Optimal Gear Design By Using Box And Random Search Methods

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ABSTRACT
The development of evolutionary algorithms plays a major role, in recent days, for optimal design of gears, so as to reduce the weight. In this study an optimal weight design (OWD) problem of gear is formulated for constrained bending strength of gear, tortional strength of shafts and each gear dimension as a NIP problem and solved it directly by keeping nonlinear constraint using Box and Random search methods, such that the number of decision {design} variables does not increase and easily get the best compromised solution. An extensive computer program in Java has been written exclusively for their purpose and is successfully used to obtain the optimal gear design.

Keywords **–** *optimal weight design (OWD), NIP problem, Box Method, Random search method, Decision {design} variables.*

1. INTRODUCTION
The most important problem that confronts practical engineers is the mechanical design, a field of creativity. In case of gear design, an infinite number of possible design solutions are found within the overall objective. Any one of these solutions is adequate because it represents a synthesis, which merely satisfies the functional requirements [1]. Here lies a conductive environment for applying cut and try technique to obtain an optimal design solution among the available solutions. The approach to solve certain design problem has so relied on the trail-and-cut methods which, because of their methodology, take considerable time to obtain the optimal solution.

In this study an optimal weight design (OWD) problem of gear is formulated for constrained bending strength of gear, tortional strength of shafts and each gear dimension as a NIP problem and solves it directly by keeping nonlinear constraint by using Box and Random Search Methods.

As a result, the number of decision (design) variables does not increase and easily get the best compromised solution. An extensive computer program in Java has been written exclusively for their purpose and is successfully used to obtain the optimal gear design.

2. ENGINEERING OPTIMIZATION
Optimization is the act of obtaining the best result under given circumstances. In design, construction and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit.

2.1 Optimization Algorithms
2.1.1 Single variable optimization algorithms
These algorithms provide a good understanding of the properties of the minimum and maximum points in a function and how optimization algorithms work iteratively to find the optimum point in a problem. The algorithms are classified into two categories, they are direct methods and gradient based methods. Direct methods do not use any derivative information of the objective function: only objective function values are used to guide the search process. However, gradient based methods use derivative information (first and/ or second order) to guide search process.

2.1.2 Multi – variable optimization algorithms
A number of algorithms for unconstrained, multi-variable optimization problems are present. These algorithms demonstrate how this search for optimum points progress in multiple dimensions.

2.1.3 Constrained optimization algorithms
Constrained optimization algorithms used in single variable and multi variable optimization algorithms repeatedly and simultaneously maintained the search effort inside the feasible search region. These algorithms are mostly used in engineering optimization problems. These algorithms are divided into two broad categories; they are direct search methods and gradient-based methods. In constraint optimization problem, equality constraints make the search process slow and difficult to converge.

2.1.4 Specialized optimization algorithms

There exist a number of structured algorithms, which are ideal for only a certain class of optimization problems. Two of these algorithms are integer programming and geometric programming. These are often used in engineering design problems. Integer programming methods can solve optimization problems with integer design variables. Geometric programming methods solve optimization problems with objective functions and constraints written in a special form.

3. BOX METHOD

The Box method is similar to the simplex method of unconstrained search except that the constraints are handled in the former method. This method was developed by M.J.Box in 1965; [2], the algorithm begins with a number of feasible points created at random. If a point is found to be infeasible, a new point is created using the previously – generated feasible points. Usually, the infeasible point is pushed towards the centroid of the previously found feasible points. Once a set of feasible points is found, the worst point is reflected about the centroid of rest of the points to find a new point, Depending on the feasibility and function value of the new point, the point is further modified or accepted. If the new point falls outside the variable boundaries, the point is modified to fall on the violated boundary. If the new point is infeasible, the point is retracted to towards the feasible points. The worst point in the simplex is replaced by this new feasible point and the algorithm continues for the next iteration. The Box Method is also called as "Complex Search Method".

3.1 BOX [Complex Search] Algorithm [2]

Step 1: Assume a bound in x (x (L), x (U)), a reflection parameter α.

Step 2: Generate an initial set of P (usually 2n) feasible points. For each point

(a) Sample n times to determine the point $x_i^{(p)}$ in the given bound.

(b) If $x^{(p)}$ is infeasible, calculate $\bar{x}$ (centroid) of current set of points and reset

$$x^{(p)} = x^{(p)} + \frac{1}{2}(\bar{x} - x^{(p)})$$

Until $x^{(p)}$ is feasible;

Else if $x^{(p)}$ is feasible, continue with (a) until P points are created

(c) Evaluate $f(x^{(p)})$ for p = 0, 1, 2…, (P-1)

Step 3: Carry out the reflection step:

(a) Select x^R such that

$f(x^R) = \max f(x^{(p)}) = F_{max}$

(b) Calculate the centroi $\bar{x}$ d (of points except x^R) and the next point

$$x^m = \bar{x} + \alpha(\bar{x} - x^R)$$

(c) If x^m is feasible and $f(x^m) \geq F_{max}$ retract half the distance to the centroid $\bar{x}$. Continue until $f(x^m) < F_{max}$
Else if x^m is feasible and $f(x^m) < F_{max}$, go to Step 5.
Else if x^m is infeasible, go to Step 4.

Step 4: Check for feasibility of the solution

(a) For all i, reset violated variable bounds:

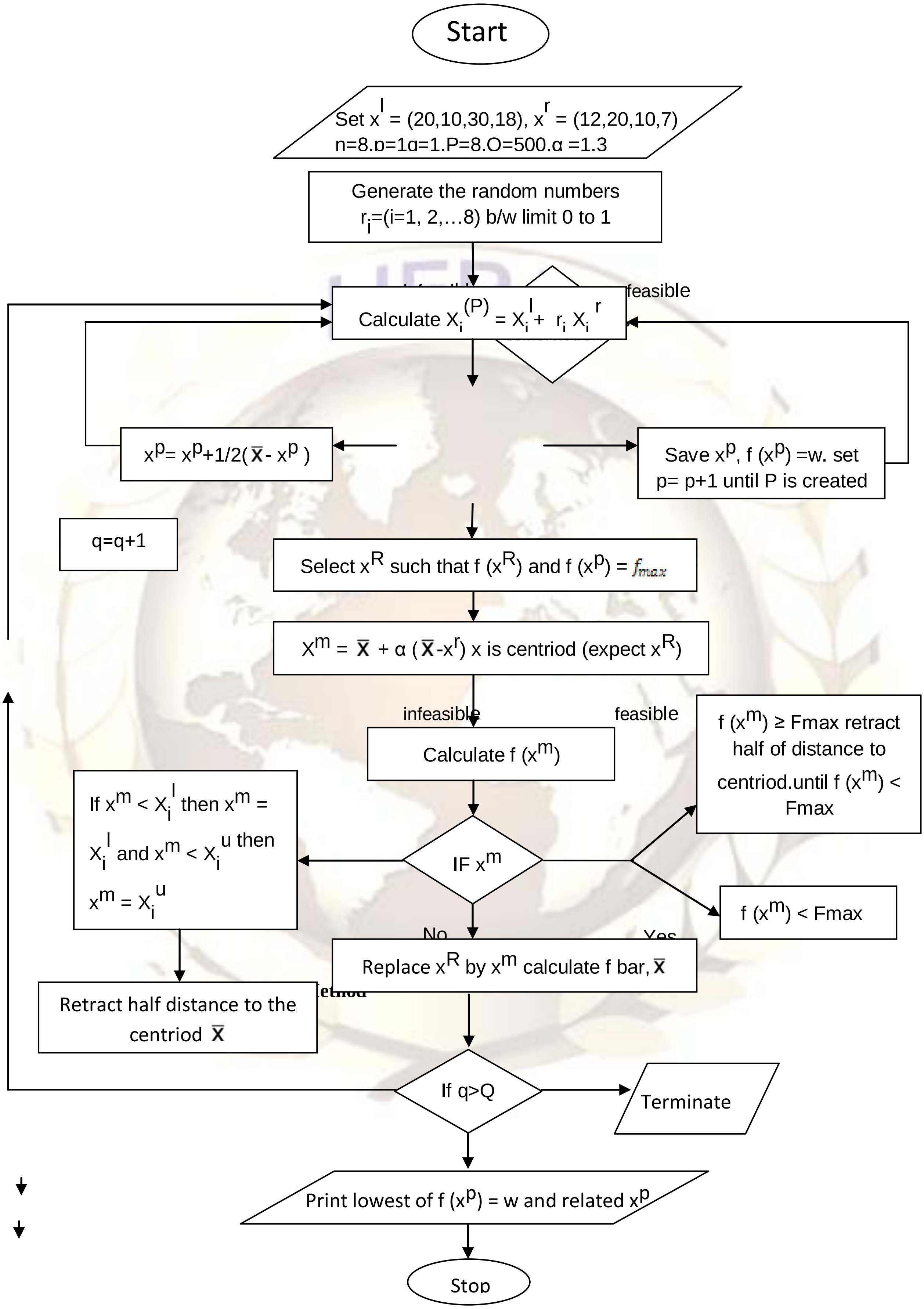
If $x_i^m < x_i^{(L)}$ set $x_i^m = x_i^{(L)}$

If $x_i^m > x_i^{(U)}$ set $x_i^m = x_i^{(U)}$

(b) If the resulting x^m is infeasible, retract half the distance to the centroid. Continue until x^m is feasible. Go to Step 3(c).

Step 5: Replace x^R by x^m. Check for termination.

Calculate $\bar{f} = \frac{1}{P}\sum_p f(x^{(p)})$ and $\bar{x} = \frac{1}{P}\sum_p x^{(p)}$


Start
Set x^l = (20,10,30,18), x^r = (12,20,10,7)
n=8.p=1q=1.P=8.Q=500.α =1.3
Generate the random numbers
r_i=(i=1, 2,…8) b/w limit 0 to 1
feasible
Calculate X_i^(P) = X_i^l + r_i X_i^r
x^p= x^p+1/2(X̄ - x^p)
Save x^p, f (x^p) =w. set
p= p+1 until P is created
q=q+1
Select x^R such that f (x^R) and f (x^p) = f_max
x^m = X̄ + α (X̄ -x^r) x is centriod (expect x^R)
infeasible
feasible
Calculate f (x^m)
f (x^m) ≥ Fmax retract half of distance to centriod.until f (x^m) < Fmax
If x^m < X_i^l then x^m = X_i^l and x^m < X_i^u then x^m = X_i^u
IF x^m
f (x^m) < Fmax
No
Yes
Replace x^R by x^m calculate f bar, X̄
Retract half distance to the centriod X̄
If q>Q
Terminate
Print lowest of f (x^p) = w and related x^p
Stop

4. RANDOM SEARCH METHOD

Like the Box method, the random search method also works with a population of points. But instead of replacing the next point in the population by a point created in a structured manner points are created either at random or by performing a unidirectional search along the random search directions. Here, we describe one such method. Since there is no specific search direction used in the method, random search methods work equally efficiently to many problems. In the Luus and jaakola method 1973; an initial point and an initial interval are chosen at random. Depending on the function values at a number of random points in the interval, the search interval is reduced at every iteration by a constant factor. Then it is increased. In the following algorithm, P points are considered at each iteration and Q such iterations are performed. Thus, if the initial interval in one variable is do and at every iteration the interval is reduced by a factor €, the final accuracy in the solution in that variable becomes (1-€) ^Q do and the required number of function evaluations is P X Q.

4.1. Random Search Algorithm [2]

Step 1 Given an initial feasible point x^0, an initial range z^o such that the minimum, x^*, lies in $(x^0 - \frac{1}{2}z^0, x^0 + \frac{1}{2}z^0)$ Choose the Parameter $0<\in<1$ For each of Q blocks, initially set q = 1 & p = 1.

Step 2 For i = 1, 2 …N, create points using a uniform distribution of r in the range (-0.5, 0.5). Set $x_i^{(p)} = x_i^{q-1} + rz_i^{q-1}$

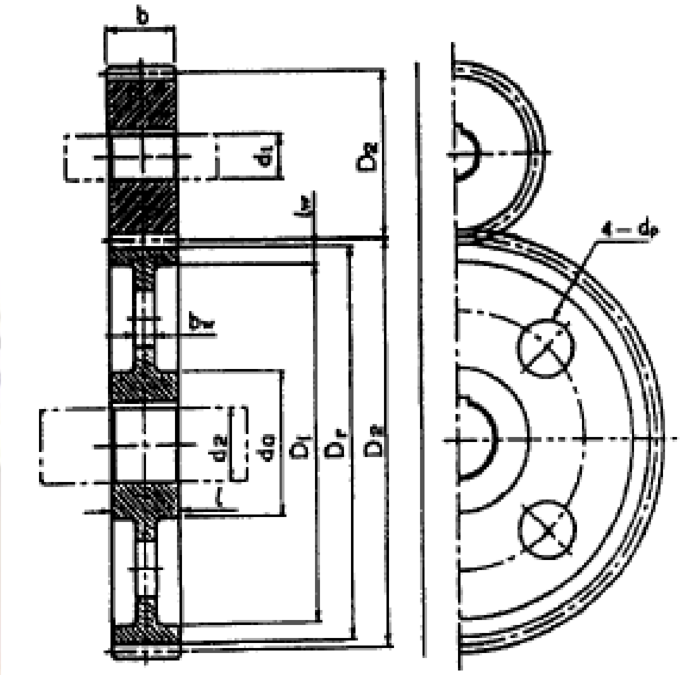
Step 3 If $x^{(p)}$ is infeasible and p < P, repeat Step-2. If $x^{(p)}$ is feasible, save $x^{(p)}$ and $f(x^{(p)})$, increment p and repeat Step-2;
Else if p = P, set x^q to be the point that has the lowest $f(x^{(p)})$, overall feasible $x^{(p)}$ including x^{q-1} and reset p = 1.

Step 4 Reduce the range via $z_i^q = (1-\in)\, z_i^{q-1}$.

Step 5 If q > Q, Terminate;
Else increment q and continue with Step-2.

The suggested values of parameters are € =0.05, P= 5(depending upon the design variables), and Q is related to the desired accuracy in the solution. It is to be noted that the obtained solution is not guaranteed to be the true optimum.

5. PROBLEM DESCRIPTION



In the present work an OWD of a gear with a minimum weight is considered in fig above. Input power of 7.5 KW, the speed of crank shaft gear (pinion) is considered to be 1500 rpm and the gear ratio is 4. Necessary conditions required for developing a mathematical model for gear design are discussed in this section as given in [4].

Preliminary Gear considerations: The following are input parameters required for preliminary gear design [6].

1. Power to be transmitted (H), KW.
2. Speed of the pinion (N1), rpm.
3. Gear ratio (a)

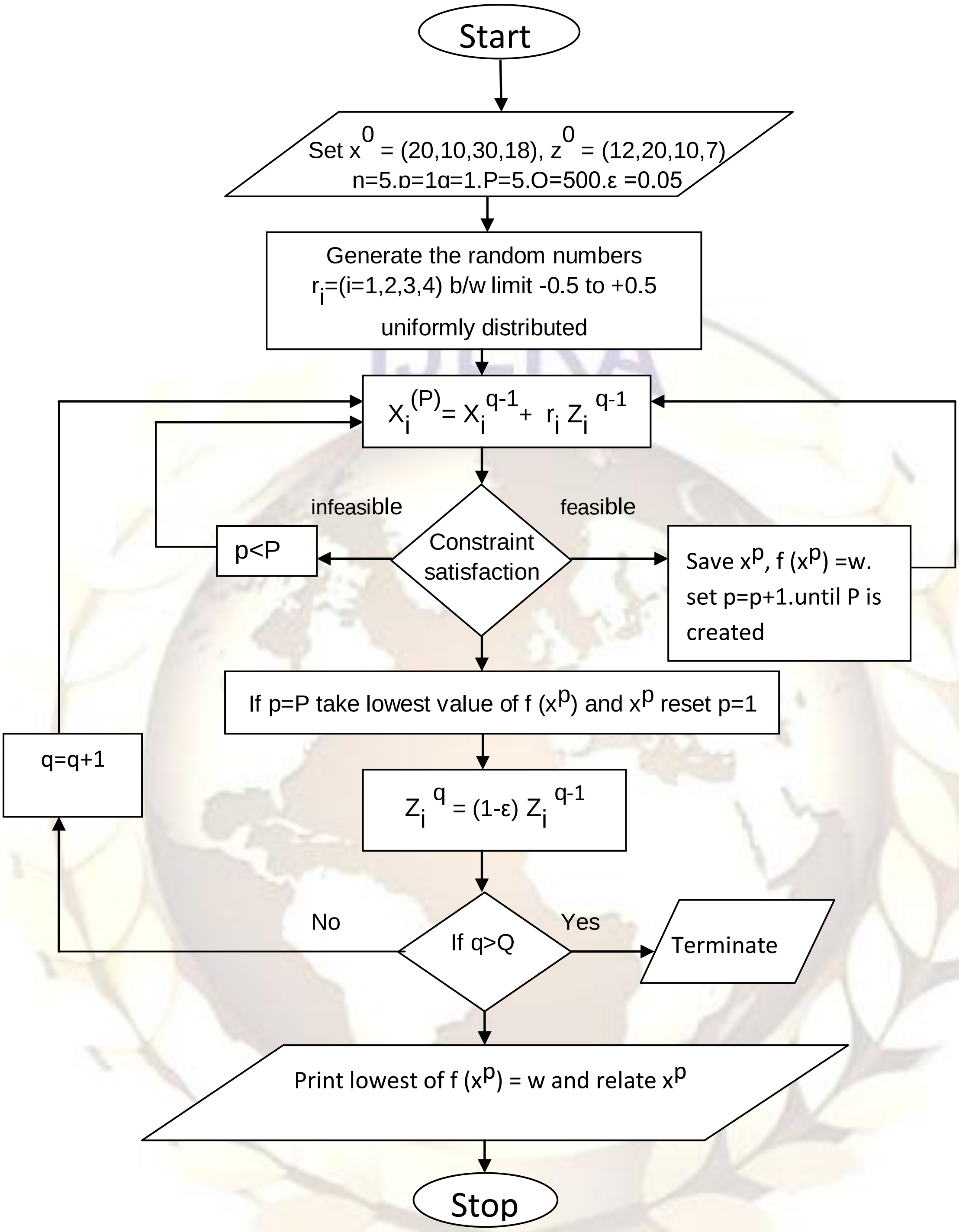


Fig4.1 Flow chart of Random Search Method

The objective function which is used to minimize the weight of gear considered can be expressed as min

$$W = \frac{\prod}{4}\frac{\rho}{1000}\{bm^2 \; z_1^2(1+a^2) - (D_i^2 - d_o^2)(1-b_w) - nd_p^{\;2}bw - (d_1^2 + d_2^2)b\}$$

The decision variables in this respect are face width (b), diameter of pinion (d_1), diameter of gear (d_2), number of teeth on pinion (z_1). Following are the constraints of the objective function. Satisfied bending strength of spur gear, tortional strength of shafts and each gear dimension with a minimized gear weight F (b,d_1,d_2,z_1, m). It is the following NIP problem:

min. F (b,d_1,d_2,z_1, m) =W (1)

subjected to. G_1(b, m) = $F_s \geq b_1$ (2)

G_2(b, m,z_1) = F_s /$F_p \geq b_2$ (3)

G_3 (d_1) = $d_1^3 \geq b_3$ (4)

$G_4(d_2)$ = d_2^3 ≥ b_4 (5)

G_5(∝, z_1) = (1+a) mz_1/2 ≤ b_5 (6)

D_r= m (az_1-2.5), l_w = 2.5m, D_i = D_r-2l_w, b_w= 3.5m, d_0 = d_2 +25, d_p= 0.25(D_i-d_0), F_s= π$f_v f_w$σbmy, F_p = $f_v f_w D_1$b (2z_1/z_1+z_2), D_1 = mz_1, D_2 = amz_1, N_2= N_1/a, z_2= $z_1 D_2$ /D_1, v = π$D_1 N_1$/60, b_1=102H/v, b_3 = 4.97×10^6 H/N_1×γ, b_4= 4.97×10^6H/ N_2×γ.

20≤ b ≤32	10≤ d_1 ≤30
30≤ d_2 ≤40	m = 2.75,3,3.5
18≤ z_1 ≤25	a =4
H=7.5	N_1 = 1500
σ =30	γ =2
b_2 = 0.193	f_v= 0.389
f_w= 0.8	y = 0.102
ρ =8	n =6

Table 5.1: The range of Variable and coefficient values

Where F (b,d_1,d_2,z_1) is the weight function, b and d_1/ d_2 are face width and diameter of pinion/gear shaft, respectively. z_1 and m are number of teeth in pinion and module, respectively, W is weight of gears. σ is allowable stress of gear 'a' is gear ratio. G_1 is bending strength of teeth (F_s: Lewis formula, pinion). G_2 is surface durability(k) .$F_p$is wear load. b_i (i=1 ……5) are constraint quantity each other. D_r is dedendum circle,D_i , d_0 and l are inside diameter of rim, out diameter of boss and length of boss. b_w, d_p and n are thickness of web, drill holl diameter and number of d_p. ρ is density of gear. f_v/ $f_w$is velocity/load factor. y is form factor. G_3 /$G_4$is cubic diameter of pinion/gear shaft, respectively. G_5 is distance between the axes ©.(b^L,d_1^L,d_2^L) and (b^U,d_1^U,d_2^U) are lower/upper limit values of the design variable, respectively. D_1/D_2 and N_1/N_2 are pitch diameter of pinion/gear and speed of pinion/gear. z_2 is number of teeth on gear. v, H and γ are pitch line velocity, input power and allowable shearing stress of shafts, respectively.

5.1. Application of Box [Complex Search] Method

Here first created random numbers depending upon the design variables (P=2n). 'n' is the number of design variables. Here 8 design variables are r1, r2.....r8.The random numbers are created between 0 to 1, after that determined $x_i^{(p)}$ in the given bound. Initial point must be feasible and then calculated $x_i^{(p)}$. It must satisfy all constraints. If it is infeasible and then calculate centroid of current set of points and reset $x^{(p)} = x^{(p)} + \frac{1}{2}(\bar{x} - x^{(p)})$ Until $x^{(p)}$ is feasible; Calculate "W" for P points, and take minimum of "W" of these points. Take the maximum" value of above set of points it is marked asF_{max}, and then calculate $x^m = \bar{x} = \alpha(\bar{x} - x^R)$. x^R means the worst points related to the maximum "W" value. Keep this x^m in "W", it should be less than maximum "W", if it is greater than "W" retract the half distance to the centroid until it is less than "W". If x^m feasible calculate "W" related to "x=[b,d1,d2,z1]".In case if x^m is infeasible check for feasibility of the solution if the design variables are out of the boundary set for with in the boundary limits. If the resulting x^m is also infeasible retract half the distance to the centroid. Continue until x^m if feasible and keep this x^m in "W" and take least "W" value, this completes one iteration. Else set k=k+1 by doing 500 iterations take least "W" of this iterations related to "x" value.

5.2. Application of Random Search Method [RSM]

Here first create random numbers depending upon the decision variables .Here 4 design variables so random numbers (r1, r2, r3, r4).The random numbers are created between(-0.5to+0.5)limit. It is uniformly distributed. After that we have to find $x_i^{(p)} = x_i^{q-1} + rz_i^{q-1}$ where (i=1, 2…4).If $x^{(p)}$ is infeasible and p < P, repeat finding $x_i^{(p)} = x_i^{q-1} + rz_i^{q-1}$ If x $^{(p)}$ is feasible, save x $^{(p)}$ and f(x $^{(p)}$), increment p and repeat same procedure until p=P. Take minimum of value of 'W' (i.e. f(x $^{(p)}$) and reset p=1. Reduce the range via $z_i^q = (1-\in)\ z_i^{q-1}$.Repeat the procedure until q>Q. Terminate; Else increment q and continue the procedure and calculate 'W' value by doing 500 iterations we will take least value of 'W' and corresponding x = [b,d_1,d_2,z_1].

6. DISCUSSION ON THE RESULTS

Here the main objective is to minimize the weight. For that Box and Random Search Methods are used. The gear module (m) values considered are 2.75mm; 3mm and 3.5mm.These have been compared with those of literature and incorporated in tables 7.1 to 7.3. By observing the tabulated values it is found that Random search method gives better results than the Box method.

Box method and Random search method [RSM] are applied to the OWD problem of the gear. The results obtained by both the methods are compared with that available in literature [4]. Among the three methods the Random search method is found to be giving good results for the problem considered can be effectively applied for single stage gear design problem. From the tables 7.1 to 7.3 even though the results of [4] give minimum values the variables are violated the constraints. Therefore the solutions presented are not feasible solutions. Hence RSM is found to be best method.

7. COMPARISON OF RESULTS

For Module m=2.75

	BY BOX METHOD	BY RANDOM SEARCH METHOD	BY LITERA TURE
WEIGHT	7560.98	7077.23	3512.6
Face width: b	26.69	23.94	24
Diameter of pinion shaft: d_1	30.0	29.88	30
Diameter of gear shaft: d_2	40.0	39.99	30**
Number of teeth(pinion): z_1	20.91=(21)	18	18
Number of teeth(gear): z_2	83.64=(84)	72	72
Module: m	2.75	2.75	2.75
Pitch circle(pinion): D_1	57.50	49.5	49.5
Pitch circle(gear): D_2	230.01	198	198
Between the axes: C	143.75	123.75	123.75
Surface durability: k	0.287	0.3747	0.374
Dedendum circle(gear): D_r	223.135	191.1	191.1
Inside diameter of rim: D_i	209.385	177.4	177.4
Thickness of web: b_w	9.625	9.63	9.63
Outside diameter of boss: d_0	65	64.99	55
Drill holls: d_p	36.09	28.10	30.6

** indicates constrained violation

Table: 7.1

For Module m=3

	BY BOX METHOD	BY RANDOM SEARCH METHOD	BY LITERATURE
WEIGHT	7560.98	7077.23	3512.6
Face width: b	26.69	23.94	24
Diameter of pinion shaft: d_1	30.0	29.88	30
Diameter of gear shaft: d_2	40.0	39.99	30**
Number of teeth(pinion): z_1	20.91=(21)	18	18
Number of teeth(gear): z_2	83.64=(84)	72	72
Module: m	2.75	2.75	2.75
Pitch circle(pinion): D_1	57.50	49.5	49.5
Pitch circle(gear): D_2	230.01	198	198
Between the axes: C	143.75	123.75	123.75
Surface durability: k	0.287	0.3747	0.374
Dedendum circle(gear): D_r	223.135	191.1	191.1
Inside diameter of rim: D_i	209.385	177.4	177.4
Thickness of web: b_w	9.625	9.63	9.63
Outside diameter of boss: d_0	65	64.99	55
Drill holls: d_p	36.09	28.10	30.6

** indicates constrained violation

Table: 7.2

For Module m=3.5

	BY BOX METHOD	BY RANDOM SEARCH METHOD
WEIGHT	10033.21	7111.95
Face width: b	25.82	23.94
Diameter of pinion shaft: d_1	30.0	26.55
Diameter of gear shaft: d_2	40.0	39.52
Number of teeth(pinion): z_1	21.98=(22)	18
Number of teeth(gear): z_2	87.92=(88)	72
Module: m	3.5	3.5
Pitch circle(pinion): D_1	76.93	63
Pitch circle(gear): D_2	307.72	252
Between the axes: C	192.325	157.5
Surface durability: k	0.273	0.333
Dedendum circle(gear): D_r	298.97	243.25
Inside diameter of rim: D_i	281.47	225.75
Thickness of web: b_w	12.25	63
Outside diameter of boss: d_0	65	64.52
Drill holls: d_p	54.11	40.30

Table: 7.3

8. CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

The gear is one of the machine elements. It transmits power with accuracy to parallel shafts, skew shafts and intermittent action gear etc. Therefore it has various uses in industrial production. When designing a gear usually the trail and cut methods are used to determine factors such as input power, rotation frequency, transmission ratio, bending strength of the gear, tortional strength of shafts and each gear dimension. However, this method does not include the method of optimal weight design [4]. The mathematical model of an optimal weight design problem of gear for minimizing objective functions includes the above mentioned design factors.

Box method and Random search method [RSM] are applied to the OWD problem of the gear. Example taken in this study is a spur gear. The results obtained by both the methods are compared with that available in literature [4]. Among the three methods the Random search method is found to be giving good results for the problem considered can be effectively applied for single stage gear design problem. From the tables 7.1 to 7.3 even though the results of [4] give minimum values the variables are violated the constraints. Therefore the solutions presented are not feasible solutions. Hence RSM is found to be best method. As a result the minimum weight of the gear considered using RSM is 7077.23.

This study can be extended using other methods like cutting plane method and feasible direction method to get faster and better values. And also the BOX and RSM Algorithms can be applied for designing optimization of the mechanical elements.

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