RESEARCH ARTICLE OPEN ACCESS

Optimization Of Hypoid Gear Using Genetic Algorithm

Alaa Mohamed[*&a], A. Khattab [a], T.A. Osman[a], Mostafa. Shazly [b]
[*] Production Engineering and Printing Technology Department, Akhbar El Yom Academy, Giza, Egypt.
[a] Mechanical Design and Production Engineering Department, Cairo University, Giza, Egypt.
[b] Mechanical Engineering Department, The British University in Egypt, Cairo, Egypt.

Abstract
Genetic algorithm (GA) is a Non-Traditional method is useful and applicable for optimization of mechanical component design. The GA is an efficient search method which is inspired from a natural genetic selection process to explore a given search space. In this work, GA is applied to minimize the volume of hypoid gear with respect to a specified set of constraints. Module, face width and number of teeth of hypoid are used as design variables and the bending stress, contact stress and a geometric limit on the face width are set as constraints. The results showed that the optimal procedure reduced the volume of a gear designed according to ANSI/AGMA 2003-B97 to 54% of its original volume. Further analysis was performed to study the effect of the design variables and the input parameters of the objective function.
Keywords: Genetic algorithms, hypoid gears, volume optimization.

I. INTRODUCTION

Gears are used in most types of machinery and vehicles for the transmission of power. The design of gears is highly complicated involving the satisfaction of many constraints such as strength, pitting resistance, bending stress, scoring wear, and interference in involutes gears and so on. The main concentration focuses on hypoid gear sets, which are used to transmit motion between Non-Intersecting and Non-parallel Shaft. However, geometrical design and strength evaluation of the hypoid gear depend on the machine tool of specific production companies because the geometrical design and strength evaluation of the hypoid gear are complex and difficult [1].

Hypoid gears are used in various automotive, rotorcraft and industrial applications to transmit power between two perpendicular shafts having a certain amount of offset. They also find, a wide range of applications in transportation equipment such as Lorries, ships, helicopters, earth moving equipments, and construction equipments. Hypoid gears are similar to spiral bevel gears except that the shaft center lines do not intersect. The shaft offset introduces several advantages to hypoid gears such as larger pinion size with fewer numbers of teeth, higher contrast ratio, and lower contact stresses. However, higher relative sliding velocity between contacting surfaces results in high power losses and wear rates are among the most common problems found in hypoid gears [2, 3].

Many numerical optimization algorithms such as GA, Simulated Annealing, Ant-Colony Optimization, and Neural Network have been developed and used for design optimization of engineering problems to find optimum design. Solving engineering problems can be complex and a time consuming process when there are large numbers of design variables and constraints. Hence, there is a need for more efficient and reliable algorithms that solve such problems. The improvement of faster computer has given chance for more robust and efficient optimization methods. Genetic algorithm is one of these methods. The genetic algorithm is a search technique based on the idea of natural selection and genetics [4].

Ki-Hun Lee, [5] studied the Optimum Design Method of Hypoid Gear by Minimizing Volume and the optimum decreases is 12.5 %.

II. PRINCIPLE OF GENETIC ALGORITHM

Genetic algorithm (GA) maintains a population of encoded solutions, and guides the population towards the optimum solutions [5]. Fitness function provides a measure of performance of an individual how fits. Rather than starting from a single point solution within the search space as in traditional optimization methods, the genetic algorithm starts running with an initial population which is coding of design variables. GA selects the fittest individuals and eliminates the unfit individuals in this way. The flow chart of a genetic algorithm is shown in Figure 1. An initial population is chosen randomly at the beginning, and fitness of initial population individuals is evaluated. Then an iterative process starts until the termination criteria have been run across. After the evaluation of individual fitness in the population, the genetic operators, selection, crossover and mutation are applied to breeding a new generation. Other genetic operators are applied as needed. The newly created individuals replace the existing generation and reevaluation is started in fitness of new individuals. The loop is repeated until an acceptable solution is found [6].

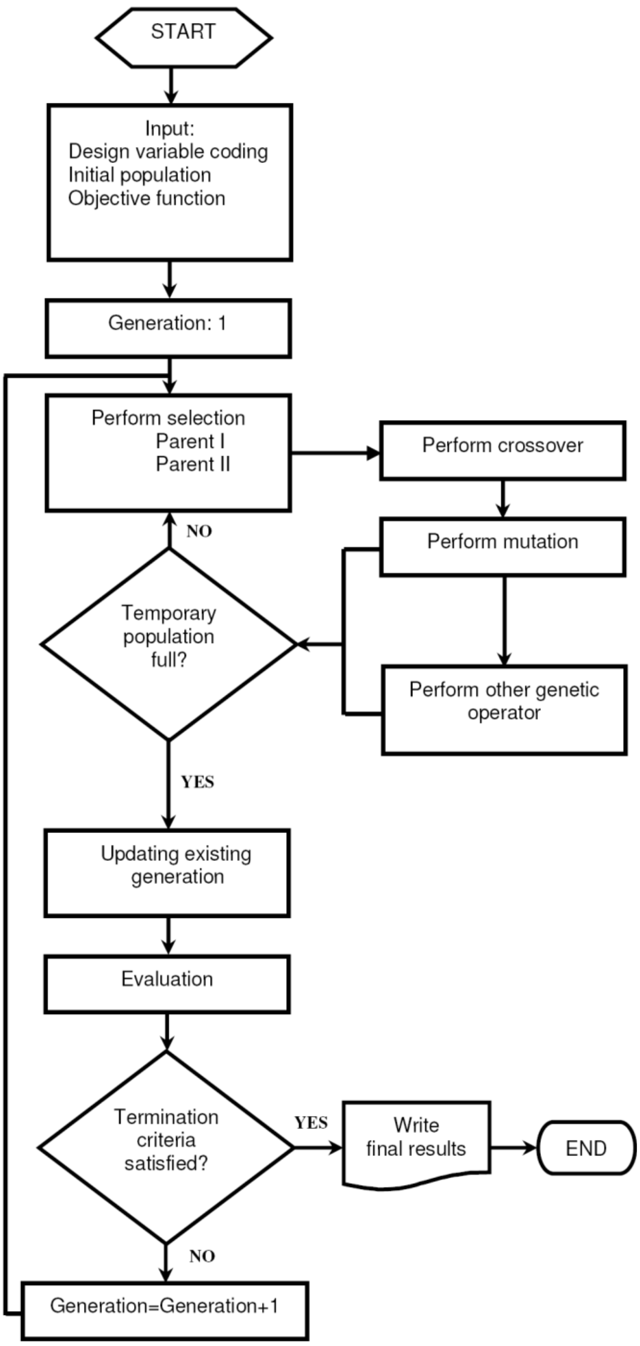

Fig. (1) Flow chart for the genetic algorithm

III. OPTIMIZATION FORMULATION

An optimization search technique based on genetic algorithms is considered in the present work to minimize the volume of hypoid gears with constraints on the bending stress, contact stress and a geometric limit on the face width. The design variables considered for optimization are the module, face width and number of teeth.

3.1 BASIC PARAMETERS

Parameters considered in the design of the hypoid gear pair include: Power, Type, Speed, Hypoid offset, Gear ratio, Pressure angle, and Shaft angle, as illustrated in table 1.

Table 1: Basic hypoid gear design parameter data

Power	95 Hp (70.84 Kw)
Type	Gleason Hypoid
Speed	1200 rpm
Hypoid offset	1.5 in (38 mm)
Gear ratio	4
Pressure angle	20 °
Shaft angle	90 °

3.2 OBJECTIVE FUNCTION

The volume considered in the calculations is pinion volume simplified to a truncated cone at the pitch cone [7-9]. Volume (V) of a truncated cone (Frustum) shown in Figure 2 is given by:

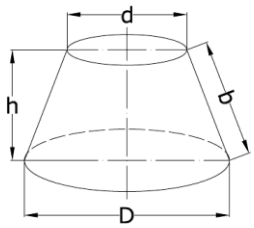

Fig. (2) Frustum

$$V=\frac{\pi}{3}.F.\cos(\gamma_p)\left[\left(\frac{n}{2P_d}\right)^2+\left(\frac{n}{2P_d}.\frac{A_o-F}{A_o}\right)^2+\frac{n}{2P_d}\left(\frac{n}{2P_d}.\frac{A_o-F}{A_o}\right)\right] \quad (1\text{ a})$$

$$V=\frac{\pi}{3}.b.\cos(\gamma_p)\left[\left(\frac{m.z_1}{2}\right)^2+\left(\frac{m.z_1}{2}.\frac{R_o-b}{R_o}\right)^2+\frac{m.z_1}{2}\left(\frac{m.z_1}{2}.\frac{R_o-b}{R_o}\right)\right] \quad (1\text{ b})$$

The optimization problem can be presented as follows:

Minimize: Volume of pinion pitch frustum (A function in number of teeth, module and face width)

Subject to:

Working contact Stress - Allowable contact stress ≤ 0 … $G_1(x)$

$$C_P\sqrt{K_0K_VK_mC_SC_{XC}\frac{2T_P}{Fd^2I}}-\frac{S_{ac}C_LC_H}{S_HK_TC_R}\leq 0$$

Working bending stress - Allowable bending stress ≤ 0 ... $G_2(x)$

$$\frac{2T_P}{Fd}\,\frac{P_dK_0K_V}{1}\,\frac{K_SK_m}{K_XJ}-\frac{S_{at}K_L}{S_FK_TK_R}\leq 0$$

3 x Face width - Cone distance ≤ 0… $G_3(x)$

$$3\times F-A\leq 0$$

3.3 CONSTRAINTS ON THE OBJECTIVE FUNCTION

Constraints are conditions that must be met in the optimum design and include restrictions on the design variables. These constraints define the boundaries of the feasible and infeasible design space domain. The constraints considered for the optimum design of minimizing pinion volume are the following:

$$\beta = \frac{S_{ac}}{S_{at}} \quad (2\ a)$$

$$\beta = \frac{\sigma_{H\lim}}{\sigma_{F\lim}} \quad (2\ b)$$

Constraints G_1 and G_2 are solved as follows:

First for G_1 the working contact stress is equated with the allowable contact stress as in Eq. (3).

$$S_{ac} = \frac{P_d C_p S_H K_T C_R}{n C_L C_H} \sqrt{\frac{2T_p}{FI} K_o K_v K_m C_s C_{xc}} \quad (3a)$$

$$\sigma_{H\lim} = \frac{Z_E S_H K_\theta Z_z}{m n Z_{NT} Z_w} \sqrt{\frac{2000.T_1}{b_{el} Z_1} K_A K_v K_{H\beta} Z_x Z_{xc}} \quad (3\ b)$$

For G_2 the working bending stress is equated with the allowable bending stress as in Eq. (4).

$$S_{at} = \frac{2T_p}{F} \frac{P_d^{\,2} K_o K_v}{n} \frac{K_s K_m}{K_x J} \frac{S_F K_T K_R}{K_L} \quad (4\ a)$$

$$\sigma_{F\lim} = \frac{2000T_1}{b} \frac{K_A K_v}{m^2 Z_1} \frac{Y_x K_{H\beta}}{Y_\beta Y_J} \frac{S_F K_\theta Y_Z}{Y_{NT}} \quad (4\ b)$$

From equation (2), (3), and (4), the following relations are obtained

$$\frac{2T_p}{F} \frac{P_d^{\,2} K_o K_v}{n} \frac{K_s K_m}{K_x J} \frac{S_F K_T K_R}{K_L} = \beta \frac{P_d C_p S_H K_T C_R}{n C_L C_H} \sqrt{\frac{2T_p}{FI} K_o K_v K_m C_s C_{xc}} \quad (5\ a)$$

$$\frac{Z_E S_H K_\theta Z_z}{m n Z_{NT} Z_w} \sqrt{\frac{2000.T_1}{b_{el} Z_1} K_A K_v K_{H\beta} Z_x Z_{xc}} = \beta \frac{2000T_1}{b} \frac{K_A K_v}{m^2 Z_1} \frac{Y_x K_{H\beta}}{Y_\beta Y_J} \frac{S_F K_\theta Y_Z}{Y_{NT}} \quad (5\ b)$$

Therefore, from Eq (5), we get

$$P_d = \frac{C_p S_H C_R K_x K_L J}{\beta C_L C_H S_F K_R K_s} \sqrt{\frac{F}{2T_p I} \frac{C_s C_{xc}}{K_o K_v K_m}} \quad (6\ a)$$

$$m = \frac{Z_E S_H Z_Z Y_\beta Y_J Y_{NT}}{\beta Z_{NT} Z_W S_F Y_Z Y_x} \sqrt{\frac{b}{2000T_1 Z_1} \frac{Z_x Z_{xc}}{K_A K_v K_{H\beta}}} \quad (6\ b)$$

Using Eq (3) and rearranging, we get

$$n = \frac{P_d C_p S_H K_T C_R}{S_{ac} C_L C_H} \sqrt{\frac{2T_p}{FI} K_o K_v K_m C_s C_{xc}} \quad (7\ a)$$

$$z_1 = \frac{Z_E S_H K_\theta Z_z}{m \sigma_{HP\lim} Z_{NT} Z_w} \sqrt{\frac{2000T_1}{b Z_1} K_A K_v K_{H\beta} Z_x Z_{xc}} \quad (7\ b)$$

IV. RESULTS and DISCUSSION

The developed CAD system along with the associated optimization module is used to study the effect of different gear design inputs on optimum gear parameters over a wide range of practical values. The results compared with results of analytical method, as shown in Table 2.

Table 2: Comparison of the results

Design variables	**Analytical method**	**Genetic algorithm**
Module (m)	5.621 mm	5.08 mm
Face width (f)	47.041 mm	34.155 mm
Number of teeth (N)	11	10
Minimum volume	92703.3 m^3	50429.6 m^3

The starting point of analytical method, m=5.621, f =47.041, N =11. The programmer, Developed in MATLAB 7.0 for analytical method has been run several times for different values of design variables. The results obtained are given in Table 2. As can be seen from the results, the genetic algorithm produced much better results than analytical method.

- **Effect of Torque on the Optimum Design Parameters**

Figs. (3) To (6) shows that the optimum design parameters (module, face width, number of teeth, and minimum pinion volume) increases with increasing the torque and decreases with decreasing the rpm

- **Effect of Material Property Ratio at Different Torque Value ()**

Figs. (7) To (10) shows that the optimum design parameters (module, face width, number of teeth, and minimum volume) increases with increasing input torque, and decreases with decreasing the material property factor (β).

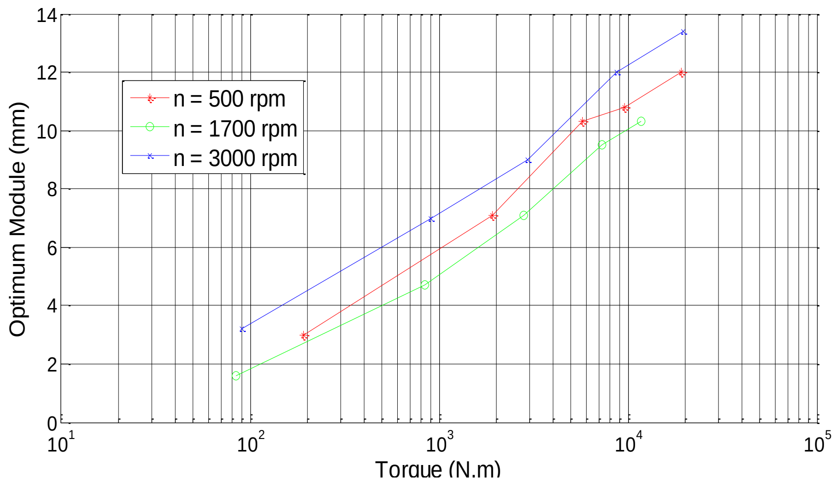

Fig. (3) Effect of torque on optimum module for max. & min. rpm, $\beta = 5$, S_{at} ($\sigma_{F\ lim}$) = 205 MPa, m_G (r) = 4, face width to cone distance ≤ 1/3

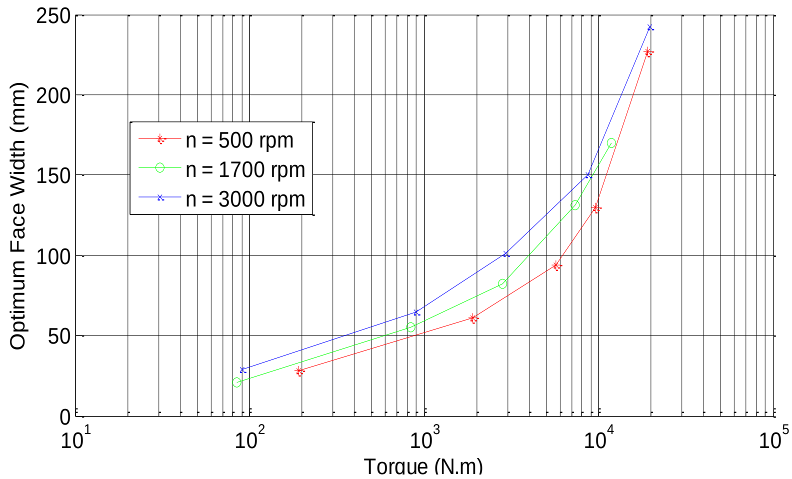

Fig. (4) Effect of torque on optimum face width for max. & min. rpm, $\beta = 5$, S_{at} ($\sigma_{F\ lim}$) = 205 MPa, m_G (r) = 4, Face Width to cone distance ≤ 1/3

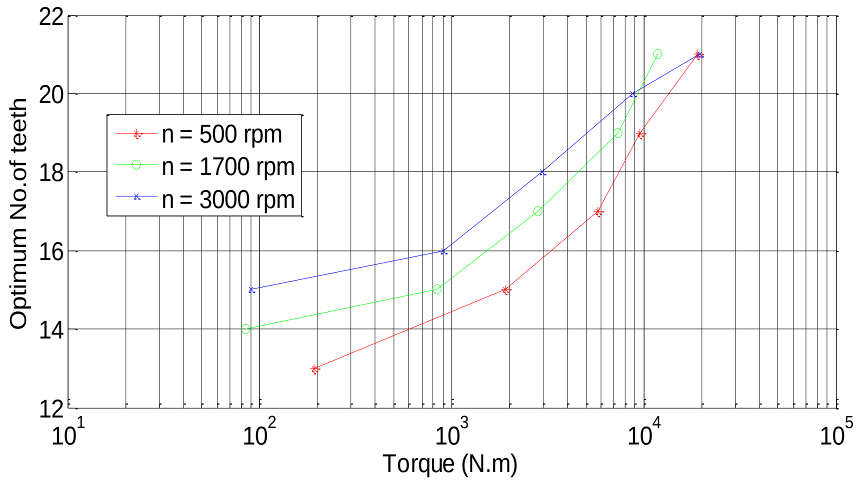

Fig. (5) Effect of torque on the optimum number of teeth for max. & min. rpm, $\beta = 5$, S_{at} ($\sigma_{F\ lim}$) = 205 MPa, m_G (r) = 4, face width to cone distance ≤ 1/3

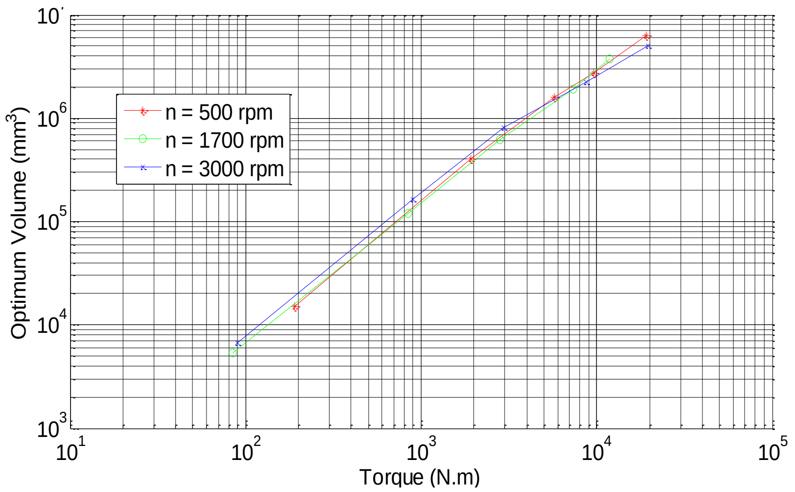

Fig. (6) Effect of torque on optimum volume for max. & min. rpm, β = 5, S_{at} ($\sigma_{F\ lim}$) = 205 MPa, m_G (r) = 4, face width to cone distance ≤ 1/3

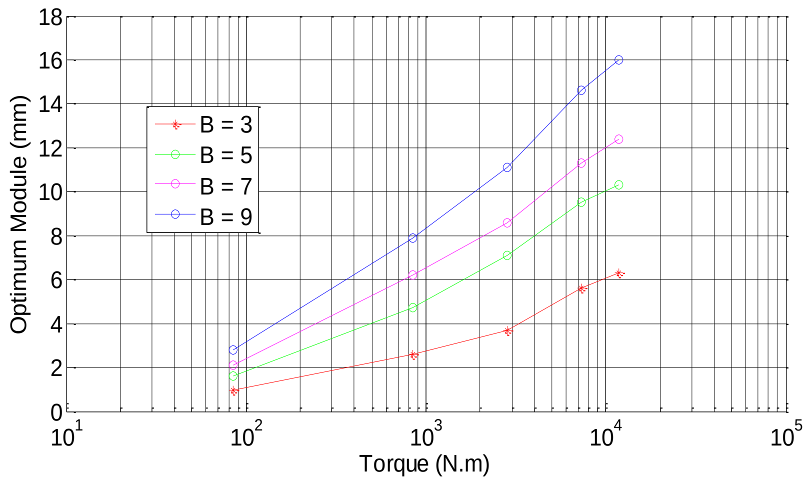

Fig. (7) Effect of torque on optimum module for different material property factor, S_{at} ($\sigma_{F\ lim}$) = 205 MPa, n_p (n_1) =1700 rpm, m_G (r) = 4

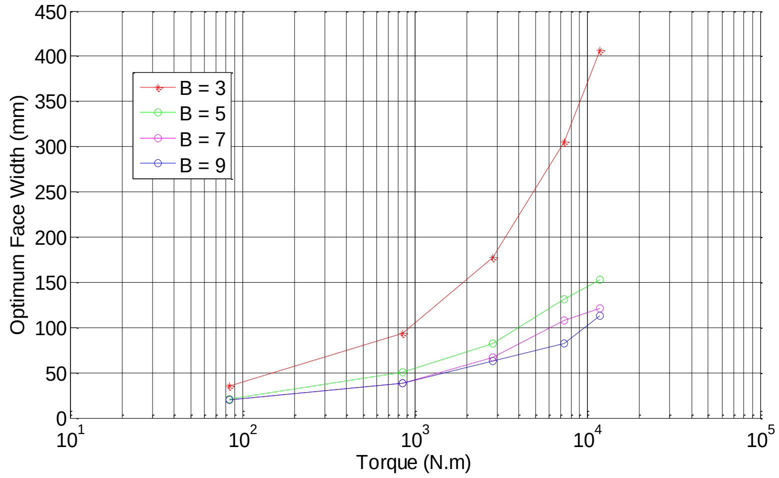

Fig. (8) Effect of torque on optimum face width for different material property factor, S_{at} ($\sigma_{F\ lim}$) = 205 MPa, n_p (n_1) =1700 rpm, m_G (r) = 4

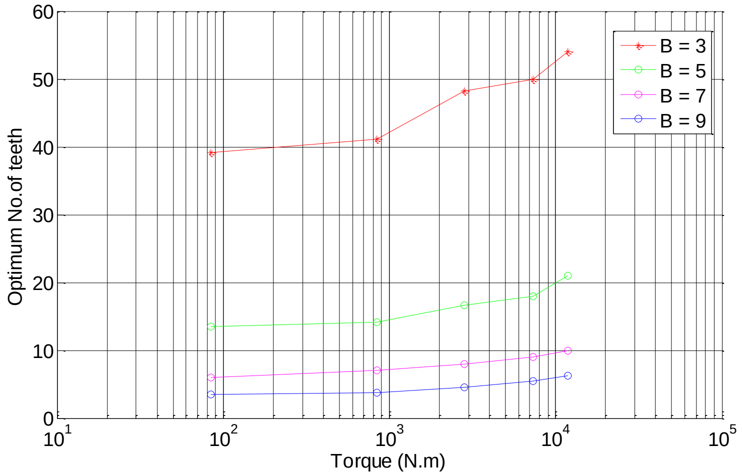

Fig. (9) Effect of torque on optimum number of teeth for different material property factor, S_{at} ($\sigma_{F\ lim}$) = 205 MPa, n_p (n_1) =1700 rpm, m_G (r) = 4

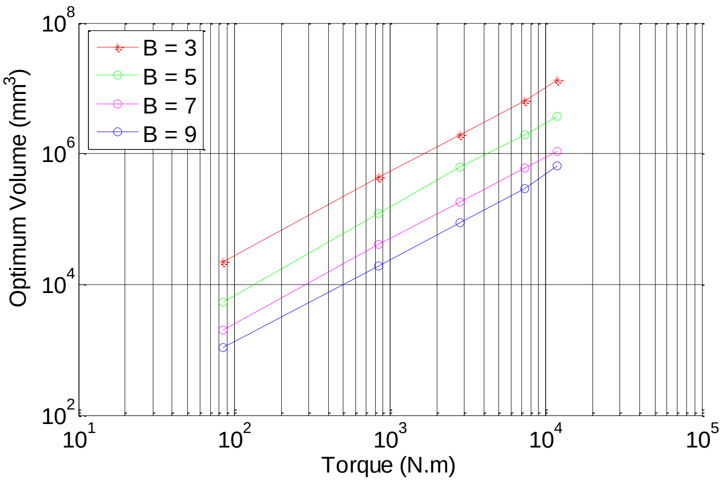

Fig. (10) Effect of torque on optimum volume for different material property factor, S_{at} ($\sigma_{F\ lim}$) = 205 MPa, n_p (n_1) =1700 rpm, m_G (r) = 4

V. CONCLUSIONS

The aim of this study was to minimize the volume of the hypoid gear system using the genetic algorithm and numerical optimization method. The results obtained showed that the genetic algorithm to provide better solution than those obtained from numerical optimization method. It can be concluded that the genetic algorithm that can be successfully and efficiently used for the hypoid gear system design.

The results showed that the optimal procedure reduced the volume of a gear designed according to ANSI/AGMA 2003-B97 to 54% of its original volume.

References

[1] Maitra, G. M., "Handbook of Gear Design", Tata McGraw-Hill, 1985.

[2] Mott, R.,"Machine Elements in Mechanical Design", Merril Publications, 1992.

[3] Niemann, G., "Machine Elements", Vol.II, Springer Int'l, 1978.

[4] Vanderplaats, G.N., Chen, X., Zhang, N., "Gear optimization." NASA Contractor Reports, Dec 1988, No.4201, IS: 0565-7059 Research Report Univ. of California, Santa Barbara, CA, USA.

[5] Ki-Hun Lee, Geun-Ho Lee, In-Ho Bae, Tae-Hyong chong. "An Optimum Design Method of Hypoid Gear by Minimizing Volume", Transactions of the Korean Society of Machine Tool Engineers, vol. 16 No. 6, 2007.

[6] Metwalli, S.M., and Mayne, R.W., " New Optimization Techniques, " The 4th ASME Design Automation conference, Chicago, V. II, ASME, Paper No.77-DAC- 9, 1977.

[7] Fang, "Optimization the dynamic behavior of Spiral Bevel Gears", Journal of Mechanical Design, Transactions of the ASME, Sep 2000, Vol. 114, pp 498-506.

[8] X.Z. Deng, Z.D. Fang, H.B. Yang, "Calculation of Hypoid Tooth Surface

Contact Stress", China Mechanical Engineering 12 (2001) 1362–1364.

[9] Lin C.Y., Tsay C.B., and Fong Z.H. "Computer Aided Manufacturing of Spiral Bevel and Hypoid Gears by Applying Optimization Techniques", Journal of Materials Processing Technology, 114: 22-35, 2001.

Abbreviations

A_o(R_o)	Outer cone distance, in (mm).
P_d (m)	Outer diametral pitch, in^{-1}.
F (b)	Face width, in (mm).
z_1	Pinion number of teeth.
γ_p	Pinion pitch angle
d (d_{e1})	Pinion outer pitch diameter, in (mm);
T_p (T_1)	Operating pinion torque, lb in (Nm).
β	Material property factor.
S_c (σ_H)	Calculated working contact stress, lb/in^2 (N/mm^2).
S_t (σ_F)	Calculated working bending stress at the root of the tooth, lb/in^2 (N/mm^2).
S_{wc} (σ_{HP})	Allowable contact stress, lb/in^2 (N/mm^2).
S_{ac} ($\sigma_{HP\ lim}$)	Material contact stress, lb/in^2 (N/mm^2).
S_{wt} (σ_{FP})	Allowable bending stress, lb/in^2 (N/mm^2).
Sat ($\sigma_{F\ lim}$)	Material bending stress, lb/in^2 (N/mm^2).
K_0 (K_A)	Overload factor.
K_v (Q_v)	Dynamic factor.
K_x (Y_β)	Tooth lengthwise curvature factor.
K_m ($K_{H\beta}$)	Load distribution factor.
K_s (Y_x)	Size factor.
K_L (Y_{NT})	Stress cycle factor.
K_T (K_θ)	Temperature factor.
K_R (Y_z)	Reliability factor.
C_{xc} (Z_{xc})	Crowning factor.
C_p (Z_E)	Elastic coefficient, [lb/in^2]$^{0.5}$ ([N/mm^2]$^{0.5}$).
C_s, (Zx)	Size factor.
C_L (ZNT)	Stress cycle factor.
C_H (Z_w)	Hardness ratio factor.
C_R (Z_z)	Reliability factor.
S_H	Contact safety factor.
S_F	Bending safety factor.
J (Y_j)	Bending strength geometry factor.
I (Z_i)	Pitting resistance geometry factor.