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Implementation of Haar Wavelet through Lifting For Denoising

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ABSTRACT
Techniques based on Haar wavelet through lifting coefficients are gaining popularities for denoising data. The idea is to transform the data in to the wavelet basis, where large magnitude wavelet coefficients are mainly the signals and the smaller magnitude coefficients represents the noise. By suitably modifying these coefficients, the noise can be removed from data .wavelet denoising is problem on implementation on computer is somewhat difficult. In order to implement on computer we will use Haar wavelet through lifting for denoising.

I. INTRODUCTION

In signal processing we have many applications which deal with noisy data. In most cases it is desirable to work with undisturbed signal. Therefore, we have many methods to obtain clean signal by suppressing the noise. One powerful approach to do this is to take advantage of the properties coming along with the Haar wavelet transformation through lifting. The wavelet transformation recombines the data in such a way that the result concentrates the main information in only a few values, the wavelet coefficients.

In signal processing we are using wavelets to denoise the signal rather than fourier methods because wavelets are located in time and frequency where as fourier basis functions are localized in frequency but not in time[1]. We have two popular thresholding methods to denoise the signal so we will use these thresholding methods along with lifting transformation to denoise the signal. In this paper presents effect of increasing the resolution of the signal on denoising using haar wavelet through lifting

II. Haar wavelet

Haar wavelet is simple. It is a just step function given Ψ (t) is

$$\Psi(t) = \begin{cases} 1 & if \;\; 0 \le t < \frac{1}{2}, \\ -1 & if \;\; \frac{1}{2} \le t < 1, \\ 0 & otherwise \end{cases}$$

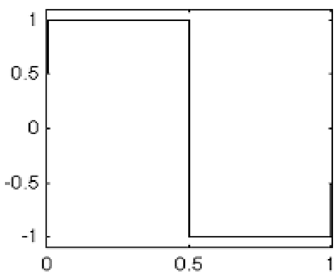

Fig1. Haar wavelet function Ψ (t)

We have wide variety of popular wavelet algorithms like Mexican Hat wavelets, Daubechies wavelets, and Morlet wavelets etc. For smoothly changing time series, these wavelet algorithms have better resolution. The Haar wavelet algorithms applied to time series only if the number of samples is power of two (e.g.2, 4, 8…) the Haar wavelet uses a rectangular window to sample the time series. Each pass over the time series generates a new time series and a set of coefficients. The new time series is the average of the previous time series over the sampling window. The coefficients represent the average change in the sample window.

2.1 Advantages of Haar wavelet transforms:

It is a simple concept.
It is faster than other wavelets.
As it can be calculated in a place without a temporary array hence it is taking less memory.
Other wavelet transforms are not exactly reversible without the edge effect which is not the case with this Haar transform

III. Denoising using wavelet thresholding

Denoising is the most important processing application that is pervasive in almost any signal. Indeed, data acquisition always comes with some kind of noise, so modeling this noise and removing it efficiently is crucial. Wavelet denoising is also called as wavelet thresholding [2] has been proposed by Donoho and Johnstone. One of the main advantages of wavelet de-noising is t hat it does not require any assumptions about the noisy signal, and can deal with signals with discontinuities and spatial variations [3]. Thresholding is a non-linear function applied to the detail coefficients to eliminate the signal components that are assumed to represent noise observations. Mainly we have two types thresholdings.

a) Soft thresholding function is defined as:

$$T_{soft} = \begin{cases} sign\big(x(t)\big)(|x(t)| - \delta) & |x(t)| > \delta \\ 0, & |x(t)| \le \delta \end{cases}$$

b) Hard thresholding function defined as:

$$T_{hard} = \begin{cases} x(t), & |x(t)| > \delta \\ 0, & |x(t)| \leq \delta \end{cases}$$

Where δ is threshold.

In Soft thresholding first setting to zero the elements whose absolute values are lower than the threshold, and then shrinking the nonzero coefficients towards zero, where as in hard thresholding which zeroes out all coefficients with magnitude smaller than the threshold value. Main drawback of soft thresholding is the large detail coefficient is always shrinked as no detail coefficient is left unchanged . This causes particular difficulty when the signal under consideration has high frequency components. The hard thresholding procedure creates discontinuities at x = ±δ, where as soft thresholding continuous function.

IV. LIFTING WAVELET TRANSFORM

In mathematical analysis, wavelets were defined as translates and dilates of one mother wavelet function and were used to analyze and represent general function. Several techniques to construct wavelet bases exist and one of these is lifting scheme. Lifting scheme wavelet algorithms are recursive, which means output of first step becomes the input to next step. The scheme does not require the information in Fourier transform because the wavelet transform can be implemented in spatial or time domain [4].

4.1 Advantages of wavelet lifting

a) It allows a faster implementation of the wavelet transform.
b) Lifting scheme also leads to a fast in-place calculations of the wavelet transform.
c) Implementation that does not require auxiliary memory.
d) With the lifting scheme the inverse wavelet transform can immediately be found by changing + into- in forward transform.

Lifting consists of three stages i.e. Split, prediction, and update

4.2 Algorithm for lifting

We start with finite sequence s_j of length 2^j.

It is transformed into two sequences, each of length 2^{j-1}. Denoted by s_{j-1} andd_{j-1}, respectively.

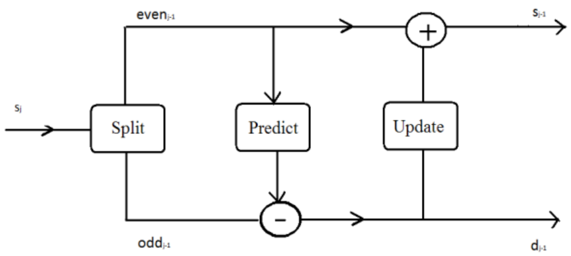

Fig2. The three steps in a lifting building block.

Split The input signal is split into two, the odd component and the even components. Thereafter it first does predict and then update [5].

Prediction If the signal contains some structure, then we can expect correlation between a sample and its nearest neighbors. If the value at sample number is 2n,we predict the value at 2n+1 is the same. So we replace the value at 2n+1 with correction to the prediction, which is the difference.

$$d_{j-1}[n] = s_j[2n+1] - s_j[2n]$$

In general, the idea is to have prediction procedure P and then compute

$$d_{j-1} = odd_{j-1} - P(even_{j-1})$$

Thus in the d signal each entry is one odd sample minus some prediction based on a number of even samples.

Update Given an even entry, we have predicted that the next odd entry has the same value, and stored the difference. We then update our even entry is replaced by average.

$$s_{j-1}[n] = s_j[2n] + d_{j-1}[n]/2$$

In general we decide on an updating procedure, and the compute

$$s_{j-1} = even_{j-1} + U(d_{j-1})$$

Invert the lifting procedure

The above equations are for direct transform and for inverted we have

$$even_{j-1} = s_{j-1} - U(d_{j-1})$$

$$odd_{j-1} = d_{j-1} + P(even_{j-1})$$

In order to get original sequence s_j we have to merge the sequences $even_{j-1}$ and odd_{j-1}

V. Simulation and analysis

The proposed Haar wavelet through lifting denoising scheme has been implemented to different signals. We have taken sine wave, step signal, and piece-regular signal. We added random noise to this signals, and by using haar wavelet lifting algorithm de-noise the signal by varying the length of the signal. We calculated SNR and MSE of original signal and de-noised signal.

First we have taken sine wave and added random noise to it, after that we denoised the signal using haar wavelet lifting algorithm the results are follows.

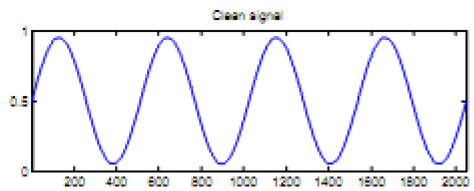

Fig 3 Original signal

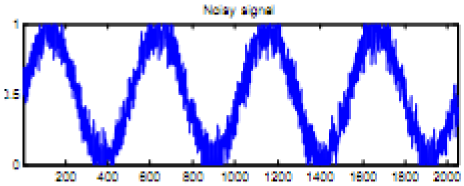

Fig 4 Noisy signal

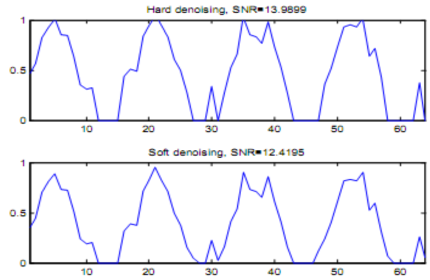

Fig 5 when n=64 hard and soft denoising results

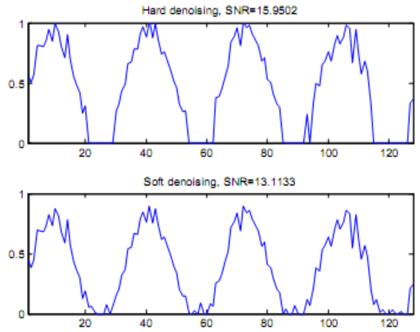

Fig 6. When n=128 hard and soft denoisnig results

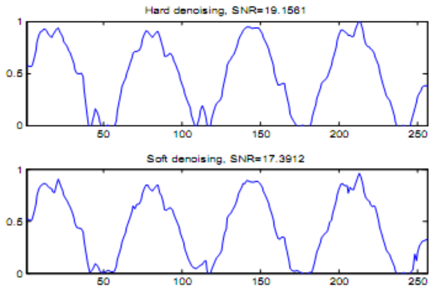

Fig 7 when n=256 hard and soft denoising results

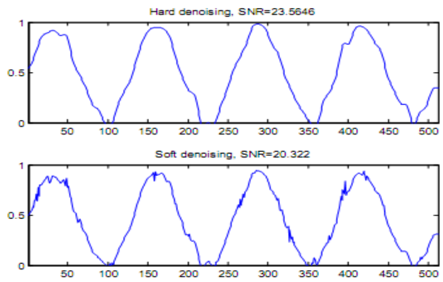

Fig 8 When n=512 hard and soft denoising results

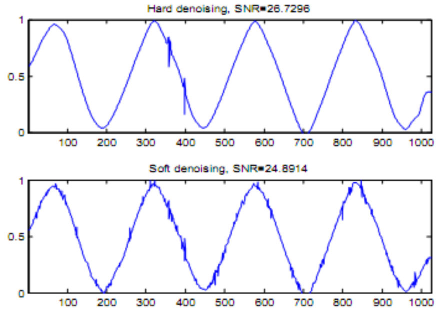

Fig9 When n=1024 hard and soft denoising results

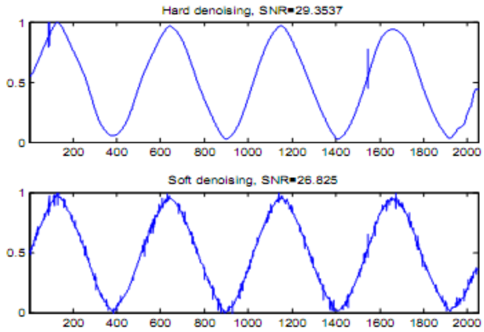

Fig10 When n=2048 hard and soft denoising results

Step signal	Soft thresholding		Hard thresholding	
	SNR	MSE	SNR	MSE
N=64	18.2850	0.01024	21.7322	0.00688
N=128	21.3450	0.00508	23.5145	0.00396
N-256	23.8531	0.00294	25.1075	0.00230
N=512	24.8697	0.00169	26.3767	0.00142
N=1024	25.7254	0.00108	29.2300	0.00072
N=2048	28.4523	0.00056	31.0862	0.00041

Table1. Comparison of SNR and MSE different resolution of the signal

Similarly when we have taken step, and piece-regular signals, and we obtained following results.

Sine wave with different resolution	Soft thresholding		Hard thresholding	
	SNR	MSE	SNR	MSE
N=64	12.4195	0.01773	13.9899	0.01479
N=128	13.1133	0.01150	15.9502	0.00830
N-256	17.3912	0.00500	19.1561	0.00408
N=512	20.3220	0.00252	23.5646	0.00173
N=1024	24.8914	0.00105	26.7296	0.00853
N=2048	26.8250	0.00059	29.3537	0.00060

Table2. Comparison of SNR and MSE different resolution of the signal

Piece-regular signal	Soft thresholding		Hard thresholding	
	SNR	MSE	SNR	MSE
N=64	12.9029	0.01196	13.0957	0.01010
N=128	13.4691	0.00770	15.0063	0.00649
N-256	15.7880	0.00422	17.6001	0.00343
N=512	17.2874	0.00248	18.6767	0.00212
N=1024	19.1550	0.00142	20.8221	0.00117
N=2048	21.9805	0.00072	23.0352	0.00064

Table3. Comparison of SNR and MSE different resolution of the signal

We know that SNR (signal to noise ratio) and MSE (mean square error) are important parameters in denoising performance. The error between reconstructed signal and original signal is smaller and denoising effect is better while SNR is higher and MSE is less. By observing fig4-9 and table1 we have high SNR and less MSE for the signal resolution 2048 .By observing table1-3 we can conclude that as the signal resolution increases we will get better denoised signal, which is having less MSE and SNR ratio is high, Hard thresholding will give better results than soft thresholding

VI. CONCLUSION

To deal with the shortcoming of the denoising the implementation on computer is solved by using Haar wavelet through lifting for denoising. This has the following characteristics:

It allows a faster implementation of the wavelet transform and as the resolution of the signal increases the SNR values increases and MSE decreases when we use the Haar wavelet through lifting denoising.

We have implemented this method on different signals, corresponding SNR and MSE values are displayed in table1-3. We observed that this method will give good results as the resolution of signal increases.

References

[1] *D.L. Donoho* and *I.M. Johnstone*, "*Ideal spatial adaptation* by wavelet shrinkage”, Biometrika, Vol.81, No.12, 1994, pp.425-455

[2] D.L. Donoho, “De-noising by soft thresholding.” IEEE Transactions on Information Theory, vol. 41, no.3, pp. 613-627, May 1995.

[3] C. Taswell, “The what, how, and why of wavelet shrinkage de-noising.” IEEE Computational Science & Engineering, vol. 2, Issue 3. pp. 12-19, May/Jun 2000.

[4] I. Daubechies and W. Sweldens, “Factoring wavelets into lifting steps,” Journal Fourier and Applications. Vol. 4, no. 3, pp. 247–269, 1998.

[5] A. Jensen and la Cour-Harbo, “A. Ripples in Mathematics: Discrete Wavelet Transform,” Germany: Springer-Verlag, 2001.

[6] W. Sweldens, “The Lifting Scheme: A Construction of Second Generation Wavelets,” SIAM Journal in Math. Analysis, vol. 29, no. 2, pp. 511-546.,1998.