

Effect of Chemical Reaction on Convective Instability in a Horizontal Porous Layer Saturated with a Couple-Stress Fluid

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ABSTRACT

The problem of onset of convective instability in a horizontal inert porous layer saturated with a couple-stress fluid subject to zero-order chemical reaction is investigated by the method of small perturbation. Modified Darcy-couple-stress model is used to describe the fluid motion. The horizontal porous layer is cooled from the upper boundary while an isothermal boundary condition is imposed at the lower boundary. Closed form solution pertaining to the equilibrium state is obtained. The resulting eigenvalue problem, subject to realistic flow boundary conditions, is solved approximately using the Galerkin method. The media Darcy-Rayleigh number, characterizing the stability of the system, is calculated as a function of the Frank-Kamenetskii number, wavenumber and the couple-stress parameter. It is found that the destabilizing effect of chemical reaction on the system is more pronounced and the aspect ratio of convection cells becomes uniform provided that the couple-stress parameter is large.

Keywords - Chemical reaction, Couple stress, Galerkin method, Natural convection, Porous media, Stability.

I. INTRODUCTION

In hydrodynamic stability, experiments have led to theory all along and the main source of arriving at a mathematical breakthrough is to have a feeling for the right result that may have been suggested from nowhere or through the application of intuitive reasoning based on experience and observation. Conventional hydrodynamic stability theory is mainly concerned with the determination of critical values of Rayleigh number and it demarcates a region of stability from that of instability. Right from the conceptualizations of turbulence, instability of fluids is being regarded at its root. Thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in geophysics, interiors of the Earth, oceanography, atmospheric physics, and so forth. A comprehensive account of thermal instability in a viscous fluid layer under varying assumptions of hydrodynamics has been

summarized in the celebrated monograph by Chandrashekar [1]. The Boussinesq approximation has been used throughout, which states that the density changes are disregarded in all other terms in the equation of motion except in the external force term.

Thermal convective instability in a layer of porous medium has merited extensive attention over the years and is now emerged as an important field of study in the general area of fluid dynamics and heat transfer. The formulation and derivation of the basic equation of a layer of fluid heated from below in a porous medium using Boussinesq approximation has been given in a treatise by Joseph [2]. When a fluid flows in an isotropic and homogenous porous medium, the gross effect is represented by Darcy's law. The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamicists and has importance in chemical technology and industry. The rotation of the Earth distorts the boundaries of a hexagonal convection cells in a fluid through a porous medium and this plays an important role in the extraction of energy in the geothermal regions. The growing volume of work pertaining to this field is well documented by Ingham and Pop [3], Vafai [4] and Nield and Bejan [5].

Buoyancy driven convection of reacting fluids within porous media occurs during oxidation of solid materials in large containers or the synthesis of ceramic material. Moreover, free convection within porous medium can remove heat from radioactive waste products or delay the thermal explosion of coal piles or waste dumps. When an exothermic reaction takes place in a fluid-saturated porous medium, the heat generated by the reaction changes the fluid density and free convection may occur. The induced natural convection, in turn, affects the rate of heat release by the reaction.

Kordylewski and Krajewski [6] were the first to perform a stability analysis based on Darcy's law with the Boussinesq approximation and a zero-order exothermic reaction. Farr *et al.* [7] carried out stability analysis on free convection in confined porous medium with zero-order exothermic reactions. Vilijoen and Hlavacek [8] and

Subramanian and Balakotaiah [9] carried out numerical studies of systems under similar conditions. Malashetty *et al.* [10] performed a linear stability analysis to study the onset of convective instability in a horizontal inert porous layer saturated with a fluid undergoing a zero-order exothermic chemical reaction. McKay [11] studied the onset of buoyancy driven convection in superposed reacting fluid and porous layers. It is found that, with chemical reactions, the fluid in the porous medium is more prone to instability as compared to the case in which chemical reactions are absent. More recently, Akbar *et al.* [12] investigated the problem of onset of convective instability in a horizontal inert porous layer saturated with a Maxwell viscoelastic fluid subject to zero-order chemical reaction by linear stability analysis. The possibility of oscillatory instability, which is inherent in viscoelastic fluid convection, is discussed.

Newtonian fluids cannot precisely describe the characteristics of fluid flow in many practical problems. Owing to the development of modern machine apparatus, the increasing use of fluids containing a microstructure such as those containing additives, suspensions, granular matter or long-chained polymers has been emphasized. These fluids deform and produce a spin field due to the microrotation of suspended particles forming a micropolar fluid. The theory of micropolar fluids was developed by Eringen [13] which takes care of local effects arising from the microstructure as well as the intrinsic motion of microfluidics. The spin field due to microrotation of the freely suspended particles sets up an anti-symmetric stress known as couple stress. The couple stress effects are considered as a consequence of the action of a deforming body on its neighbourhood. The couple-stress fluid is a special case of micropolar fluid when microrotation balances with the natural vorticity of the fluid. Since flow behaviour of these kinds of non-Newtonian fluids cannot be described accurately by the classical continuum theory, many micro-continuum theories have been proposed [14-16]. Stokes micro-continuum theory is the simplest theory of fluids that allows for the presence of couple stresses, body couples and non-symmetric tensors. The governing equations of the couple-stress fluid flows are similar to the classical Navier-Stokes equations with an increase in the order of the equation by two. The structure of the equations facilitates a comparison with the results for the classical Newtonian fluid which is nonpolar. This couple-stress fluid model has implications for fluids such as animal blood, liquid crystals, polymer-thickened oils, synthetic fluids, synovial fluids present in synovial joints and theory of lubrication.

A good number of fluid flow problems that are present in the realm of viscous fluid theory have

been investigated in the context of the couple-stress fluid theory in the last couple of decades. Srivastava [17] investigated the problem of peristaltic transport of a couple-stress fluid under a zero Reynolds number and long wavelength approximation. Since the long chain hyaluronic acid molecules are found as additives in synovial joints, Walicki and Walicka [18] modeled synovial fluid as couple-stress fluid in human joints. Sharma and Sharma [19] considered the thermal instability of a couple-stress fluid with suspended particles. The magnetic field and rotation are found to have stabilizing effects on the stationary convection and introduce oscillatory modes in the system. Malashetty *et al.* [20] analysed the onset of double diffusive convection in a couple-stress fluid saturated horizontal porous layer using linear and weak nonlinear stability analyses. Sarvanan and Premalatha [21] studied the effect of couple stress on the onset of thermo-vibrational convection in a porous medium. These diverse flow problems indicate the continuing interest among researchers in couple-stress fluid flows.

In the present paper, the convective instability of a chemically reacting couple-stress fluid in a porous medium heated from below is investigated using modified Darcy model. We assume that the fluid is undergoing a zero-order exothermic chemical reaction and that there exists a local thermal equilibrium between the fluid and the solid phases. Closed form solution for temperature distribution of basic undisturbed state corresponding to the thermal boundary condition is first obtained and the ignition conditions are indicated. A linear stability analysis is then performed and the resulting eigenvalue problem is solved by the Galerkin technique.

II. MATHEMATICAL FORMULATION

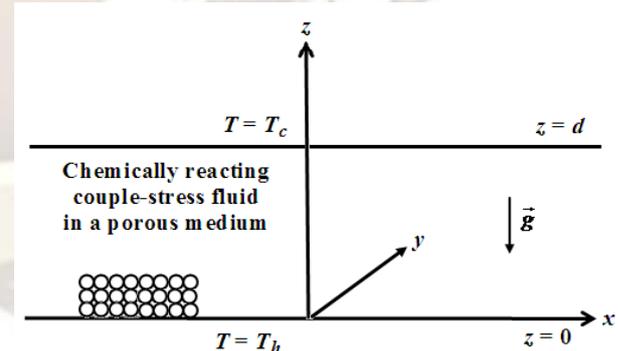


Figure 1. Physical configuration.

We consider a horizontal constant porosity layer of finite thickness bounded between $z = 0$ and $z = d$ (with z -axis directed vertically upward) and of infinite extent in the horizontal xy plane. The inert porous layer is saturated with a chemically reactive couple-stress fluid subject to weakly exothermic chemical reactions and is cooled from the top at a temperature of T_c . If the temperature in the whole

domain of interest varies slightly from T_c , a zero-order reaction can be assumed. Moreover, it is assumed that local thermal equilibrium exists between the solid matrix and the saturated fluid. The system of equations describing the problem under consideration is the following:

$$\nabla \cdot \mathbf{q} = 0 \quad (1)$$

$$\frac{\rho_0}{\varepsilon} \left[\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon} (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p + \rho \mathbf{g} - \frac{1}{k} (\mu - \mu_c \nabla^2) \mathbf{q} \quad (2)$$

$$M \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \kappa \nabla^2 T + Q \exp\left(\frac{-E}{RT}\right) \quad (3)$$

$$\rho = \rho_0 [1 - \beta(T - T_c)] \quad (4)$$

where $\mathbf{q} = (u, v, w)$ is the mean filter velocity, t the time, p the pressure, ρ the fluid density, ρ_0 the reference density, \mathbf{g} the acceleration due to gravity, μ the fluid viscosity, ε the porosity, k the permeability of the porous medium, κ the effective thermal diffusivity, μ_c the couple-stress viscosity, M the ratio of the specific heat of the solid due to porous medium and that of the fluid at constant pressure, β the coefficient of thermal expansion, T the temperature, Q the product of the heat of reaction, a pre-exponential factor and reactant concentration, E the activation energy, R the universal gas constant, ∇ the vector differential operator and (x, y, z) are the spatial coordinates.

The thermal boundary conditions are given by

$$T(x, y, d) = T_c \quad (5)$$

$$T(x, y, 0) = T_h \quad (6)$$

where $T_h > T_c$. We assume that the couple-stress fluid in the porous medium is subject to high-activation energy. With this approximation, Eq. (3) can be simplified to

$$M \frac{\partial \theta}{\partial t} + (\mathbf{q} \cdot \nabla) \theta = \kappa \nabla^2 \theta + \delta \exp(\theta) \quad (7)$$

where $\delta = \frac{Q \exp\left(-\frac{E}{RT_c}\right)}{T_r}$, $\theta = \frac{T - T_c}{T_r}$,

$$T_r = \frac{RT_c^2}{E} \text{ with } T_r \text{ being the prescribed reference temperature for a reacting fluid. Eqs. (5) and (6), in terms of } \theta, \text{ reduce to}$$

$$\theta = 0 \text{ at } z^* = 1 \quad (8)$$

$$\theta = \theta_h \text{ at } z^* = 0 \quad (9)$$

where $\theta_h = \frac{T_h - T_c}{T_r}$ and the asterisk denotes a dimensionless quantity.

At an undisturbed state, the variables are taken to be

$$\mathbf{q} = (u, v, w) = (0, 0, 0), \theta = \theta_b(z), p = p_b(z), \rho = \rho_b(z). \quad (10)$$

The quiescent state solutions are therefore given by the following equations

$$\frac{dp_b}{dz} - \rho_b g = 0 \quad (11)$$

$$\rho_b = \rho_0 [1 - \beta T_r \theta_b] \quad (12)$$

$$\kappa \frac{d^2 \theta_b}{dz^2} + \delta \exp(\theta_b) = 0. \quad (13)$$

Eq. (13) can be recast in the dimensionless form

$$\frac{d^2 \theta_b}{dz^{*2}} = -F \exp(\theta_b) \quad (14)$$

where $F = \frac{\delta d^2}{\kappa}$. The dimensionless number F is

the Frank-Kamenetskii number. On integration, Eq. (14) leads to (after suppressing the asterisk) the following general solution

$$\theta_b = \log\left(\frac{C_1}{2F}\right) + \log\left[1 - \left(\frac{1 - C_2 \exp(-\sqrt{C_1} z)}{1 + C_2 \exp(-\sqrt{C_1} z)}\right)^2\right] \quad (15)$$

where C_1 and C_2 are the integration constants to be determined. Application of the following boundary conditions

$$\theta_b = \theta_h \text{ at } z = 0 \text{ and } \theta_b = 0 \text{ at } z = 1 \quad (16)$$

gives C_1 implicitly through the following equation

$$\exp(\sqrt{C_1}) \left[\frac{1 - \sqrt{1 - \frac{2F}{C_1}}}{1 + \sqrt{1 - \frac{2F}{C_1}}} \right] = \left[\frac{1 - \sqrt{1 - \frac{2F \exp(\theta_h)}{C_1}}}{1 + \sqrt{1 - \frac{2F \exp(\theta_h)}{C_1}}} \right] \quad (17)$$

and C_2 by the relation

$$C_2 = \exp(\sqrt{C_1}) \left[\frac{1 - \sqrt{1 - \frac{2F}{C_1}}}{1 + \sqrt{1 - \frac{2F}{C_1}}} \right]. \quad (18)$$

The reaction in the fluid is self-sustainable and the nature of the lower boundary changes to adiabatic beyond a certain value of the Frank-Kamenetskii number F . We consider only Frank-Kamenetskii numbers below this critical value. The critical value of the Frank-Kamenetskii number F is found to be 0.878455 when $\theta_h = 1.19$. Computations for C_1 and C_2 were performed for selected values of θ_h and for different values of F

up to its ignition value. The results of these computations for $\theta_h = 1$ are presented in Fig. 2. It is seen that, for small values of F , the basic temperature profile is well-nigh linear. However, the basic temperature profile turns out to be more and more nonlinear as the value of F is increased in that the heat generated due to the chemical reaction also increases.

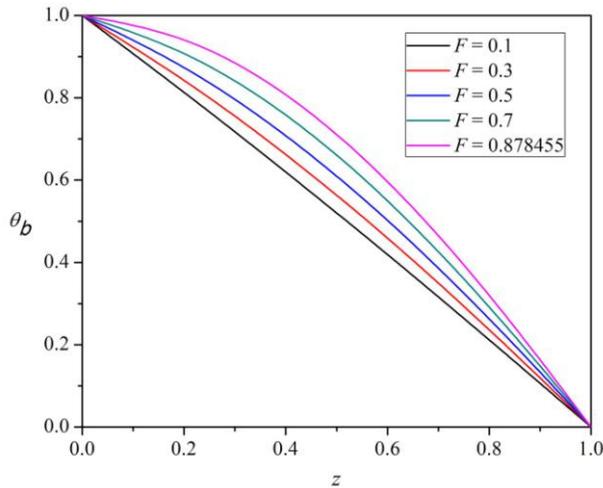


Figure 2. Basic temperature profile for different values of F and $\theta_h = 1$.

III. STABILITY ANALYSIS

We now perform a linear stability analysis by letting

$$\mathbf{q} = \mathbf{q}_b + \mathbf{q}' = (u', v', w'), \quad p = p_b + p', \quad (19)$$

$$\rho = \rho_b + \rho', \quad \theta = \theta_b + \theta'$$

where the primes indicate infinitesimally small perturbations. On substituting (19) into Eqs. (1), (2), (4) and (7), neglecting the nonlinear terms, incorporating the quiescent state solutions and eliminating the pressure term, we obtain the following equations

$$\frac{\rho_0}{\varepsilon} \frac{\partial}{\partial t} (\nabla^2 w') = \beta \rho_0 g T_r \nabla_1^2 \theta' - \frac{\mu}{k} \nabla^2 w' + \frac{\mu_c}{k} \nabla^4 w' \quad (20)$$

$$M \frac{\partial \theta'}{\partial t} + \frac{d\theta_b}{dz} w' = \kappa \nabla^2 \theta' + \delta \exp(\theta_b) \theta' \quad (21)$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. According to the normal

mode analysis, convective motion is assumed to exhibit horizontal periodicity [1]. Then the perturbed quantities can be expressed as

$$\begin{bmatrix} w' \\ \theta' \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \end{bmatrix} \exp[i(lx + my) + \sigma t] \quad (22)$$

where l and m are the wavenumbers in the x and y directions respectively and σ is the growth rate. Substitution of (22) into Eqs. (20) and (21) leads to the following equations

$$\frac{\rho_0}{\varepsilon} \sigma (D^2 - k_h^2) W = -\beta \rho_0 g T_r k_h^2 \Theta - \frac{\mu}{k} (D^2 - k_h^2) W + \frac{\mu_c}{k} (D^2 - k_h^2)^2 W \quad (23)$$

$$M \sigma \Theta + \frac{d\theta_b}{dz} W = \kappa (D^2 - k_h^2) \Theta + \delta \exp(\theta_b) \Theta \quad (24)$$

where $D = \frac{d}{dz}$ and $k_h^2 = l^2 + m^2$. Non-dimensionalizing Eqs. (23) and (24) using the transformations

$$W^* = \frac{d}{\kappa} W, \quad \sigma^* = \frac{\sigma M d^2}{\kappa} \sigma, \quad \alpha = k_h d, \quad z^* = \frac{z}{d} \quad (25)$$

we obtain (after suppressing the asterisks) the equations in the following dimensionless form

$$\left(\frac{\sigma}{M Va} \right) (D^2 - \alpha^2) W = -Ra \alpha^2 \Theta - (D^2 - \alpha^2) W + \Gamma (D^2 - \alpha^2)^2 W \quad (26)$$

$$\sigma \Theta + \left(\frac{d\theta_b}{dz} \right) W = (D^2 - \alpha^2) \Theta + F \exp(\theta_b) \Theta \quad (27)$$

where $Va = \frac{\varepsilon \mu d^2}{\rho_0 k \kappa}$ is the Vadasz number (the so-

called Darcy-Prandtl number), $Ra = \frac{\beta \rho_0 g T_r d k}{\mu \kappa}$ is

the media Darcy-Rayleigh number and $\Gamma = \frac{\mu_c}{\mu d^2}$

is the couple-stress parameter. Noting that the principle of exchange of stabilities is valid [10, 20], we arrive at the following stability equations

$$\left[1 - \Gamma (D^2 - \alpha^2) \right] (D^2 - \alpha^2) W + Ra \alpha^2 \Theta = 0 \quad (28)$$

$$(D^2 - \alpha^2) \Theta + F \exp(\theta_b) \Theta - \frac{d\theta_b}{dz} W = 0. \quad (29)$$

The realistic flow boundary conditions are

$$W = DW = \Theta = 0 \quad \text{at } z = 0, 1. \quad (30)$$

It is no longer possible to obtain a closed form solution to the system of equations (28) – (30) inasmuch as the differential equation (29) consists of variable coefficients. We therefore employ a numerical method and the relevant details of it are given in the following section.

IV. METHOD OF SOLUTION

The system comprising Eqs. (28) and (29) and the homogeneous boundary conditions (30) is

an eigenvalue problem, with Ra being the eigenvalue. An approximate solution of this eigenvalue problem can be obtained by the well-known Galerkin method [22]. To this end, we let

$$W = \sum A_i W_i, \quad \Theta = \sum B_i \Theta_i \quad (31)$$

where A_i and B_i are constants and W_i and Θ_i are trial functions. Appealing to the Galerkin method, one obtains the following system of homogenous algebraic equations

$$\left. \begin{aligned} C_{ji} A_i + D_{ji} B_i &= 0 \\ E_{ji} A_i + F_{ji} B_i &= 0 \end{aligned} \right\} \quad (32)$$

where

$$C_{ji} = \Gamma \left[\langle W_j D^4 W_i \rangle + \alpha^4 \langle W_j W_i \rangle - 2\alpha^2 \langle W_j D^2 W_i \rangle \right] - \left[\langle W_j D^2 W_i \rangle - \alpha^2 \langle W_j W_i \rangle \right]$$

$$D_{ji} = -Ra \alpha^2 \langle W_j \Theta_i \rangle, \quad E_{ji} = \langle \Theta_j \frac{d\theta_b}{dz} W_i \rangle$$

$$F_{ji} = - \left[\langle \Theta_j D^2 \Theta_i \rangle - \alpha^2 \langle \Theta_j \Theta_i \rangle + F \langle \Theta_j \exp(\theta_b) \Theta_i \rangle \right]$$

$$\text{and } \langle f g \rangle = \int_0^1 f g dz.$$

The following trial functions are chosen guided by the boundary conditions and variational considerations

$$\left. \begin{aligned} W_i &= z^{i+1} - 2z^{i+2} + 3z^{i+3} \\ \Theta_i &= z^i - z^{i+1} \end{aligned} \right\} \quad (33)$$

V. RESULTS AND DISCUSSION

The problem of Rayleigh-Bénard convection in a couple-stress fluid saturated densely packed porous medium with chemical reaction is studied using method of small perturbation. Closed form solution for the basic quiescent state is first obtained. The resulting eigenvalue problem is solved numerically by the Galerkin technique. The convergence of the results is achieved, in general, by considering three or four terms in the series expansion of Eq. (31). The error in the approximate solutions estimated by the difference between the solution with three trial functions and that with four trial functions is about 0.01%. The results obtained for fourth order are presented graphically through Figs. 3 and 4. For the problem under consideration it is well known that stationary instability is preferred to oscillatory instability. It is well known that rigid-rigid boundaries offer most stabilizing effect against the fluid motion and the least suppression is offered by free-free boundaries [1].

Fig. 3 depicts the variation of critical media Darcy-Rayleigh number Ra_c as a function of the couple-stress parameter Γ for different values of the Frank-Kamenetskii number F . Physically the

parameter F is the ratio of the characteristic flow time to the characteristic reaction time. We observe from this figure that Ra_c decreases with an increase in F indicating that the effect of chemical reaction is to advance the onset of convection. This feature of chemical reaction on the onset of convection should be comprehended in conjunction with the implications of the basic temperature profile for the stability of the system. The couple-stress parameter Γ is indicative of the concentration of suspended particles. We find that Ra_c increases with an increase in Γ and hence its effect is to reinforce the stability the system. Furthermore, it is worth noting that the destabilizing effect of chemical reaction is more pronounced when Γ is large.

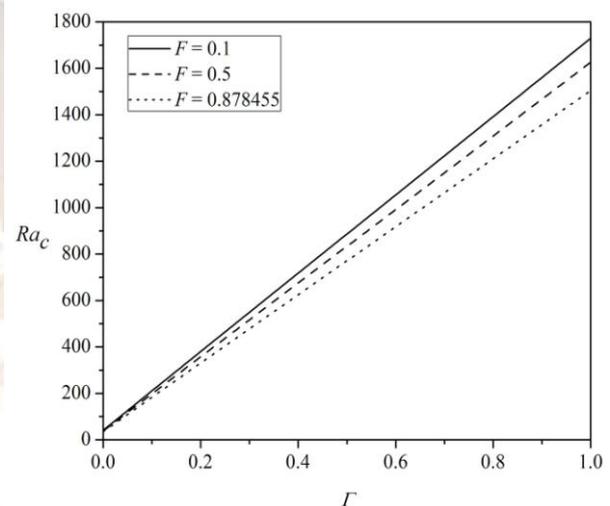


Figure 3. Plot of Ra_c as a function of the couple-stress parameter Γ for different values of F .

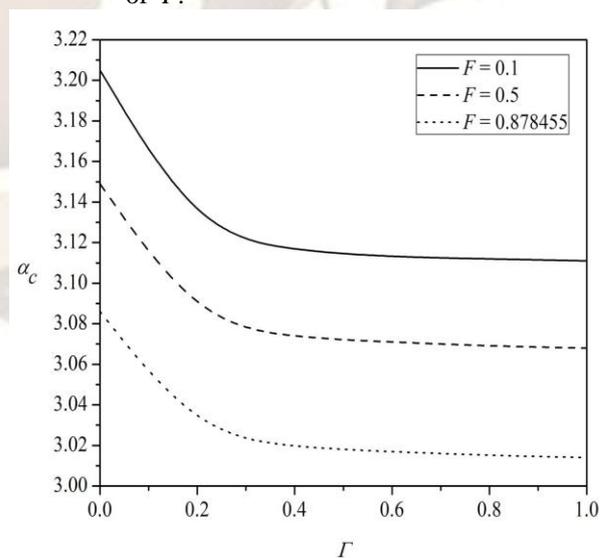


Figure 4. Plot of α_c as a function of the couple-stress parameter Γ for different values of F .

Fig. 4 shows the variation of critical wavenumber α_c with the couple-stress parameter Γ for a range of values of the Frank-Kamenetskii number F . It is seen that the critical wavenumber, characterizing the size of convection cell pattern at the onset of instability, is sensitive to the variation in Γ and F . Indeed, as both Γ and F increase, the critical wavenumber α_c decreases monotonically. Meaning, the effect of both Γ and F is to enlarge the size of convection cell at the onset of convection. Noticeably, it is follows from Fig. 4 that, the size of convection cells tends to be uniform when Γ is large. This future of the aspect ratio of convection cells could be ascribed to the stabilizing influence of the presence of couple stresses. It is advantageous to mention that the effect of internal heating on the tessellation of the convection cells depends largely on the type of boundary combinations used in the convective stability analyses (Maruthamanikandan, [23]). The problem at hand would offer useful insights into heat transfer applications if the radiation and magnetic effects are factored in [24, 25]. We believe that the findings of the present study could have considerable bearing on the heat transfer mechanisms concerning fluids with couple stresses.

VI. CONCLUSIONS

On the basis of Stokes micro-continuum theory, the combined effect of couple stresses signifying non-Newtonian characteristics of the fluid and chemical reaction on a densely packed porous medium heated from below is examined. The modified Darcy equation that includes the time derivative and the inertia term is used to model the momentum equation. The central conclusions of the present study are the following:

- (i) The effect of couple stresses is to enhance the stability of the system.
- (ii) Chemical reaction, giving rise to a nonlinear basic temperature distribution, destabilizes the system.
- (iii) The destabilizing effect of chemical reaction on the threshold of convection in a porous layer is more pronounced when the couple-stress parameter Γ is large.
- (iv) The stabilizing effect of couple stresses is not attenuated by the countervailing influence of chemical reaction and vice versa.
- (v) The dimension of convection cells is considerably influenced by the presence of chemical reaction.
- (vi) The aspect ratio of convection cells becomes uniform provided that the couple-stress parameter Γ is large.

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