) \subseteq B \). Since \( f \) is an IF\( G \)OM in \( Y \), \( f^{-1}(\alpha(\beta)) \subseteq f^{-1}(B) \). Put \( A = f(C) \). Then \( A \) is an IF\( G \)OM in \( \alpha(\beta) \) and \( \alpha(\beta) \subseteq A \). Thus \( \alpha(\beta) \subseteq A \) and \( f^{-1}(A) \subseteq B \). That is \( f^{-1}(A) \subseteq B \).

Sufficiency: Let \( B \subseteq X \) be an IF\( S \). If \( f(B) = 0 \), then there is nothing to prove. Suppose that \( \alpha(\beta) \subseteq B \). Then \( f \) is an IF\( S \) of \( f^{-1}(\alpha(\beta)) \), there exists an IF\( G \)OM in \( Y \) such that \( \alpha(\beta) \subseteq B \). Therefore there is an IF\( G \)OM in \( Y \) such that \( \alpha(\beta) \subseteq B \). Then \( f \) is an IF\( G \)OM in \( Y \) and \( \alpha(\beta) \subseteq B \).

Theorem 3.24: Let \( f : X \rightarrow Y \) be a bijective mapping, where \( X \) is an IF\( G \)OM in space. Then the following are equivalent:

(i) \( f \) is an IF\( M \)GOM.
(ii) \( f \) is an IF\( M \)GOM.
(iii) \( f \) is an IF\( M \)GOM.

Proof: (i) \( \Rightarrow \) (ii) Let \( \alpha(\beta) \in Y \) and \( \beta \) be the IF\( S \) in \( Y \). Then there exists an IF\( G \)OM in \( X \) such that \( \alpha(\beta) \subseteq B \). Since every IF\( G \)OM in \( Y \) is an IF\( G \)OM in \( X \), \( f \) is an IF\( G \)OM in \( Y \). Then by hypothesis, \( f(C) \) is an IF\( G \)OM in \( Y \). Now \( \alpha(\beta) \subseteq f(C) \). Put \( B = f(C) \). This implies \( \alpha(\beta) \subseteq f(B) \). (ii) \( \Rightarrow \) (iii) Let \( \alpha(\beta) \subseteq Y \) and the IFN of \( \alpha(\beta) \) there exists an IF\( G \)OM in \( Y \) such that \( \alpha(\beta) \subseteq B \).

Sufficiency: Let \( \alpha(\beta) \subseteq B \). Then \( f^{-1}(\alpha(\beta)) \subseteq \alpha(\beta) \). That is \( f^{-1}(\alpha(\beta)) \subseteq \alpha(\beta) \).

Theorem 3.25: If \( f : X \rightarrow Y \) be a bijective mapping then the following are equivalent:
Theorem 3.26: If $f: X \to Y$ be a bijective mapping then the following are equivalent:

(i) $f$ is an IFMGβCM

(ii) $f(A)$ is an IFGβOS in $Y$ for every IFGβOS $A$ in $X$

(iii) for every IFP $c(\alpha, \beta) \in Y$ and for every IFGβOS $B$ in $X$ such that $f^{-1}(c(\alpha, \beta)) \in B$, there exists an IFGβOS $A$ in $Y$ such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B$.

Proof: (i) $\implies$ (ii) is obvious, since $f$ is bijective.

(ii) $\implies$ (iii) Let $c(\alpha, \beta) \in Y$ and let $B$ be an IFGβOS in $X$ such that $f^{-1}(c(\alpha, \beta)) \in B$. This implies $c(\alpha, \beta) \in f(B)$. By hypothesis, $f(B)$ is an IFGβOS in $Y$. Let $A = f(B)$. Therefore $c(\alpha, \beta) \in f(B) = A$ and $f^{-1}(A) = f^{-1}(f(B)) \subseteq B$.

(iii) $\implies$ (i) Let $B$ be an IFGβCS in $X$. Then $B^c$ is an IFGβOS in $X$. Let $c(\alpha, \beta) \in Y$ and $f^{-1}(c(\alpha, \beta)) \in B^c$. This implies $c(\alpha, \beta) \in f(B^c)$. By hypothesis there exists an IFGβOS $A$ in $Y$ such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B^c$. Put $A = f(B^c)$. Then $c(\alpha, \beta) \in f(B^c)$ and $A = f(f^{-1}(B^c)) \subseteq f(B^c)$. Hence by Theorem 2.18, $f(B^c)$ is an IFGβOS in $Y$. Therefore $f(B)$ is an IFGβCS in $Y$. Thus $f$ is an IFMGβCM.

REFERENCES