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Linear and Nonlinear Behavior Analysis of a Flexible Shaft Supported By Hydrostatic Squeeze Film Dampers

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Abstract

Linear and non linear models of a hydrostatic squeeze film damper are presented and numerically simulated by a step by step method on a modal basis, in order to study the non-linear dynamic behaviour of a flexible shaft. The Reynolds equation is solved at each step in order to evaluate the film forces. The equations of motion are then integrated by using the Newmark method with a variable step in order to obtain speeds and the position for the next step. The non-linear hydrostatic forces are determined by the application of the boundary conditions, and the integration of the pressure field is determined by resolution of Reynolds equation, by applying the central finite difference method. The aim of this research is to study the effect of pressure ratio, viscosity, and rotational speeds on the vibratory responses and the transmitted bearing forces. The results are discussed, analysed and compared to a linear approach which is restricted to only small vibrations around the equilibrium position. The results show good agreements between linear and non-linear methods when the unbalance force is small, and then the linear model may be used for small vibrations in order to reduce compilation time during the iterative process.

KEYWORDS: Journal Hydrostatic Bearing, Linear and Non-Linear Dynamic Behaviour, Squeeze Film Dampers, Rotor Dynamic.

I. Introduction

Hydrostatic squeeze film dampers mounted on rolling-element bearings can be used in high-speed gas turbine engines and power turbines to attenuate the unbalance responses and bearing transmitted forces. These types of journal bearings are provide excellent, low friction characteristics and are better than conventional ball bearings or sleeve bearings in many industrial applications. Many researchers have investigated the influence of hydrostatic squeeze film dampers, on the dynamic behavior of flexible shafts, in order to control the vibration of high speed flexible shafts. Bonneau and Frene [1] presented an approach for an active squeeze film damper with a variable clearance squeeze film damper to study the flexibility of the shaft coupled with the nonlineair behaviour of the fluid bearings. They showed that it is possible to monitor the damping effect by controlling the clearance squeeze film dampers or the viscosity squeeze film damper. In order to suppress flexural vibrations of high-speed rotor systems, a compact damper incorporating ER fluid was designed by Seungchul et al [2]. Based on the system model, a semi- active artificial intelligent (AI) feedback controller was developed, taking into account the stiffening effect of the point damper in flexible rotor applications. Pecheux et al [3] numerically investigated the application of squeeze film dampers for active control of flexible rotor dynamics using viscosity change of a negative ER fluid to control the dynamic behavior of the shaft. A novel numerical method to compute Floquet Multipliers was presented to predict the nonlinear response of rotor with elastically supported SFD reported in literature (Qin et al. 2009)[4]. This method can begin integration from any point near stable trajectory and avoid the numerical oscillation in the first periods of integration. Chang – Jian et al [5] investigated theoretically the nonlinear dynamic behavior of a hybrid squeeze film damper – mounted rigid rotor lubricated with couple stress fluid. The numerical results show that due to the nonlinear factors of oil film force, the trajectory of the rotor demonstrates a complex dynamic with rotational speed ratio. Bouzidane and Thomas [6] have numerically simulated the effect of a negative electro-rheological fluid (NERF) within a four-pad hydrostatic bearing. Using a linear assumption for modal analysis, they investigated the effects of electro-rheological fluids, recess pressure and eccentricity ratio on load carrying capacity, flow and the equivalent dynamic characteristics. Similar type

NER-HSFD was applied to control the dynamic behavior of a flexible shaft by applying an electric field in order to modify the viscosity of the NER fluid in the hydrostatic bearing, and thus control its damping They also studied the effects of electro-rheological fluids on the variation of the unbalance response and the transmissibility of a NER hydrostatic squeeze film damper [7-9]. The objective of this research is to investigate the dynamic behavior of a flexible shaft, to reduce the transient responses and bearings transmitted forces. Two different models (linear, nonlinear) have been developed and presented to specify the nonlinear behavior of the flexible shaft.The effects of the pressure ratio, viscosity and rotational speed on the transient amplitude–speed response and the transmitted forces are studied, for a flexible shaft supported partly by a hydrostatic squeeze film damper, and the results are discussed by using linear model.

II. Mathematical Modeling

Figure 1 shows a four-pad hydrostatic squeeze film damper made of four identical plane hydrostatic bearing pads with indices 1, 2, 3 and 4 respectively indicating the lower, right, upper and left characteristics of the thrusts. The hydrostatic journal is fed with an incompressible fluid through recesses in the bearings, which are themselves supplied with external pressure P_S through capillary restrictor-type hydraulic resistances.

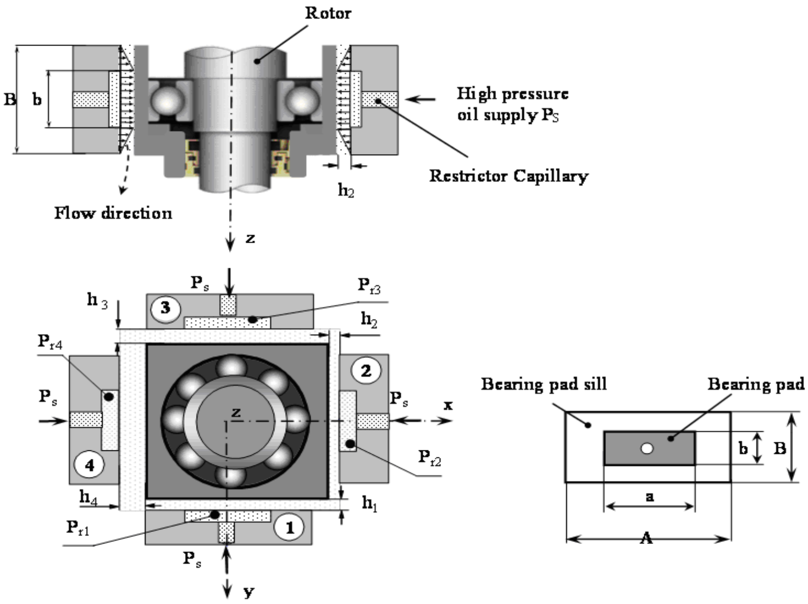


Figure 1: Four-Pad Hydrostatic Squeeze Film Damper geometry and nomenclature

2.1 Reynolds Equation

The Reynolds equation allows for the computation of the pressure distribution P_i (x_i, z_i, t). If we consider that the fluid flow is incompressible, laminar, isoviscous, and inertialess fluid, the Reynolds equation may be written as [7]:

$$\frac{\partial}{\partial x_i}\left(\frac{\partial P_i(x_i,z_i,t)}{\partial x_i}\right)+\frac{\partial}{\partial z_i}\left(\frac{\partial P_i(x_i,z_i,t)}{\partial z_i}\right)=12\frac{\mu}{h_i^3}\dot{h}_i \tag{1}$$

where: $P_i(x_i,z_i,t)$ is the hydrostatic pressure field of the i^{th} hydrostatic bearing pad; h_i is the film thickness of the i^{th} hydrostatic bearing pad ($h_i \neq f(x_i,z_i)$), (x_i, z_i, y_i) is the coordinate system used in the Reynolds equation, i=1, 2,3 and 4; $\dot{h}_i(dh_i/dt)$ is the squeeze velocity of the i^{th} hydrostatic bearing pad.

2.2 Reynolds Boundary Conditions

In order to solve the Reynolds equation (Eq.1) and evaluate the film forces of hydrostatic bearing, it is assumed that: (i) the recess depth is considered very deep (hp = 30 h_0); (ii) at the external boundary, nodal pressures are zero; (iii) the nodal pressures for node on the recess are constant and equal to P_{ri},; (iv) flow of lubricant through the restrictor is equal to the journal bearing input flow; (v) negative pressure is set to zero during the interactive process to take care of oil film cavitations (when the thickness film increasing).

- The film thickness h_i ($h_i = f(x,y) \neq f(x_i,z_i)$) is obtained as follows:

$$\begin{cases} h_1 = h_0 - x\,;\; h_3 = h_0 + x \\ h_2 = h_0 - y\,;\; h_4 = h_0 + y \end{cases} \qquad (2)$$

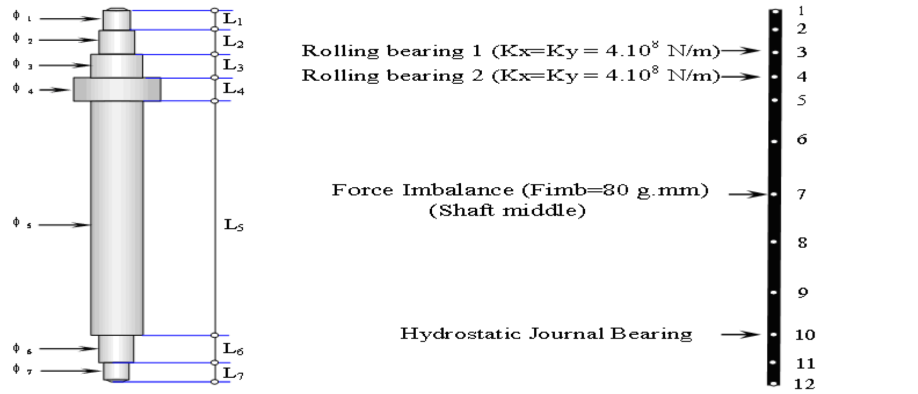
where (x , y) is the coordinate system used to describe the rotor motion; h_0 is the film thickness at the centred position of the hydrostatic squeeze film damper.

2.4 Shaft Mode

Figure 2 shows a rotating flexible shaft supported at one end by two rolling bearings and at the other end by a four-pad hydrostatic journal bearing. The rotor is modelled with typical beam finite elements including gyroscopic effects. Each element has four degrees of freedom per node [10]. The governing equations of motion could be written as follows:

$$[[M_t]+[J_r]]\{\ddot{\delta}\}+\dot{\varphi}[G]\{\dot{\delta}\}+[[K]+\ddot{\varphi}[G]]\{\delta\}=\{F_{imb}\}+\{F_{gr}\}+\{F_{nl}\} \qquad (3)$$

where: $[M_t]$ and $[J_r]$ are the translational and rotary mass matrices of the shaft, $[G]$ is the gyroscopic matrix, $[K]$ is the stiffness matrix of the shaft and rolling bearings, $\{\delta\}$ is the node displacement vector, $\{F_{imb}\}$ are the imbalance forces, $\{F_{gr}\}$ are the gravity forces, and $\{F_{nl}\}$ are the non-linear hydrostatic bearing forces. $\ddot{\varphi}, \dot{\varphi}\ (\varphi = \dot{\varphi} + \ddot{\varphi}\, t)$ and $\varphi\ (\varphi = \varphi_0 + \dot{\varphi}_0 t + 0.5\ddot{\varphi} t)$ represent the angular acceleration, angular velocity and angular displacement respectively



Shaft representation b) Finite element model of shaft- bearings system

Figure 2: Model of flexible shaft-bearings system

1.3 Forces of Hydrostatic Bearing

2.3.1 Nonlinear Model

The nonlinear hydrostatic force of the i[th] hydrostatic bearing pad can be obtained by integrating the pressure over the bearing area:

$$F_{pi} = \int_{S_i} P_i\, ds_i = \iint P_i\, dx_i\, dz_i \qquad (4)$$

where s_i and ds_i are the contact surface and element on the surface of the i[th] bearing pad, respectively.

2.3.2 Linear Model

The linear model is based on small displacements and small velocity assumptions [8]. Therefore, the linear hydrostatic force of the i[th] hydrostatic bearing (F_{pi}) can be expressed as follows:

$$F_{Pi} = -K_{pi} h_i - C_{pi} \dot{h}_i \qquad (5)$$

where: K_{pi} and C_{pi} represent the stiffness and damping of the i[th] hydrostatic bearing pad, and F_{pi} is the hydrostatic force of the i[th] hydrostatic bearing pad.

III. Mathematical Modeling

The linear and non-linear dynamic behavior of the flexible rotor is simulated step by step on a modal basis. At each step the Reynolds equation (Eq.1) is solved to evaluate the film forces, and then the equations of motion (Eq.3) are integrated using the Newmark method with a variable step to obtain speeds and the position for the next step. Computation of the pressure distribution was done through resolution of the Reynolds equation by applying the centered finite difference method. The shaft is divided into 11 beam elements and 12 nodes. Every node has four degrees of freedom including two translations and two rotations. The bearing characteristics are the following: bearing pad length A is 0.09 m; bearing pad width B is 0.015; dimension ratio a/A=b/B is 0.5; film thickness h_0 is 0.07 mm. The capillary diameter d_c is 1.2 mm; l_c the capillary length is 58 mm; the pressure supply is 2.5 MPa. The rotor system is subjected to an imbalance value F_{imb}=80.10^{-6} kg.m located at the middle of the shaft (node 7). A linear variation of rotation speed between 5000 and 60 000 rpm over a 20 second interval was made.

The table 1 presents the stiffness, damping coefficients and film thickness with pressure ratio. Note that the pressure ratio $\beta_0 \left(P_{ri} / P_s\right)$ is defined as the ratio of the recess pressure of the ith hydrostatic bearing pad over the supply pressure when the eccentricity of the journal equal to zero (e = 0; $\varepsilon = e/h_0 = 0$).The values of stiffness and damping coefficients were determined numerically using small perturbations of the shaft position, at the equilibrium position of the shaft [8].

Pressure ratio β_0	K_{pi} [N/m]	C_{pi} [N.s/m]	**h_0** [μm]
0.67	35454680	54780	56.549
0.50	32310120	25873	71.604
0.35	24366980	13243	88.014

Table 1 Stiffness and damping coefficients with pressure ratio

IV. Results and discussions

To check the validity of numerical analysis of the flexible shaft- hydrostatic journal bearing system, the results of linear models were computed and compared with those obtained by the nonlinear model, for small vibrations around the static equilibrium position with ε = 0.06 (the unbalance eccentricity). As mentioned above, Figure 3 shows the comparison of linear and nonlinear results for computing the vibration amplitudes (at the middle of the shaft and inside the four-pad HSFD) and transmitted force. The results obtained by the non-linear model and linear model are in a very good agreement (almost identical). It should be noted that the main interest to use the non-linear model, in this study, is to validate the calculation of the dynamic characteristics obtained using of the numerical methods.

4.1 Influence of the pressure ratio

The influence of pressure ratio and rotational speed on the transient amplitude–speed response and the transmitted forces by using linear method are presented in figure 4. These graphs show that a decrease in pressure ratio from 0.67 to 0.35 increase the displacement amplitude in the journal bearing and reduce the displacement amplitude in the middle of the shaft and bearing transmitted forces. It should be noted that the effect of the pressure ratio is very different accordingly with the operating speed. When increasing the pressure ratio, the damping in the journal bearing decreases. This decrease of damping can be explained by the increase of the film thickness (clearance, h_0) [8]. This leads to more displacement in the HSFD and more dissipated energy. Consequently, a decrease of the pressure ratio results an increase of vibration response across the entire frequency range. It can be observed that the response amplitudes in the middle of the shaft at the first critical speed are larger than those at the second critical speed. It should also be noted a reverse effect. The vibrations in the middle of the shaft and the bearing transmitted forces decrease with the pressure ratio, when the operating speed is close to the critical speeds due to the increase of the film thickness. When the speed is very different from the critical speed, the variation of pressure ratio has no effect on amplitude of the bearing transmitted forces. These results are similar to those of Pecheux *et al.* (1997) [3] and Bonneau *et al.* (1997) [1].

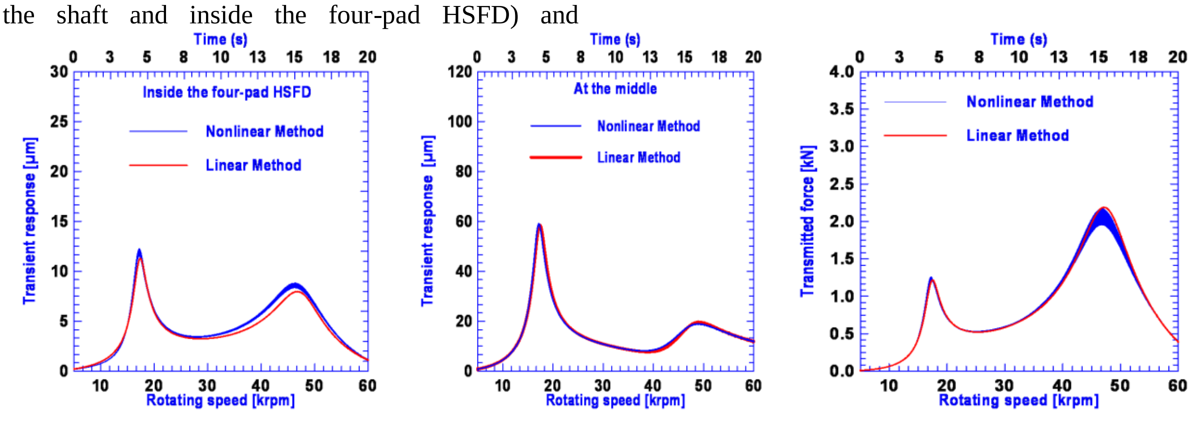


Figure 3 Comparison of linear and nonlinear models: vibratory responses and transmitted force.

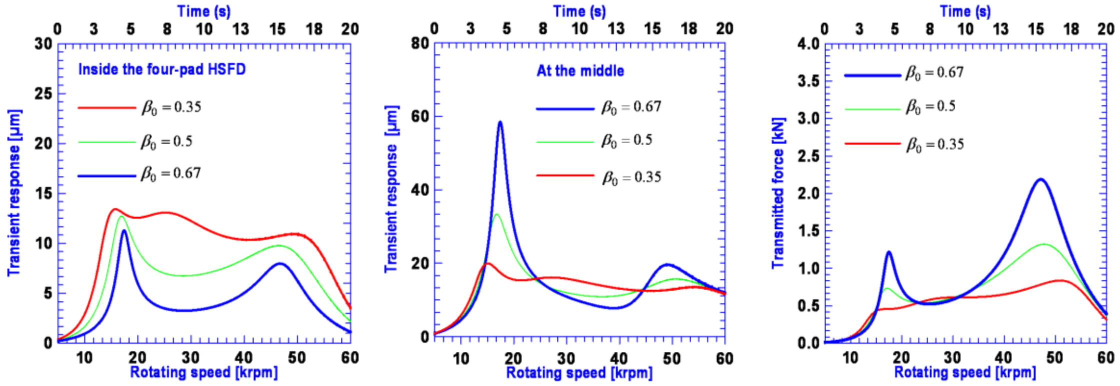


Figure 4: Influence of pressure ratio on the transient response of the shaft at the middle, inside the HSFD, and force transmitted vs. speed and time (linear method)

4.2 Influence of the viscosity

Two viscosities (0.05; 0.195 Pa.s) are used in order to study the effect of viscosity on the dynamic behavior of the flexible shaft supported by a hydrostatic journal bearing. Figure 5 presents the influence of viscosity and rotational speed on the transient amplitude–speed response and the bearing transmitted forces obtained by using linear method. As shown, the increase of the viscosity leads to a significant decrease of the displacement amplitude in the journal bearing due to the increase of damping. However, the increase of damping causes large vibrations amplitude at the middle of shaft and large transmitted force amplitudes around the critical speed. When the speed is very far from the critical speed, the variation of viscosity has no effect on vibration amplitudes at the middle of shaft and transmitted forces amplitudes. Note that the effect of the viscosity is very different accordingly with the operating speed. It should be noted a low viscosity causes to larger displacements in the HSFD and leads to higher dissipated energy to reduce vibrations amplitude in the middle of shaft and transmitted forces amplitudes.

V. Conclusion

In this paper, a hydrostatic squeeze film damper is designed and proposed to reduce the vibratory responses of the shaft and bearings transmitted forces. The results of the numerical simulation can be summarized as follows:

- To reduce the vibration in the middle of the flexible shaft and the bearing transmitted forces, the results have revealed that the vibration inside the HSFD must be kept large in order to dissipate a lot of energy when operating close to critical speeds. This can be achieved by decreasing of pressure ratio and viscosity. The results obtained with this type of journal bearing are close to those of SFD (Bonneau et al (1997) [1])
- When the rotor operates close to the critical speed, the vibration of a flexible shaft and the bearing transmitted forces due to a rotation unbalance are reduced for low pressure ratio and viscosity. It is therefore highly recommended that the fluid viscosity and pressure ratio be decreased at critical speeds. In the other hand, a high viscosity and pressure ratio are required for reducing rotor vibration and bearing transmitted forces at speeds very far from the critical speeds.

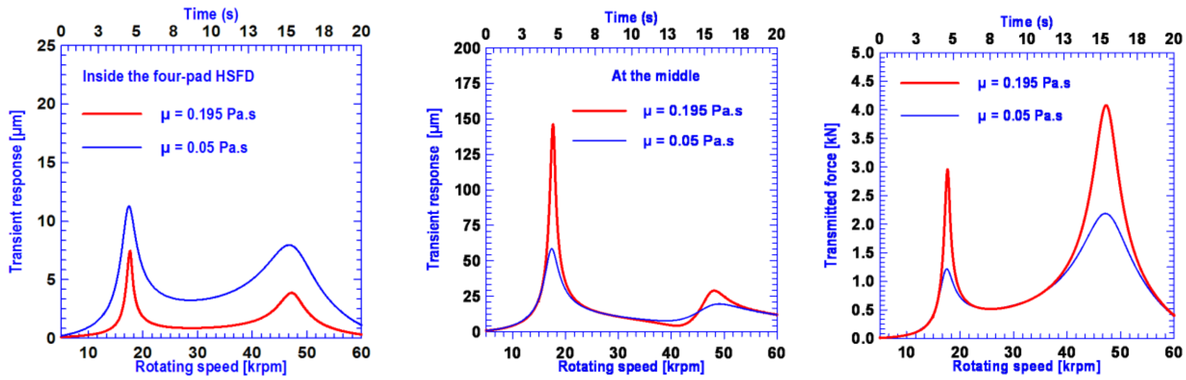


Figure 5: Influence of viscosity on the transient response of the shaft at the middle, inside the HSFD, and force transmitted vs. speed and time (linear method)

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