RESEARCH ARTICLE OPEN ACCESS

Effect of MHD on Jeffery-Hamel Flow in Nanofluids by Differential Transform Method

J.C. Umavathi, and M. Shekar
Department of Mathematics, Gulbarga University, Gulbarga, Karnataka – 585 106, India

Abstract

In this paper, the problem of Jeffery-Hamel flow of nanofluid with magnetic effect is analyzed. The basic governing equations which are highly nonlinear are solved analytically using a semi-numerical-analytical technique called differential transform method (DTM) and numerically using Runge-Kutta shooting method (RKSM). The principle of differential transformation is briefly introduced, and then applied for the aforementioned problem. Results for velocity filed in a divergent channel are presented for various values of nanoparticle solid volume fraction, Hartmann number and Reynolds number. The values obtained by DTM and RKSM are agree to the order of 10^{-5}. Further the values obtained by DTM are justified by comparing with the values obtained by Moghimi et al. [1,2] and Sheikholeslami et al. [3] and the values agree to the order of 10^{-5}, 10^{-5} and 10^{-4} respectively.

Keywords: Jeffery-Hamel flow, MHD, nanofluid, Differential Transform method.

I. INTRODUCTION:

The study of flows in converging/diverging channel is very important due to its engineering and industrial applications. Such applications include exchanging heat transfer of heat exchangers for milk flowing, cold drawing operation in polymer industry, extrusion of molten polymers through converging dies, pressure driven transport of particles through a symmetric converging/diverging channel and many others [4-6]. The well known Jeffery-Hamel problem deals with the flow of an incompressible viscous fluid between the non-parallel walls. This flow situation was initially formulated by Jeffery [7] and Hamel [1]. Later, this problem is extensively studied by the various researchers. A survey of information on this can be found in the references [6,8]. Apart from using numerical methods, the Jeffery-Hamel flow problem was solved by other techniques including the Homotopy analytical method (HAM), the Homotopy perturbation method (HPM), the Adomain decomposition method (ADM) and the spectral-Homotopy analysis method. Recently, the three analytical methods such as Homotopy analysis method, Homotopy perturbation method and Differential transformation method (DTM) were used by Joneidi et al. [9] to find the analytical solution of Jeffery-Hamel flow.

A large number of theoretical investigations dealing with magnetohydrodynamic (MHD) flows of viscous fluids have been performed during the last decades due to their rapidly increasing applications in many fields of technology and engineering, such as MHD power generation, MHD flow meters, and MHD pumps [10]. Many mathematic models have been proposed to explain the behavior of the viscous MHD flow under different conditions [11–14]. The classical Jeffery–Hamel problem was extended in [15] to include the effects of external magnetic field on conducting fluid. Motsa et al. [16] found the solution of the nonlinear equation for the MHD Jeffery–Hamel problem by using novel hybrid spectral-homotopy analysis. Recently, Moghimi et al. [1] studied the MHD Jeffery–Hamel flows in non-parallel walls by using homotopy analysis method. Moghimi et al. [2] also solved the Jeffery–Hamel flow problem by using the homotopy perturbation method. More recently, the effects of magnetic field and nanoparticle on the Jeffery-Hamel flow using a powerful analytical method called the Adomian decomposition method were studied by Sheikholeslami et al. [3].

Nanoparticles are made from various materials, such as oxide ceramics (Al_2O_3, CuO), nitride ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu, Ag, Au), semiconductors, carbon nanotubes and composite materials such as alloyed nanoparticles. Nanofluids consist of a base fluid and ultrafine nanoparticles aim to achieve the maximum possible thermal properties at the minimum possible concentrations (preferably<1% by volume) by uniform dispersion and stable suspension of nanoparticles (preferably<10 nm) in host fluids [17]. Many studies on nanofluids are being conducted by scientists and engineers due to their diverse technical and biomedical applications. Examples include nanofluid coolant: electronics cooling, vehicle cooling, transformer cooling, super powerful and small computers cooling and electronic devices cooling; medical applications: cancer therapy and safer surgery by cooling and process industries; materials and chemicals: detergency, food and drink, oil and gas. Ultra high-performance cooling is necessary for many industrial technologies. However, poor thermal conductivity is a drawback in developing energy-efficient heat transfer fluids necessary for ultra high–performance cooling. Numerous models and methods were proposed by different authors to study convective flows of nanofluids, and we mention here the papers by Choi [18]. Duangthongsuk and Wongwises [19], Yacob et al. [20], etc.

The concept of differential transformation was first proposed by Zhou [21] in solving linear and nonlinear initial valued problems in electrical circuit analysis. Later Chen and Ho [22] applied the differential transformation method to second order eigen-value problems and the transverse vibration of a twisted beam under axial loading. The differential transform method obtains a semi-analytical solution in the form of a polynomial. It is different from the traditional higher order Taylor's series method, which requires symbolic competition of the necessary derivatives of the functions. The Taylor series method is computationally taken long time for large orders. With this method, it is possible to obtain highly accurate results or exact solutions for differential equations. This method is well-addressed in [23–26].

The aim of this paper is to introduce the differential transform technique as an alternative to existing methods in solving singular two-point boundary value problems. To our knowledge, the combined effect of nanoparticles and magnetic field in the Jeffery-Hamel flow problem using DTM is not addressed yet.

II. PROBLEM STATEMENT AND GOVERNING EQUATION:

Consider the steady fully developed flow of an incompressible conducting viscous fluid between two rigid plane walls that meet at an angle 2α as shown in Fig. 1. The rigid walls are considered to be divergent if $\alpha > 0$ and convergent if $\alpha < 0$. We assume that the velocity is purely radial and depends on r and θ so that $\mathbf{v} = (u(r,\theta),0)$ only and further there is no magnetic field in the $z-$direction. The continuity equation, the Navier–Stokes equations and Maxwell's equations in polar coordinates are [3]

$$\frac{\rho_{nf}}{r}\frac{\partial(ru)}{\partial r} = 0 \tag{1}$$

$$\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{u}{r^2}\right) - u\frac{\partial u}{\partial r} - \frac{1}{\rho_{nf}}\frac{\partial P}{\partial r} - \frac{\sigma B_0^2}{\rho_{nf} r^2}u = 0 \tag{2}$$

$$\frac{1}{\rho_{nf}}\frac{\partial P}{\partial \theta} - \frac{2\mu_{nf}}{\rho_{nf} r^2}\frac{\partial u}{\partial \theta} = 0 \tag{3}$$

where $u = u(r, \theta)$ is the velocity, P is the pressure, B_0 is the electromagnetic induction, σ is the conductivity of the fluid, μ_{nf} is the effective viscosity, ρ_{nf} is the effective density and K_{nf} is the effective thermal conductivity of nanofluid.
The boundary conditions are

at the centerline of the channel: $\frac{\partial u}{\partial \theta} = 0$,

at the boundary of the channel: $u = 0$.

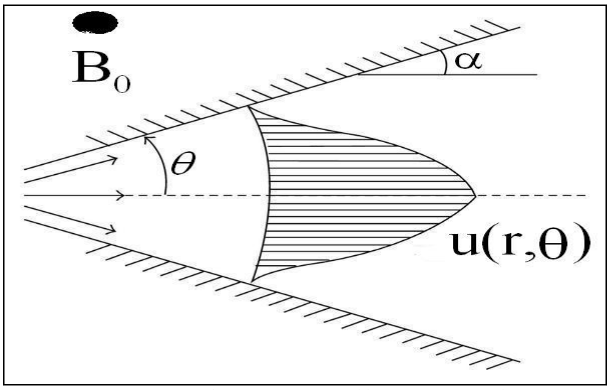

Fig. 1 Geometry of the MHD Jeffery–Hamel flow in convergent/divergent cannel with angle 2α.

We consider throughout the paper, the thermophysical properties of the nanofluid as they are given in [27], see the Table 1. Also the density of the nanofluid ρ_{nf} and the viscosity of the nanofluid μ_{nf} are given by the expressions (see [3,28])

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{4}$$

where ϕ is the solid volume fraction, the subscripts nf, f and s respectively are the thermo-physical properties of the nanofluids, base fluid and the solid nanoparticles.

Considering only radial flow, the continuity Eq. (1) implies that

$$u(r, \theta) = \frac{f(\theta)}{r} \tag{5}$$

Following [4] we define the dimensionless parameters

$$F(\eta) = \frac{f(\theta)}{f_{\max}} \quad \text{where } \eta = \frac{\theta}{\alpha} \tag{6}$$

Substituting Eqs. (4) - (6) into Eqs. (2) and (3) and eliminating the pressure term yields the nonlinear ordinary differential equation

$$F'''(\eta) + 2\alpha \operatorname{Re} A(1-\phi)^{2.5} F(\eta)F'(\eta) + \left(4-(1-\phi)^{2.5}M^2\right)\alpha^2 F'(\eta) = 0 \tag{7}$$

where α is the is the angle between two plates, A is a parameter, Re is a Reynolds number and M is the

Hartmann number based on the electromagnetic parameter are introduced as follows:

$$A = 1-\phi+\frac{\rho_s}{\rho_f}\phi, \qquad \mathrm{Re}=\frac{f_{\max}\rho_f\,\alpha}{\mu_f}=\frac{U_{\max}\rho_f\, r\,\alpha}{\mu_f},$$

$$M^2=\frac{\sigma B_0^2}{\mu_f}. \tag{8}$$

where $U_{\max}$ is the velocity at the center of the channel ($r=0$) and the Reynolds number .

$$\mathrm{Re}=\begin{pmatrix}\text{divergent-channel:} & \alpha>0,\ f_{\max}>0\\ \text{convergent-channel:} & \alpha<0,\ f_{\max}<0\end{pmatrix}$$

According to the relation (5) and (6), the boundary conditions will be

$$F(0)=1, \qquad F'(0)=0, \tag{9}$$

$$F(1)=0. \tag{10}$$

Physically, these boundary conditions mean that maximum values of velocity are observed at centerline $\eta=0$ as shown in Fig. 1. Thus, rate of velocity is zero at $\eta=0$. Also, the no-slip condition at a solid boundary is considered.

2.1 Basic Idea of Differential transformation method (DTM)

Suppose $u(y)$ is analytic in a domain *D*, then it will be differentiated continuously with respect to y in the domain of interest. The differential transform of function $u(y)$ is defined as

$$U(k)=\frac{1}{k!}\left[\frac{d^k u(y)}{dy^k}\right]_{y=0} \tag{11}$$

where $u(y)$ is the original function and $U(k)$ is the transformed function which is called the T-function.

The differential inverse transform of $U(k)$ is defined as follows:

$$u(y)=\sum_{k=0}^{\infty}U(k)\,y^k \tag{12}$$

In real applications, the function $u(y)$ by a finite series of (12) can be written as

$$u(y)=\sum_{k=0}^{n}U(k)\,y^k \tag{13}$$

and Eq. (12) implies that $u(y)=\sum_{k=n+1}^{\infty}U(k)\,y^k$ is neglected as it is small. Usually, the values of *n* are decided by a convergence of the series coefficients.

The fundamental mathematical operations performed by differential transform method are listed in Table 2.

III. SOLUTION WITH DIFFERENTIAL TRANSFORM METHOD:

Now Differential Transformation Method has been applied to solving Eq. (7). Taking the differential transformation of Eq. (7) with respect to k, and following the process as given in Table 2 yields:

$$\bar F(k+3)=-\frac{k!}{(k+3)!}\Big(\big(4-(1-\phi)^{2.5}M^2\big)\alpha^2(k+1)\bar F(k+1) + 2\alpha\,\mathrm{Re}\,A(1-\phi)^{2.5}\sum_{r=0}^{k}(k-r+1)\bar F(k-r+1)\bar F(r)\Big) \tag{14}$$

where $\bar F(k)$ is the differential transform of $F(\eta)$ and $k=0,1,2,3,\ldots\ldots\ldots\ldots n$ represents the number of term of the power series.

From a process of inverse differential transformation, it can be shown that the solution of each sub-domain take $n+3$ terms for the power series.

The transforms of the boundary conditions are

$$\bar F[0]=1, \bar F[1]=0, \qquad \bar F[2]=a_1 \tag{15}$$

Using the conditions as given in Eq. (10), one can evaluate the unknown a_1. By using the DTM and the transformed boundary conditions, above equation that finally lead to the solution of a system of algebraic equations. For copper-water nanofluid with $\mathrm{Re}=100$, $M=1000$, $\phi=0.1$ and $\alpha=5^\circ$ we have found $a_1=$ -1.8328675214 .

3.1 Convergence of DTM solution

The validation of the present results has been verified with the classical case of a Newtonian fluid ($\phi=0$), see Moghimia et al. [1,2] and Sheikholeslami et al. [3]. Also the convergence of the present method is observed by comparing our results with those of numerical method using the Runge-Kutta shooting method (RKSM). The non-dimensional velocity $F(\eta)$ for different values of Reynolds number Re, Hartmann number M and angle between the plates α are shown in Tables 3–5. These tables agree very well the results between DTM-RKSM, DTM-HAM (Moghimi et al. [1]), DTM-HPM (Moghimi et al. [2]) and DTM-ADM (Sheikholeslami et al. [3]). In particular, Table 6 gives a snap shot of the velocity values at different points inside the diverging channel while Table 6 also demonstrates on the convergence rate of the DTM. In general, fifteen terms of the DTM approximation are sufficient to give a match with the numerical results up to five decimal places. This table shows that the DTM converges more easily for these type problems. The numerical results of copper-water nanofluid with and without magnetic effect are presented in Table 7 obtained by DTM and RKSM which show a favorable agreement, thus give confidence that the results obtained are accurate. Moreover, the values of $F(\eta)$ are presented in Table 7 is for future reference also.

IV. RESULTS AND DISCUSSION:

The problem of Jeffery-Hamel flow of nanofluid with high magnetic field is discussed in a divergent channel. The governing equation for the posed problem is highly nonlinear and hence closed form solutions cannot be obtained. The objective of this work is to find the solutions of the Jeffery-Hamel flow by DTM and estimating the order of error. This paper also highlights the solution procedure of DTM.

The graphical results depicted in Figs. 2–6 are broadly in line with those given in [6] but now modified for nanofluid by the effects of the applied magnetic field. Numerical simulations show that for fixed Hartmann numbers, the fluid velocity decreases with Reynolds numbers in divergent channel.

Figure 2 display the velocity profiles for silver (Ag), copper (Cu), Diamond, TiO_2 and SiO_2 nanoparticles. It is observed that the values for silver and copper show closer values and Diamond, TiO_2 and SiO_2 are also close to each other. The optimal velocity is observed for SiO_2 nanoparticle and minimal is for silver nanoparticle.

The result obtained for different solid volume fraction is illustrated in Fig. 3. As the solid volume fraction ϕ increases the fluid velocity decreases. That is, the fluid velocity is more for base fluid $(\phi = 0)$ and less for nanofluid. Physically speaking, as the nanoparticles add to the pure fluid the density of the fluid increases and then the fluid becomes denser, so that it can have more difficult movement through the channel.

Figure 4 predicts the magnetic effect (Hartmann number M) of viscous fluid and nanofluid on the velocity for divergent channel with fixed Reynolds number. The results show moderate increases in the velocity with increasing Hartmann number for both viscous and nanofluid and, in line with observations for viscous fluid $(\phi = 0)$, no back flow is observed for all Hartmann numbers. It can be seen in Fig. 4 that without magnetic field $(M = 0)$ at $\alpha = 5^\circ$ and $\mathrm{Re} = 50$ backflow starts, with Hartmann number increasing this phenomenon is eliminated. By increasing Reynolds number, the backflow expands so greater magnetic field is needed in order to eliminate it.

Figure 5 illustrates the effect of Reynolds number Re on the fluid velocity for fixed Hartmann number for both viscous fluid and nanofluid. As Reynolds number increases the fluid velocity decreases for both viscous and nanolfuids. Here also back flow is excluded for small Reynolds number and it observed for large Reynolds.

Figure 6 give a comparison between the DTM and the numerical approximations (RKSM) for various values of α. Of particular note here is that an exact match between the two set of approximate solutions is obtained with only fifteen terms of the DTM solution series. These findings firmly establish the DTM as an accurate and efficient alternative to the other semi-analytical methods. We also observed from Fig. 5 that as the angle between the plates α increases the fluid velocity increases.

V. CONCLUSIONS:

In this paper we have used the DTM to find the analytical solution of magnetohydrodynamic Jeffry-Hamel flow problem in the nanofluid. It was found that DTM is a powerful method for solving problems consisting 3^{rd} order nonlinear differential equations. A reliable algorithm is presented based on the DTM to solve highly nonlinear equations. A comparison was made between the available results of obtained by different methods such as Homotopy analytical method, Homotopy perturbation method and Adomian decomposition method solutions, numerical results from the RKSM and the present approximate solutions. The numerical study indicates that the DTM gives more accurate in comparison to shown methods. The method has been applied directly without requiring linearization, discretization, or perturbation. The obtained results certify the reliability of the algorithm and give it a wider applicability to nonlinear differential equations. The influence of various physical parameters on the velocity was discussed in detail. The basic conclusions are as follows:

- Increasing Reynolds numbers leads to decreases the velocity field.
- Increasing Hartmann number will lead to backflow reduction.
- Also the results show that fluid velocity of nanofluid is less when compared to fluid without nanoparticles.
- The results obtained in this paper are nearly similar to the results obtained by Sheikholeslami et al. [3] in the absence of different nanoparticles.

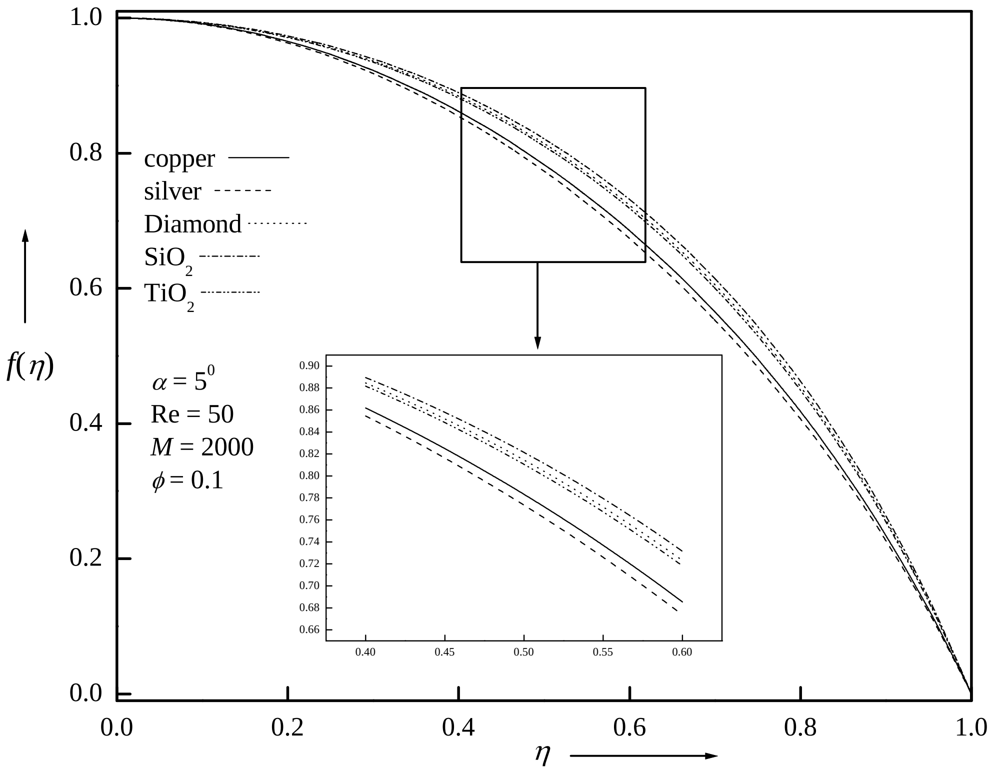

Fig. 2 Velocity profiles for several nanoparticles.

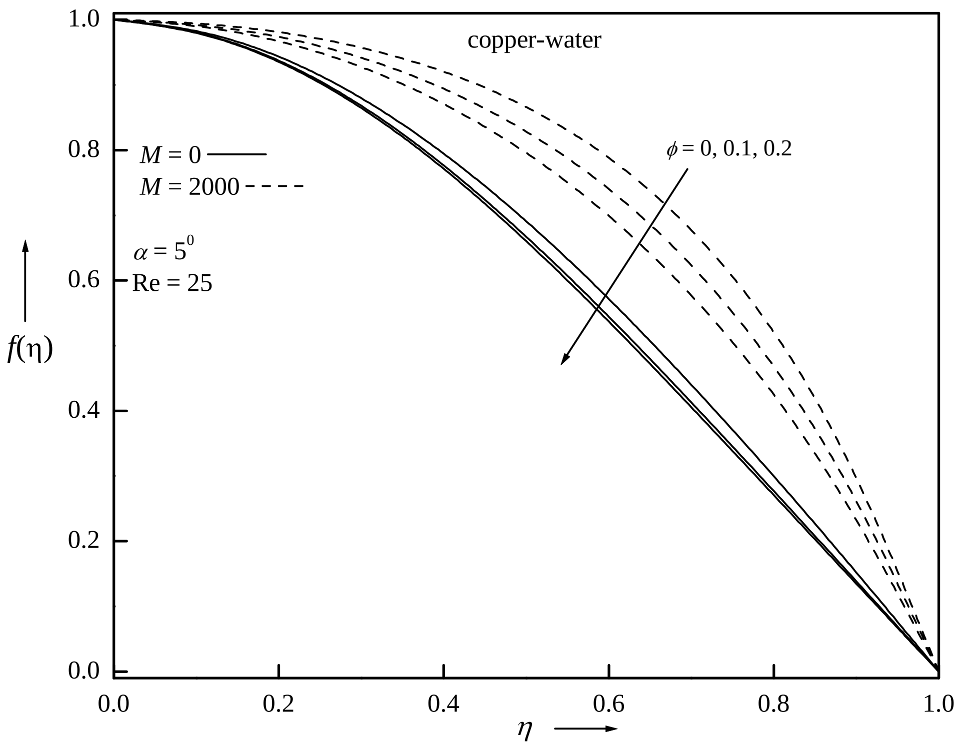

Fig. 3 Velocity profiles for several values of solid volume fraction.

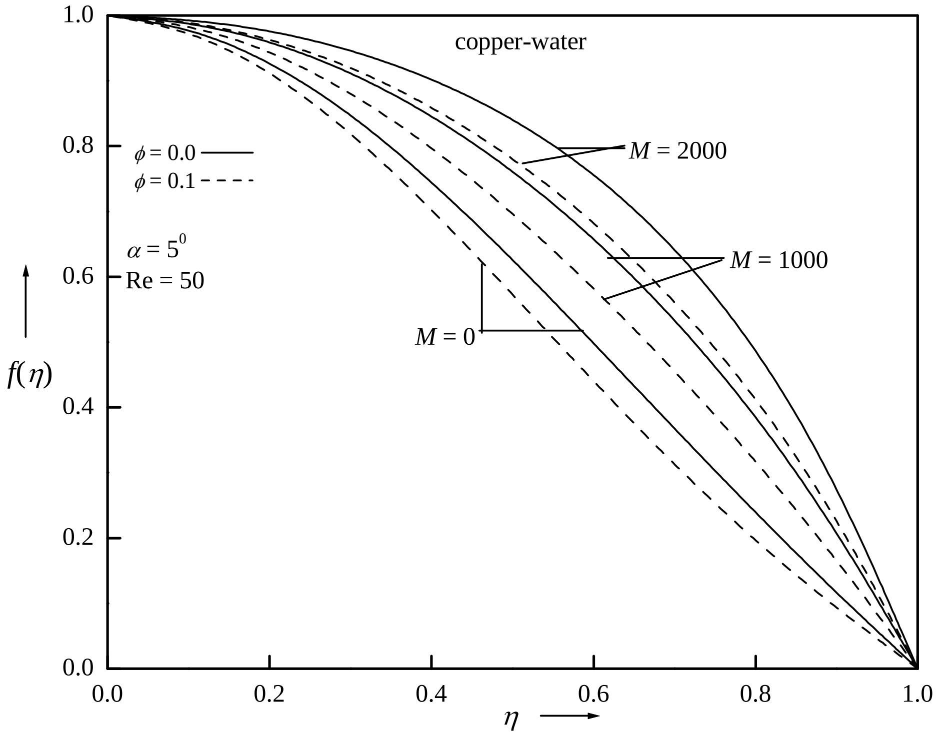

Fig. 4 Velocity profiles for several values of Hartmann number and solid volume fraction.

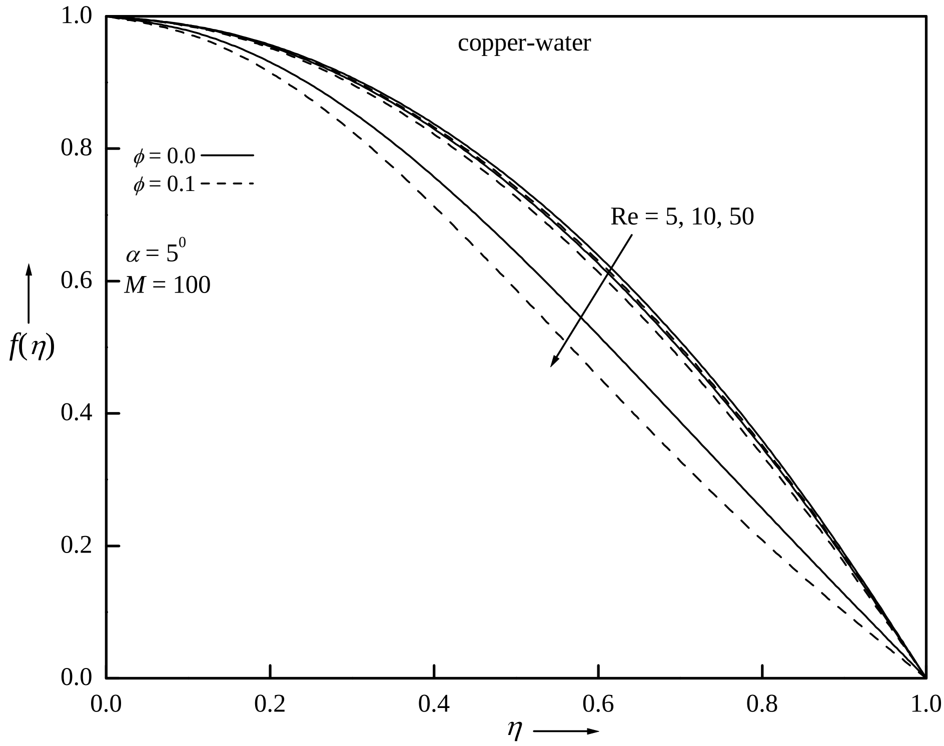

Fig. 5 Velocity profiles for several values of Reynolds number and solid volume fraction.

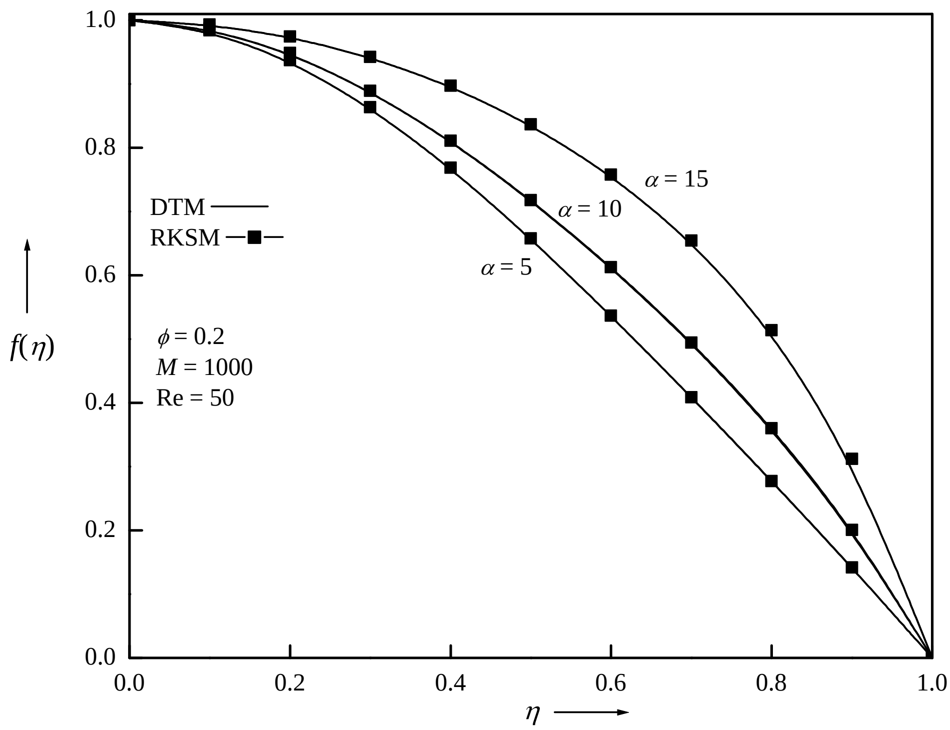

Fig. 6 Comparison results between DTM and RKSM for several values of α .

Table 1. Thermo-physical properties for pure water and various types of nanoparticles.

Property	Pure water	Silver (Ag)	Copper (Cu)	Diamond	SiO_2	TiO_2
$\rho\left(\mathrm{kg/m^2}\right)$	997.1	10500	8933	3510	2200	4250
$\mu\left(\mathrm{N\,m/s}\right)$	1 x 10^{-3}	-	-	-	-	-
$k\left(\mathrm{W/mK}\right)$	0.613	429	400	1000	1.2	8.9538
$\beta\left(1/\mathrm{K}\right)$	207 x 10^{-6}	18 x 10^{-6}	17 x 10^{-6}	1.0 x 10^{-6}	5.5 x 10^{-6}	0.17 x 10^{-6}

Table 2. The operations for the one-dimensional differential transform method.

Original function	Transformed function
$y(x) = g(x) \pm h(x)$	$Y(k) = G(k) \pm H(k)$
$y(x) = \alpha\, g(x)$	$Y(k) = \alpha\, G(k)$
$y(x) = \frac{dg(x)}{dx}$	$Y(k) = (k+1)G(k+1)$
$y(x) = \frac{d^2 g(x)}{dx^2}$	$Y(k) = (k+1)(k+2)G(k+2)$
$y(x) = g(x)h(x)$	$Y(k) = \sum_{l=0}^{k} G(l)H(k-l)$
$y(x) = x^m$	$Y(k) = \delta(k-m) = \begin{cases} 1, \text{ if } k = m \\ 0, \text{ if } k \neq m \end{cases}$

Table 3. Comparison results between DTM – HAM (Moghimia et al. [1]) and RKSM solution for velocity when $\phi = 0$, $\mathrm{Re} = 50$, $\alpha = -5^{\circ}$, $M = 1000$.

η	DTM	Moghimi et al. [1] HAM	Numerical (RKSM)	Error (DTM-Numerical)	Error (HAM-Numerical)
0	1	1	1	0.00E+00	0.00E+00
0.1	0.99675670	0.99675704	0.99675875	6.63E-05	6.65E-05
0.2	0.98649151	0.98649281	0.98649971	8.56E-05	8.76E-05
0.3	0.96751661	0.96751931	0.96753507	7.98E-05	8.35E-05
0.4	0.93673804	0.93674218	0.93677065	6.34E-05	6.83E-05
0.5	0.88920852	0.88921354	0.88925841	4.49E-05	4.99E-05
0.6	0.81746155	0.81746646	0.81752984	2.85E-05	3.26E-05
0.7	0.71062383	0.71062756	0.71070737	1.58E-05	1.85E-05
0.8	0.55338810	0.55339006	0.55347566	6.90E-06	8.20E-06
0.9	0.32514224	0.32514237	0.32520872	1.71E-06	2.05E-06
1	0	0	0	0.00E+00	0.00E+00

Table 4. Comparison results between DTM – HPM (Moghimia et al. [2]) and RKSM solution for velocity when $\phi = 0$, $\mathrm{Re} = 50$, $\alpha = 7.5^{\circ}$, $M = 1000$.

η	DTM	Moghimi et al. [2] HPM	Numerical (RKSM)	Error (DTM-Numerical)	Error (HPM-Numerical)
0	1	1	1	0.00E+00	0.00E+00
0.1	0.9937611	0.9937607	0.9937652	9.46E-05	1.39E-04
0.2	0.9747903	0.9747886	0.9748064	1.25E-04	1.62E-04
0.3	0.9422833	0.9422794	0.9423182	1.25E-04	1.51E-04
0.4	0.8947504	0.8947431	0.8948089	1.08E-04	1.26E-04
0.5	0.8297590	0.8297471	0.8298433	8.43E-05	9.62E-05
0.6	0.7434936	0.7434756	0.7436019	5.85E-05	6.58E-05
0.7	0.6300130	0.6299866	0.6301378	3.49E-05	3.88E-05
0.8	0.4799708	0.4799338	0.4800960	1.61E-05	1.78E-05
0.9	0.2783235	0.2782789	0.2784181	4.10E-06	4.50E-06
1	0	0	0	0.00E+00	0.00E+00

Table 5. Comparison results between DTM – ADM (Sheikholeslami et al. [3]) and RKSM solution for velocity when $\phi = 0$, $\mathrm{Re} = 25$, $\alpha = 5^{\circ}$.

	$M = 0$				
η	DTM	Sheikholeslami et al. [3] (ADM)	Numerical (RKSM)	Error (DTM-RKSM)	Error (ADM-RKSM)
0	1.000000	1.000000	1.000000	0.00E+00	0.00E+00
0.1	0.986669	0.986637	0.986667	2.00E-06	3.00E-05
0.2	0.947251	0.947127	0.947244	7.00E-06	1.17E-04
0.3	0.883404	0.883146	0.883392	1.20E-05	2.46E-04
0.4	0.797674	0.797259	0.797654	2.00E-05	3.95E-04
0.5	0.693202	0.692638	0.693176	2.60E-05	5.38E-04
0.6	0.573389	0.572716	0.573359	3.00E-05	6.43E-04
0.7	0.441558	0.440850	0.441526	3.20E-05	6.76E-04
0.8	0.300645	0.300013	0.300618	2.70E-05	6.05E-04
0.9	0.152962	0.152552	0.152944	1.80E-05	3.92E-04
1	0.000000	0.000000	0.000000	0.00E+00	0.00E+00
	$M = 500$				
0	1.000000	1.000000	1.000000	0.00E+00	0.00E+00
0.1	0.990221	0.992695	0.990223	2.00E-06	-2.47E-03
0.2	0.960939	0.970544	0.960944	5.00E-06	-9.60E-03
0.3	0.912287	0.912273	0.912297	1.00E-05	2.40E-05
0.4	0.844405	0.832683	0.844422	1.70E-05	1.17E-02
0.5	0.757317	0.743421	0.757341	2.40E-05	1.39E-02
0.6	0.650758	0.643816	0.650789	3.10E-05	6.97E-03
0.7	0.523953	0.515303	0.523987	3.40E-05	8.68E-03

0.8	0.375331	0.361234	0.375364	3.30E-05	1.41E-02
0.9	0.202153	0.194730	0.202176	2.30E-05	7.45E-03
1	0.000000	0.000000	0.000000	0.00E+00	0.00E+00

Table 6. Comparison results between DTM and RKSM for fixed $\phi = 0.1$, $\mathrm{Re} = 5$, $\alpha = 5^\circ$, $M = 100$.

η	5 terms	10 terms	15 terms	20 terms	30 terms	RKSM
1	0	0	0	0	0	0
0.9	0.181193	0.187290	0.187278	0.187278	0.187278	0.187280
0.8	0.346815	0.355144	0.355131	0.355131	0.355131	0.355134
0.7	0.495699	0.503963	0.503952	0.503952	0.503952	0.503954
0.6	0.626815	0.633813	0.633805	0.633805	0.633805	0.633807
0.5	0.739270	0.744534	0.744528	0.744528	0.744528	0.744529
0.4	0.832308	0.835827	0.835823	0.835823	0.835823	0.835824
0.3	0.905313	0.907335	0.907333	0.907333	0.907333	0.907333
0.2	0.957802	0.958709	0.958709	0.958708	0.958708	0.958709
0.1	0.989433	0.989661	0.989661	0.989661	0.989661	0.989661
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Table 7. Comparison results between DTM and RKSM when $\phi = 0.2$, $\mathrm{Re} = 25$, $\alpha = 5^\circ$.

	$M = 0$			$M = 200$		
η	DTM	Numerical (RKSM)	Error (DTM-RKSM)	DTM	RKSM	Error (DTM-RKSM)
0	0	0	0.00E+00	0	0	0.00E+00
0.1	0.134828	0.134724	1.04E-04	0.151112	0.151074	3.80E-05
0.2	0.270828	0.270717	1.11E-04	0.296318	0.296276	4.20E-05
0.3	0.406256	0.406152	1.04E-04	0.435188	0.435147	4.10E-05
0.4	0.538121	0.538031	9.00E-05	0.566012	0.565975	3.60E-05
0.5	0.662374	0.662302	7.20E-05	0.686043	0.686013	3.00E-05
0.6	0.774186	0.774133	5.30E-05	0.791797	0.791774	2.20E-05
0.7	0.868370	0.868337	3.30E-05	0.879439	0.879425	1.40E-05
0.8	0.939943	0.939927	1.60E-05	0.945255	0.945248	7.00E-06
0.9	0.984743	0.984738	4.00E-06	0.986132	0.986130	2.00E-06
1	1.000000	1.000000	0.00E+00	1.000000	1.000000	0.00E+00

REFERENCES:

[1]. S.M. Moghimi, G. Domairry, Soheil Soleimani, E. Ghasemi, H. Bararnia, Application of homotopy analysis method to solve MHD Jeffery–Hamel flows in non-parallel walls, Advances in Engineering Software 42 (2011) 108–113.

[2]. S.M. Moghimia, D.D. Ganji, H. Bararnia, M. Hosseini, M. Jalaal, Homotopy perturbation method for nonlinear MHD Jeffery–Hamel Problem, Computers and Mathematics with Applications 61 (2011) 2213–2216.

[3]. M. Sheikholeslami, D.D. Ganji, H. R. Ashorynejad, H.B. Rokni, Analytical investigation of Jeffery-Hamel flow with high magnetic field and nanoparticle by Adomian decomposition method, Appl. Math. Mech. -Engl. Ed. 33 (2012) 25–36

[4]. G. Hamel, Spiralförmige Bewgungen Zäher Flüssigkeiten. Jahresber. Deutsch. Math. Verein. 25 (1916) 34–60.

[5]. M. Hamadiche, J. Scott, D. Jeandel, Temporal stability of Jeffery–Hamel flow. J. Fluid Mech. 268 (1994) 71–88.

[6]. L.E. Fraenkel, Laminar flow in symmetrical channels with slightly curved walls. I: On the Jeffery–Hamel solutions for flow between plane walls. Proc Roy Soc A 267 (1962) 119–38.

[7]. G.B. Jeffery, The two-dimensional steady motion of a viscous fluid. Phil. Mag. 6 (1915) 455–465.

[8]. Q. Esmaili, A. Ramiar, E. Alizadeh, D.D. Ganji, An approximation of the analytical solution of the Jeffery–Hamel flow by decomposition method, Physics Letters A 372 (2008) 3434–3439

[9]. A.A. Joneidi, G. Domairry, M. Babaelahi, Three analytical methods applied to Jeffery–Hamel flow, Commun. Nonlinear Sci. Numer. Simulat. 15 (2010) 3423–3434.

[10]. G.W. Sutton, A. Sherman, Engineering Magnetohydrodynamics, McGraw-Hill, New York (1965).
[11]. J.C. Umavathi, M.S. Malashetty, Magnetohydrodynamic mixed convection in a vertical channel, Int. J. Non-Linear Mechanics 40 (2005) 91-101.
[12]. J. Prathap Kumar, J.C. Umavathi, Basavaraj M. Biradar, Mixed convection of magneto hydrodynamic and viscous fluid in a vertical channel, Int. J. Non-Linear Mechanics 46 (2011) 278–285.
[13]. A. Ishak, R. Nazar, I. Pop, MHD boundary-layer flow of a micropolar fluid past a wedge with constant wall heat flux. Commun. Nonlinear Sci. Numerical Simulation, 14 (2009) 109–118.
[14]. K.V. Prasad, D. Pal, P.S. Datti, MHD power-law fluid flow and heat transfer over a nonisothermal stretching sheet. Commun. Nonlinear Sci. Numerical Simulation, 14 (2009) 2178–2189.
[15]. W.I. Axford, The Magnetohydrodynamic Jeffrey–Hamel problem for a weakly conducting fluid, Q. J. Mech. Appl. Math. 14 (1961) 335–351.
[16]. S.S. Motsa, P. Sibanda, F.G. Awad, S. Shateyi, A new spectral-homotopy analysis method for the MHD Jeffery–Hamel problem, Computers & Fluids 39 (2010) 1219–1225.
[17]. S. Kakaç, S. Özerinç, A.G. Yazıcıoğlu, Enhanced thermal conductivity of nanofluids: A state-of-the-art review. Microfluid Nanofluid 8 (2010) 145-170.
[18]. S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticle. In: Siginer, D.A.,Wang H.P. (eds.), Developments and Applications of Non-Newtonian Flows, ASME FED, vol. 231/MD 66, pp. 99–105 (1995)
[19]. W. Duangthongsuk, S. Wongwises, Effect of thermophysical properties models on the predicting of the convective heat transfer coefficient for low concentration nanofluid. Int. Commun. Heat Mass Transf. 35 (2008) 1320–1326.
[20]. N. Yacob, A. Ishak, R. Nazar, I. Pop, Falkner-Skan problem for a static and moving wedge with prescribed surface heat flux in a nanofluid. Int. Commun. Heat and Mass Transfer, 38 (2011) 149–153.
[21]. K. Zhou, Differential transformation and its applications for electrical circuits, Huazhong Univ. Press; 1986. (in Chinese)
[22]. C.K. Chen, S.H. Ho, Solving partial differential equations by two dimensional differential transform method, Appl. Math. Comput. 106 (1999) 171–179.
[23]. A.S.V. Ravi Kanth, K. Aruna, Solution of singular two-point boundary value problems using differential transformation method, Physics Letters A 372 (2008) 4671–4673.
[24]. M.M Rashidi, The modified differential transform method for solving MHD boundary-layer equations, Comp. Phys. Commun. 180 (2009) 2210–2217.
[25]. H. Yaghoobi, M. Torabi, The application of differential transformation method to nonlinear equations arising in heat transfer, Int. Comm. Heat and Mass Transfer 38 (2011) 815–820.
[26]. J.C. Umavathi, A.S.V. Ravi Kanth, M. Shekar, Comparison study of differential transform method with finite difference method for magnetoconvection in a vertical channel, Heat Transfer Asian Research, vol. 42 (2013) 243–258.
[27]. H.F. Oztop, E. Abu-Nada, Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, Int. J. Heat and Fluid Flow 29 (2008) 1326–1336.
[28]. H.C. Brinkman, The viscosity of concentrated suspensions and solution, J. Chem. Phys 20 (1952) 571-581.