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Some Statistical Characteristics of MRC Signal Envelope in Rayleigh Fading Environment

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ABSTRACT
In this paper Maximal Ratio Combining (MRC) technique over Rayleigh fading channel is described and some statistical characteristics of received signal envelope are analyzed. For analytical and numerical evaluation of system performance, the probability density functions (pdf) of received signal envelope after MRC are analyzed like particular solutions of corresponding differential equation, while the existence of singular solution is considered and analyzed under different conditions.
Keywords – Maximal Ratio Combining (MRC) technique, probability density function (pdf), Rayleigh fading channel, singular solution

I. INTRODUCTION

Wireless communication systems are subject to a complex propagation environment, like multipath fading and shadowing. Considerable efforts have been devoted to the channel modelling and characterization of these effects, resulting in a range of statistical channel models [1]. Various techniques for reducing fading effect and influence of co-channel interference (CCI) have been proposed [2]. Diversity reception is a very simple and effective approach based on the idea of providing the receiver with multiple faded replicas of the same information-bearing signal [1]. The goal of diversity techniques is to increase channel capacity and to upgrade transmission reliability without increasing transmission power and bandwidth [2]. Space diversity is an efficient method for amelioration systems quality-of-service (QoS) when multiple receiver antennas are used [3].

There are several types of combining techniques which entail various tradeoffs between performance and complexity. One of the least complex techniques is selection combining (SC). Switch and stay combining (SSC) is also low-complexity and very efficient technique that reduces fading and CCI influence. Combining techniques like equal gain (EGC) and maximal ratio (MRC) require all or some of the amount of the channel state information (CSI) available at the receiver [4-5].

The rapid fluctuations of the instantaneous received signal power due to multipath effects are usually described with Rayleigh, Rician, Nakagami-*m*, Nakagami-*q* or Weibull distribution [3]. This paper discusses the case of Rayleigh distribution, which models radio transmission in urban areas [6] where the random fluctuations of the instantaneous received signal power are very frequent and fast, and where line-of-sight (los) between transmitter and receiver does not exist.

In the case of all distributions considered in the statistical theory of telecommunications, the pdfs of the received signal envelope are functions of several variables [1-3]. In the process of determination and analysis of integral characteristics, one of the variables is treated as a parameter, while the others are set to certain constant values of interest in practice. In this way, one obtains the received signal pdf curves family. The analysis of the position of the maximums for curves family can be performed analytically, using the first derivative of function, and also numerically. In such a case the same procedure is repeated for the situation where the second variable is treated as a parameter, and the others are set to constant values. This process gives a new family of curves, while the maximum position is determined by a new envelope. In papers [7-10] the position of maximums and the analytical expression for the integral characteristic of Nakagami-*m* distribution of received signal envelope are determined. Some integral characteristics of Rayleigh, Weibull and Rician distribution are presented in [11-15].

The remainder of this paper is as follows. After this Introduction, some characteristics of Rayleigh fading channel model and MRC receiver operating over it are presented in Section II. The procedure of determining the integral properties in the case of MRC reception is described in Section III. For a fixed value of Rayleigh random process variance, the pdf of signal envelope after MRC, depending on the received signal level, is analyzed for different number of diversity branches. Then the Rayleigh random process variance is treated as parameter, while the value of the received signal level is set to the constant values. In that case, the pdf of signal envelope after MRC is analyzed also for different number of diversity branches. In such a way, two series of families of curves are obtained and for each of them equation of the envelope of curves maximums is considered. In both cases these envelopes are straight lines, in the logarithmic scale, whose direction coefficients and values on ordinate-axis are determined analytically and numerically. Also, in both cases, the differential equations describing the complete dynamics of signal transmission process are determined, whereby the envelopes of the pdf curves families represent their singular solutions. Finally, some conclusions are given in Section IV

II. RAYLEIGH FADING CHANNEL MODEL

Rayleigh fading model assumes that the received signal envelope vary randomly according to a Rayleigh distribution, presenting the radial component of the sum of two uncorrelated Gaussian random variables. Rayleigh fading model is used in urban environment, where many objects scatter radio signal before it arrives to the receiver. The probability density function (pdf) of Rayleigh distribution is given with [3]:

$$p_z(z)=\frac{z}{\sigma^2}\cdot\exp\left(-\frac{z^2}{2\sigma^2}\right) \tag{1}$$

where *z* presents the received signal level and σ^2 is the Gaussian random process variance, so σ presents the standard deviation.

The Rayleigh distribution is usually used to model multipath fading with no direct line-of-sight (los) path and it typically agrees very well with experimental data for mobile systems where no los path exists between the transmitter and receiver antennas. It also applies to the propagation of reflected and refracted paths through the troposphere and ionosphere, and ship-to-ship radio links.

MRC is the optimal combining scheme, regardless of fading statistics, but it requires knowledge of all channel fading parameters. In the case of MRC technique, with *M* branches, the received signal envelope is described by [3, 14]:

$$p_{MRC}(z,\sigma)=\frac{z^{M-1}}{2^M\sigma^{2M}\,\Gamma(M)}\cdot\exp\left(-\frac{z}{2\sigma^2}\right) \tag{2}$$

where Γ(.) is Gamma function. In communication model proposed in this paper it is assumed that all diversity branches are independent, identically distributed Rayleigh faded paths. The average fading power and the severity of fading are also assumed to be equal in all the branches.

III. ANALYTICAL AND NUMERICAL RESULTS

Graphic presentation of dependence of Rayleigh pdfs after MRC, versus received signal level, for fixed value of standard deviation σ, in logarithmic scale, for different number of diversity branches, is shown in Fig 1.

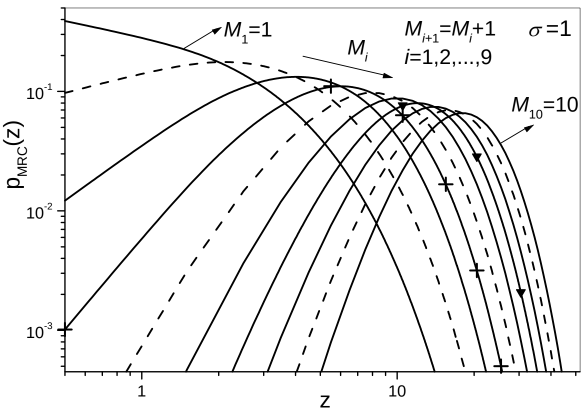

Fig. 1. The received signal pdf versus signal level, in logarithmic scale, for fixed values of σ, with number of diversity branches taking values from 1 to 10.

Analysis of this dependence shows that the increase of number of diversity branches results in smaller values of the pdf maximums, reached for higher values of received signal level, as shown in Fig 1. It can also be concluded that all the maximums lie on a straight line, and that the envelope of maximums is also a straight line, in logarithmic scale. In order to determine the values of received signal level when the maximums are reached, as well as the values of maximums and direction coefficient of envelope, the first derivative of (2) relative to received signal level is determined. Equalization the first derivative with zero obtains:

$$z_{p\max}=2\sigma^2(M-1) \tag{3}$$

Substituting (3) into (2) yields:

$$p_{MRC,\,z\max}=\frac{(M-1)^{M-1}}{2\sigma^2\cdot\Gamma(M)}\cdot\exp(1-M) \tag{4}$$

From (4) we get:

$$\log\left(\max p_{MRC}(\sigma)\right)=k_1\log\sigma+n_1 \tag{5}$$

where the direction coefficient is:

$$k_1=-2 \tag{6}$$

and the value on the ordinate axis is:

$$n_1=(M-1)\log(M-1)-\log\Gamma(M)+1-M-\log 2 \tag{7}$$

The direction coefficient of envelope has the value -2, while the value on the ordinate axis depends on number of diversity branches. The envelope determines a certain singular solution of differential equation which can describe the dynamics of this process, while the received signal pdf is its particular solution:

$$p'_{MRC}(z)+p_{MRC}(z)\left(\frac{1}{2\sigma^2}-\frac{M-1}{z}\right)=0 \tag{8}$$

The influence of standard deviation value is presented in Fig 2, showing the translation of the pdf curves family maximums envelope. For higher values of σ, the value on the ordinate axis n_1 is smaller, as shown in Fig 2, while the direction coefficient k_1 has the same value.

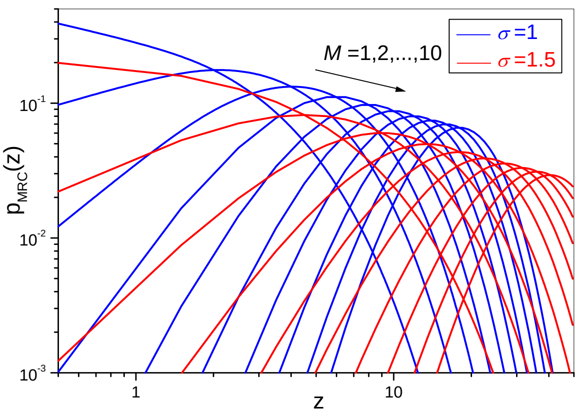

Fig. 2. The received signal pdf versus signal level, in logarithmic scale, for different values of σ, with number of diversity branches taking values from 1 to 10.

Graphic presentation of dependence of Rayleigh pdfs after MRC, versus standard deviation, for a fixed value of the received signal level, in logarithmic scale, for different number of diversity branches is shown in Fig 4.

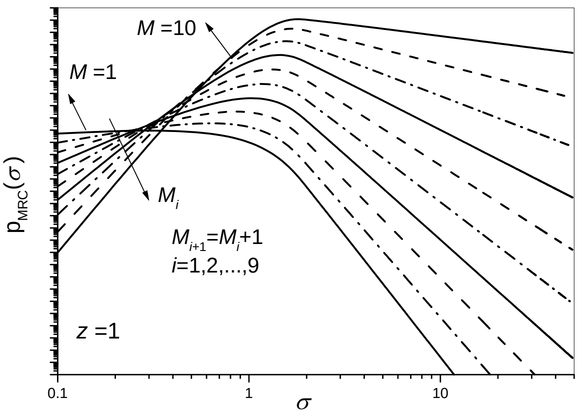

Fig. 3. The received signal pdf versus standard deviation, in logarithmic scale, for fixed values of z, with number of diversity branches taking values from 1 to 10.

The analysis of this dependence shows that the increase of number of diversity branches results in higher values of the pdf maximums, reached for higher values of standard deviation, as shown in Fig 4. In order to determine the values of standard deviation when the maximums are reached, as well as the values of maximums and direction coefficient of envelope, the first derivative of (2) relative to standard deviation power is determined. Equalization the first derivative with zero obtains:

$$\sigma_{p\max}=\sqrt{\frac{z}{2M}} \tag{9}$$

Substituting (9) into (2) yields:

$$p_{MRC,\,\sigma\max}=\frac{M^M}{\Gamma(M)}\cdot\frac{1}{z}\cdot\exp(-M) \tag{10}$$

From (10) we get:

$$\log\left(\max p_{MRC}(z)\right)=k_2\log z+n_2 \tag{11}$$

where the direction coefficient is:

$$k_2=-1 \tag{12}$$

and the value on the ordinate axis is:

$$n_2=M\log(M)-\log\Gamma(M)-M \tag{13}$$

The direction coefficient of envelope has the value -2, while the value on the ordinate axis depends on number of diversity branches. The envelope determines a certain singular solution of differential equation which can describe the dynamics of this process, while the received signal pdf is its particular solution:

$$p'_{MRC}(\sigma)+p_{MRC}(\sigma)\left(\frac{2M}{\sigma}-\frac{z}{\sigma^3}\right)=0 \tag{14}$$

The influence of received signal level is presented in Fig 4, showing the translation of the pdf curves family maximums envelope. For higher values of z, the value on the ordinate axis n_2 is smaller, as shown in Fig 4, while the direction coefficient k_2 has the same value.

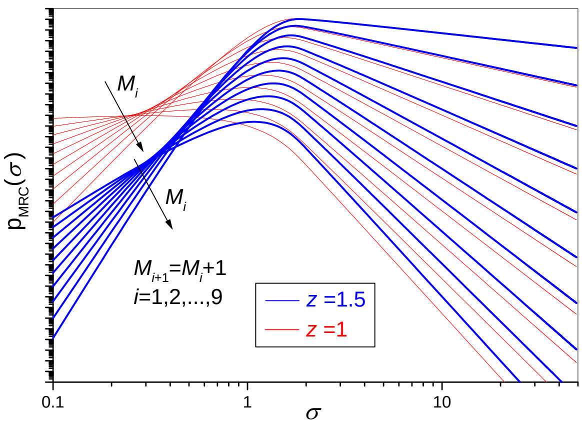

Fig. 4. The received signal pdf versus standard deviation, in logarithmic scale, for different values of z, with number of diversity branches taking values from 1 to 10.

IV. CONCLUSION

This paper presents some properties of the envelope of the pdf curves family in Rayleigh fading channel when MRC receiver is applied. The obtained results show that the position of the maximums of these pdfs is uniquely determined by the maximums envelope's equation, which presents the singular solution of corresponding differential equation describing the complete dynamics of the signal transmission process.

In such a way, the boundary conditions for radio link transmission, with given propagation conditions, could be defined, while some system performance measures could be evaluated, since they are all related to the received signal pdf.

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