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Classical, Smart And Modern Controller Design Of Inverted Pendulum

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Abstract

The Inverted Pendulum is a very popular plant for testing dynamics and control of highly non-linear plants. In the Inverted Pendulum Control problem, the aim is to move the cart to the desired position and to balance a pendulum at desired location. This paper represents stabilization of pendulum using PID, SVFB and Fuzzy Logic Control. In the mathematical model proposed here, a single rule base is used for angle of pendulum.

The SVFB and fuzzy logic control scheme successfully fulfils the control objectives, it is found that FLC is found to be the best. The simulation results of all the controllers are compared with PID, SVFC and FLC.

Keyword-Inverted pendulum,Mathematical modelling ,PID controller,State variable feedback controller(SVFB),Fuzzy Logic Controller(FLC)

1. INTRODUCTION:

The inverted pendulum is unstable[1,2,4,6] in the sense that it may fall any time in any direction unless a suitable control force is applied(9,11,13). If the designer works it right, he can get the advantages of several effects(7,8). The control objective of the inverted pendulum is to swing up[3,9,10] the pendulum hinged on the moving cart by a linear motor[12] from stable position (vertically down state) to the zero state(vertically upward state) and to keep the pendulum in vertically upward state in spite of the disturbance[5,13]. The inverted pendulum may be viewed as a classical problem in dynamics and control theory[12] and is widely used as a benchmark[16] for testing control algorithms such as PID controllers, state feedback controller, fuzzy logic controller etc[15,17].

In the field of engineering and technology the importance of benchmark needs no explanation. They make it easy to check whether a particular algorithm yields the requisite results. Several work has been reported on the inverted pendulum for its stabilization. Attempts have been made in the past to control it using classical control [14]. The purpose of the present research[18] is to do a comparison between three different methods of control as PID,SVFB and Fuzzy logic. The work was made under MATLAB simulation. To achieve our goal it was necessary to implement the mathematical model of the inverted pendulum and after that the fuzzy control,PID,SVFB controls were implemented and results were compared.

2 MATHEMATICAL MODEL OF THE PLANT

Defining displacement of the cart as x, the angle of the rod from the vertical (reference) line as θ , assuming the force applied to the system be F , centre of gravity of the pendulum rod is at its geometric centre and l be the half length of the pendulum rod, the physical model of the system is shown in fig (1),the other patameters are reffered in Table(1)

The Lagrangian of the entire system is given as,

$$L= \frac{1}{2}(m\dot{x}^2 \ +2ml\dot{x}\dot{\theta}\cos\theta+ml^2\dot{\theta}^2+M\dot{x}^2)+\frac{1}{2}I\dot{\theta}^2 \ ]-mgl\cos\theta$$

The Euler-Lagrange's equation for the system is

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right)-\frac{\delta L}{\delta x}+b\dot{x}=F \qquad (1)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}}\right)-\frac{\delta L}{\delta \theta}+d\dot{\theta}=0 \qquad (2)$$

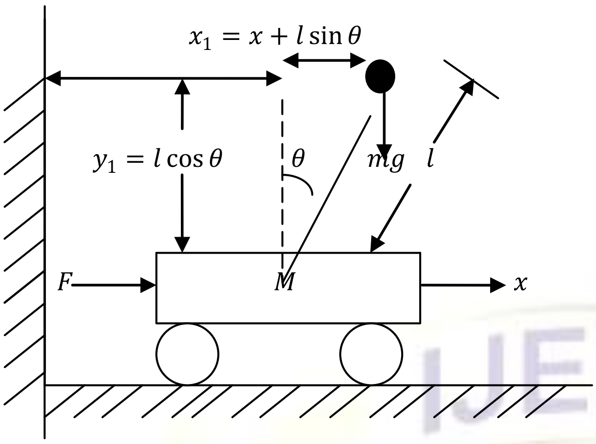


Fig 1 : The Inverted Pendulum System

The dynamics of the entire system using above equations are given by

$$(I + ml^2)\ddot{\theta} + ml\cos\theta\,\ddot{x} - mgl\sin\theta + d\dot{\theta} = 0 \qquad (3)$$

$$(M + m)\ddot{x} + ml\cos\theta\,\ddot{\theta} - ml\sin\theta\,\dot{\theta}^2 + b\dot{x} = F \qquad (4)$$

In order to derive the linear differential equation model, the non linear differential equation obtained need to be linearized. For small angle deviation around the upright equilibrium (fig.1) point, assumption made are

$$\sin\theta = \theta, \cos\theta = 1,\ \dot{\theta}^2 = 0$$

Using above relation , equation (5) and (6) is derived.

$$r\ddot{\theta}+q\ddot{x}-k\theta+d\dot{\theta}=0 \qquad (5)$$

$$\ddot{p}x + q\ddot{\theta} + b\dot{x} = F \qquad (6)$$

Where, $(M + m)$= p, mgl=k, ml=q, $I + ml^2$= r

Table 1.
Parameters of the system from feedback instrument .U.K.

Parameter	**Value**	**unit**
Cart mass(M)	1.206	Kilo gram
Mass of the pendulum(m)	0.2693	Kilo gram
Half Length of pendulum(l)	0.1623	meter
Coefficient of frictional force(b)	0.005	Ns/m
Pendulum damping coefficient(q)	0.005	Mm/rad
Moment of inertia of pendulum(I)	0.099	kg/m^2
Gravitation force(g)	9.8	m/s^2

Eq (5&6) are the linear differential equation model of the entire system.
After taking Laplace transform and substituting the parameter value (table 1), we got

$$\frac{\theta(s)}{F(s)} = \frac{-0.2783\ S^2}{s\,(s+2.026)\,(s-1.978)\,(s+0.03402)} \qquad (7)$$

$$\frac{X(s)}{F(s)} = \frac{0.68843\ (s+2.014)\,(s-1.967)}{s\,(s+2.026)\,(s-1.978)\,(s+0.03402)} \qquad (8)$$

The system matrix and output vector found by state space analysis are

$$A=\begin{bmatrix} 0 & 1.0000 & 0.0000 & 0.0000000 \\ 0 & 0.0000 & 1.0000 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 1.0000 \\ 0 & -0.1343 & 4.0126 & -0.0133 \end{bmatrix},\ B=\begin{bmatrix} 0.0000 \\ 0 \\ 0.0000 \\ 1.0000 \end{bmatrix}$$

3 PID Controller design

PID controllers are a family of controllers. The reason PID controllers are so popular is that PID gives the designer a larger number of options and those options mean that there are more possibilities for changing the dynamics of the system in a way that helps the designer.

4 Simulation and results.

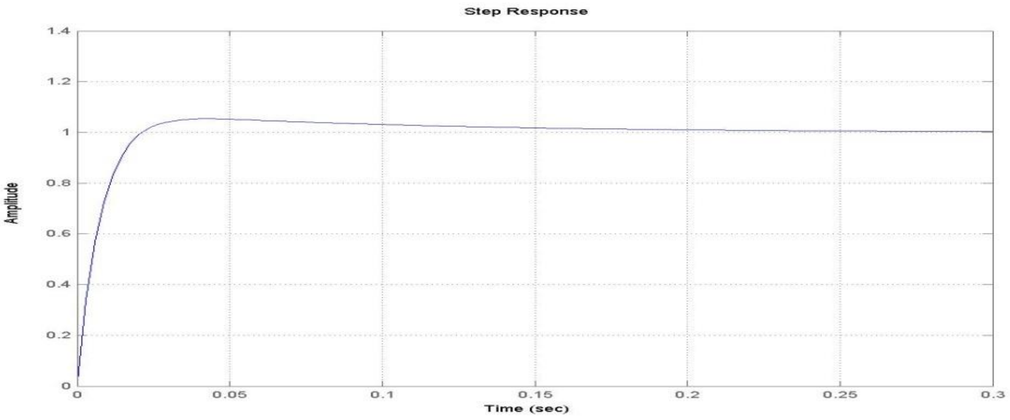


Fig 2: Response of the cart with $K_p = 1000$ $K_d = 100$ $K_i = 40$

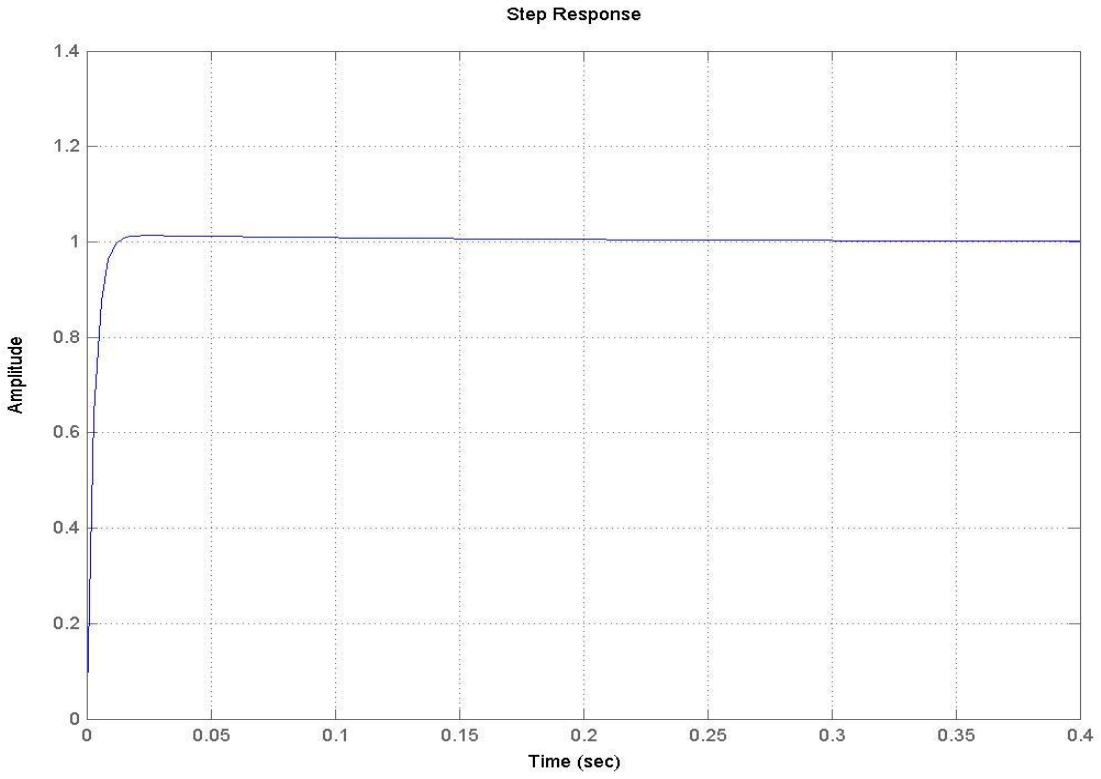


Fig 3: Response of the cart with $K_d = 110$ $K_P = 1087$ $K_I = 49$

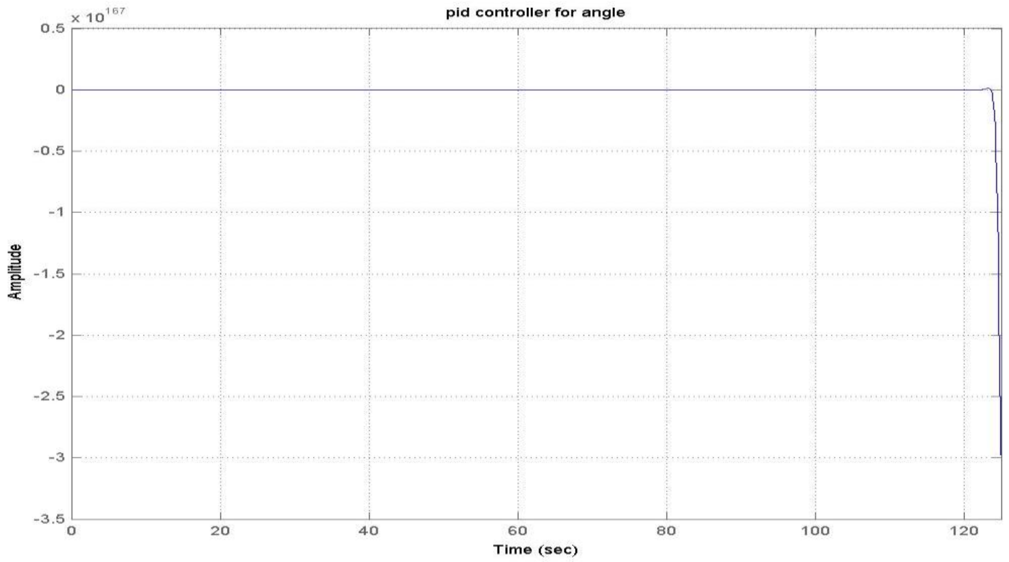


Fig 4: Response of the pendulum with K_d = -50 K_P = 400 K_I = 18

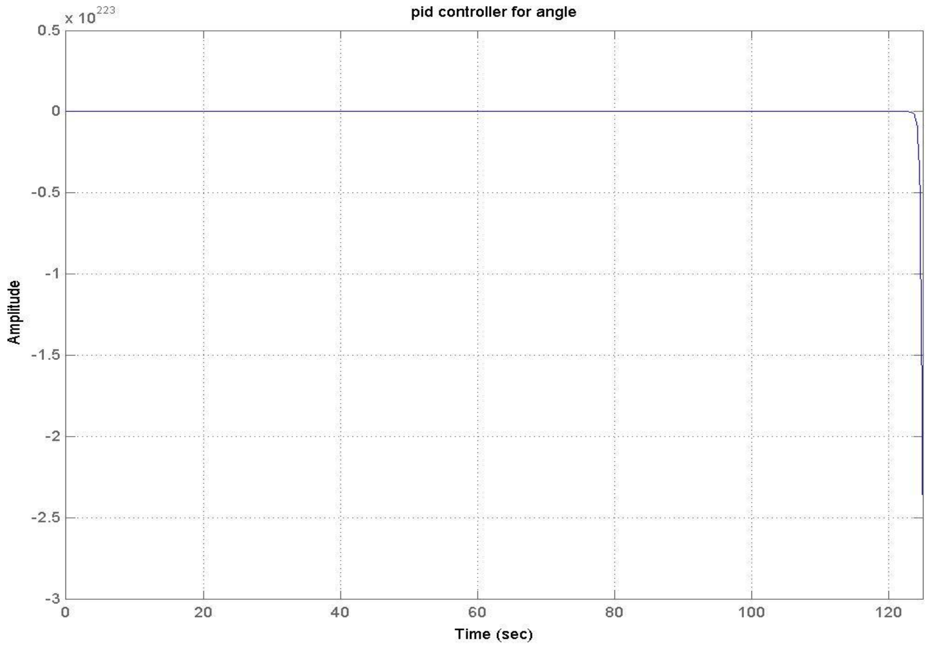


Fig 5: Response of the pendulum with K_d = -80 K_P = -291 K_I=-10

5 DESIGNING THE STATE FEEDBACK CONTROLLER:

State feedback technique through a gain matrix has been a well-known method for pole assignment of a linear system. The technique could encounter a difficulty in eliminating the steady-state errors remained in some states if system is completely controllable.

CONDITIONS:

We assign different types of time domain specification as given in that closed loop system has an overshoot of 10% and settling time of 1 sec.

The dominant poles are at

$- \xi w_n \pm j w_n \sqrt{(1- \xi^2)}$

$\cong -4 \pm j5.45531$

The third and fourth pole is placed 5 & 10 times deeper into the s-plane than the dominant poles.

Hence the desired characteristics equation is

$s^4 + 68s^3 + 845.7604s^2 + 9625.6s + 36608.32 = 0$ (3.11)

Let $k = [k_1 \;\; k_2 \;\; k_3 \;\; k_4]$

$$A - BK = \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1 & -0.1343-k_2 & 4.0126-k_3 & -0.0133-k_4 \end{matrix}$$

Closed loop characteristics equation

$S^4 + (0.0133+k_4)s^3 + (-4.0126+k_3)s^2 + (0.1343+k_2)s + k_1 = 0$ (9)

Comparing the coefficient of equation we got

$K = [36608.32 \quad 9625.4657 \quad 849.7726 \quad 67.9867]$

6 SIMULATION AND RESULTS FOR DIFFERENT INITIAL CONDITION OF THE CART- POLE SYSTEM BY STATE FEEDBACK CONTROLLER

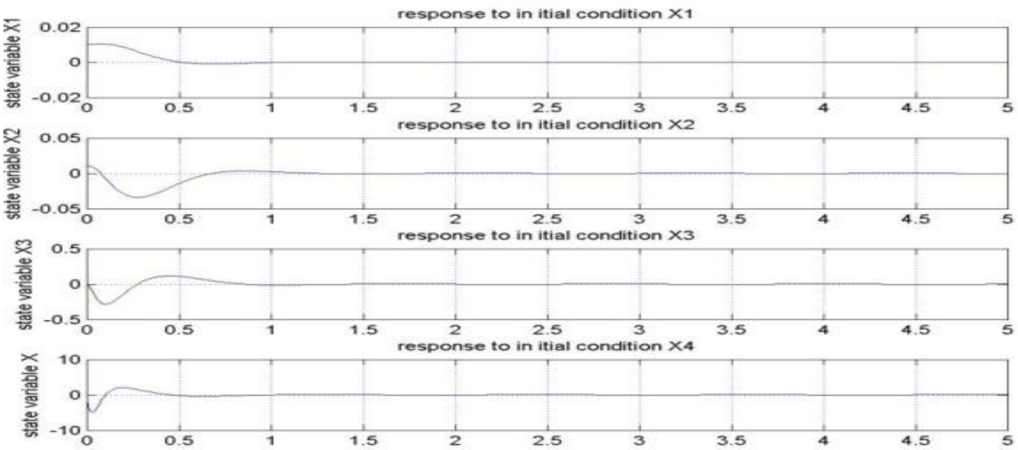


Fig 6:Initial condition response

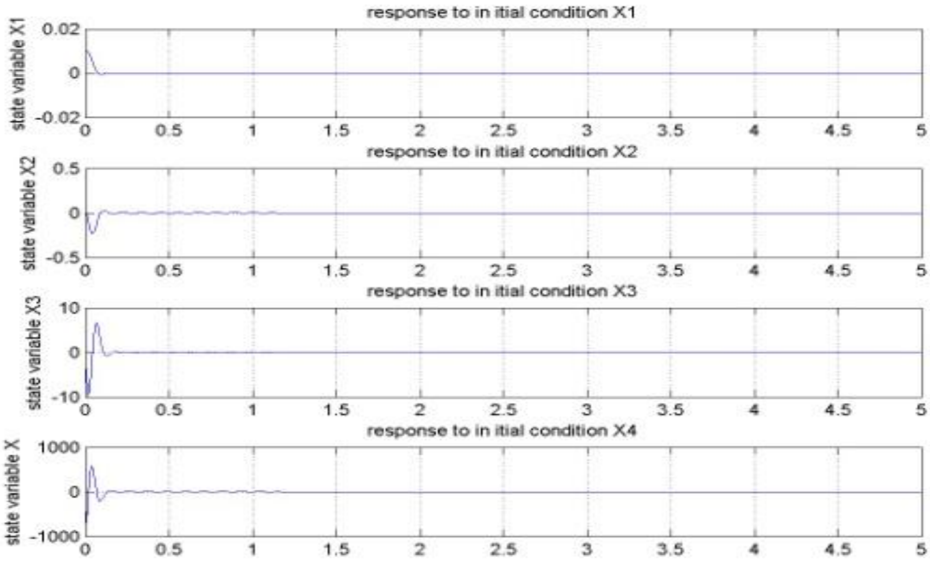


Fig 7:Initial condition response

7 :RESPONSE OF THE SYSTEM BY STATE FEEDBACK CONTROLLER:

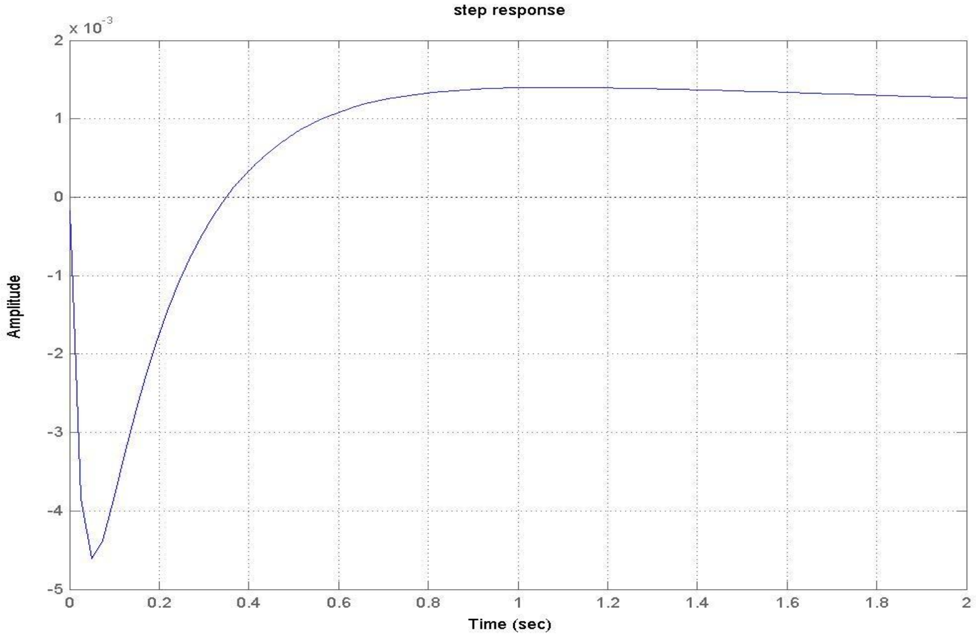


Fig 8: Step response

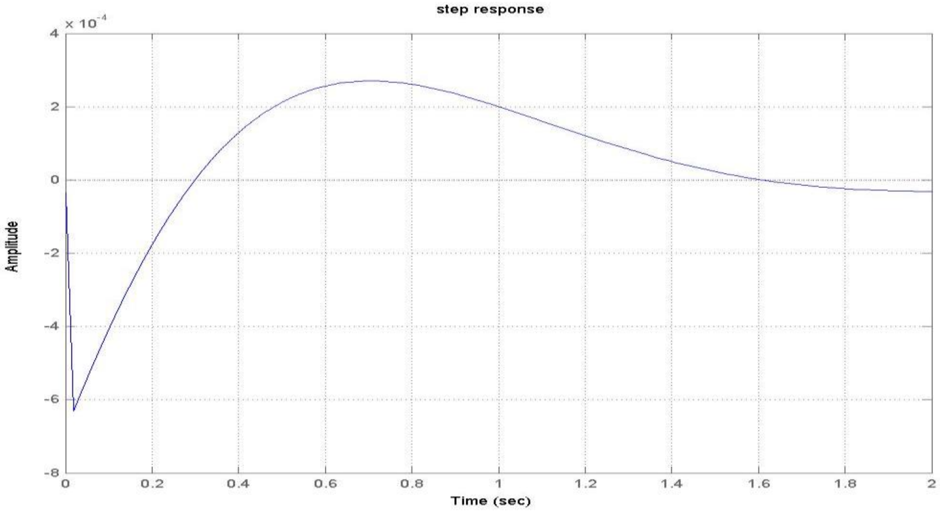


Fig 9: Step response

8 :FUZZY LOGIC

Study of Fuzzy logic is a study of a kind of logic. We are all familiar with some of the principles of logic. Fuzzy logic builds on traditional logic and extends traditional logic so that fuzzy logic can solve some long standing problems in traditional logic.

8.1 DESIGN OF FUZZY LOGIC CONTROLLER FOR INVERTED PENDULUM:

The fuzzy logic controller is to be designed for stabilizing the cart pole system. The fuzzy logic controller is constructed by considering the angular position θ and angular velocity $\dot{\theta}$ of the pendulum. and u is the output of the controller. The membership functions are defined for the input θ and $\dot{\theta}$ and output force F. For both input and output variables, seven linguistics – negative large (NL), negative medium (NM), negative small (NS), zero (ZE), positive small (PS), positive medium (PM) and positive large (PL) are assigned. The membership plot are as shown in Figure (10)

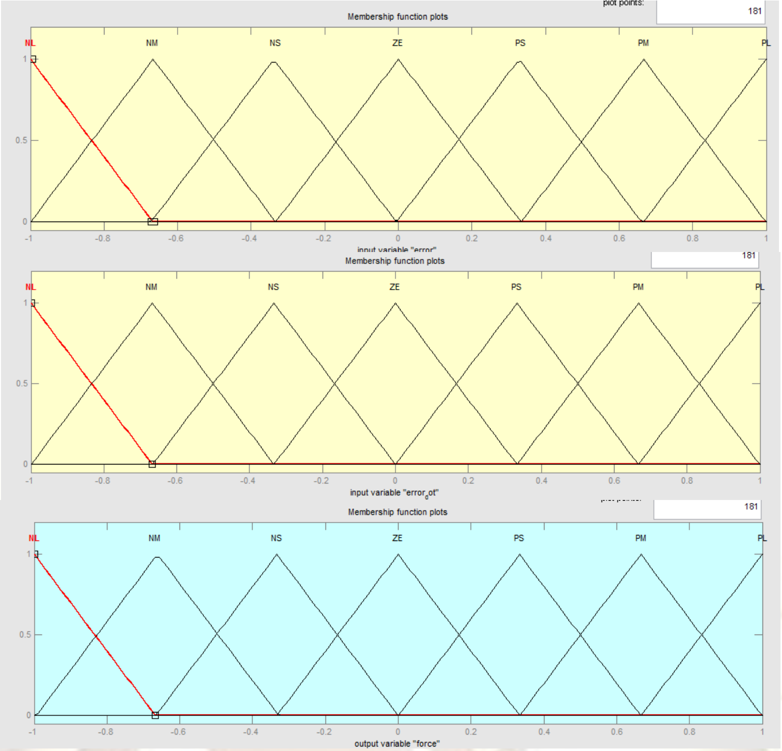


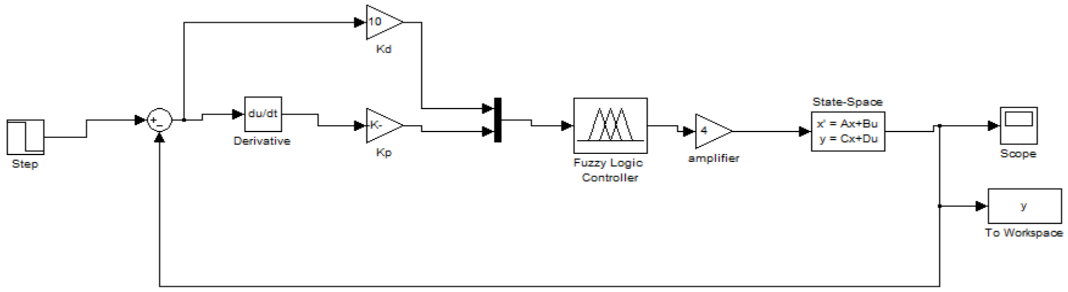
Fig 10: Membership function of θ,$\dot{\theta}$ and F

On defining the membership functions, fuzzy rule base is formed in a Fuzzy Associative Memory (FAM) table as shown in Table 2.

TABLE 2
Control Rules (FAM Table)

θ $\dot{\theta}$	NL	NM	NS	ZE	PS	PM	PL
NL	NL	NL	NL	NM	NS	NS	ZE
NM	NL	NM	NM	NM	NS	ZE	ZE
NS	NM	NM	NS	NS	ZE	ZE	PS
ZE	NS	NS	ZE	ZE	ZE	PS	PS
PS	NS	ZE	ZE	PS	PS	PM	PM
PM	ZE	ZE	PS	PM	PM	PM	PL
PL	ZE	PS	PM	PM	PL	PL	PL

8.2 SIMULINK BLOCK DIAGRAM OF FLC



8.3. RESPONSE OF FUZZY LOGIC CONTROLLER

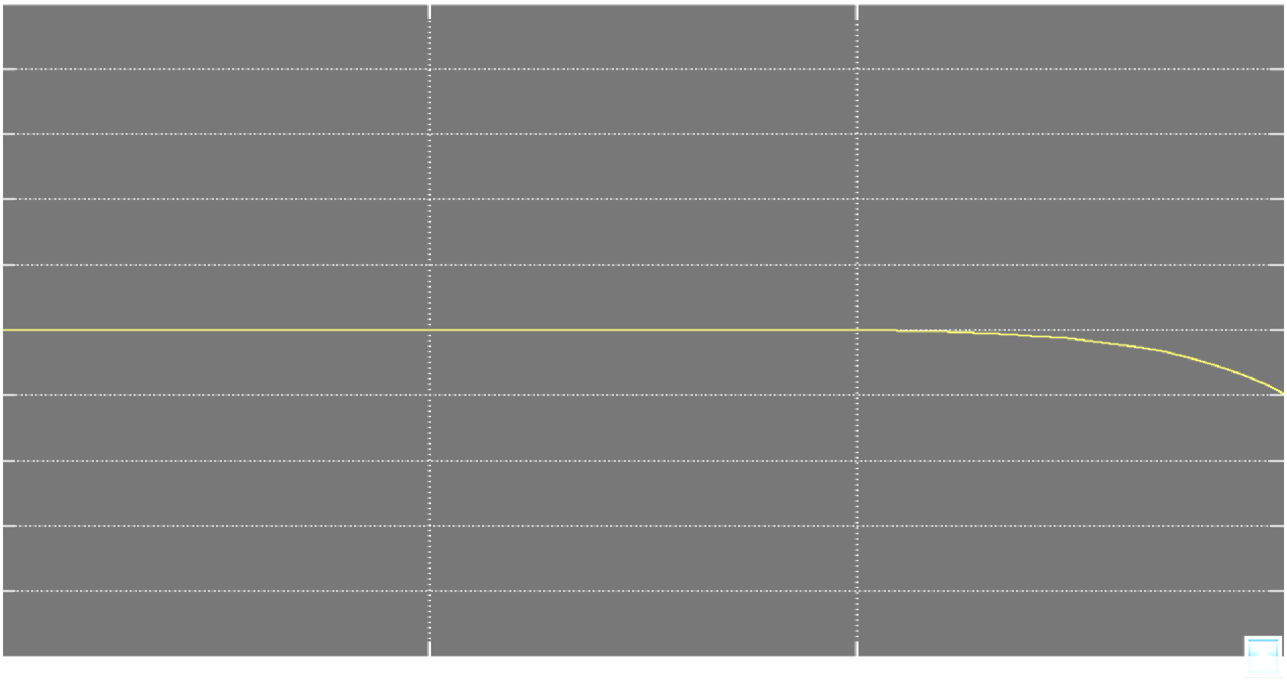


Fig 12: Response of FLC when K_d=10 K_P=2.5K_I=0.8

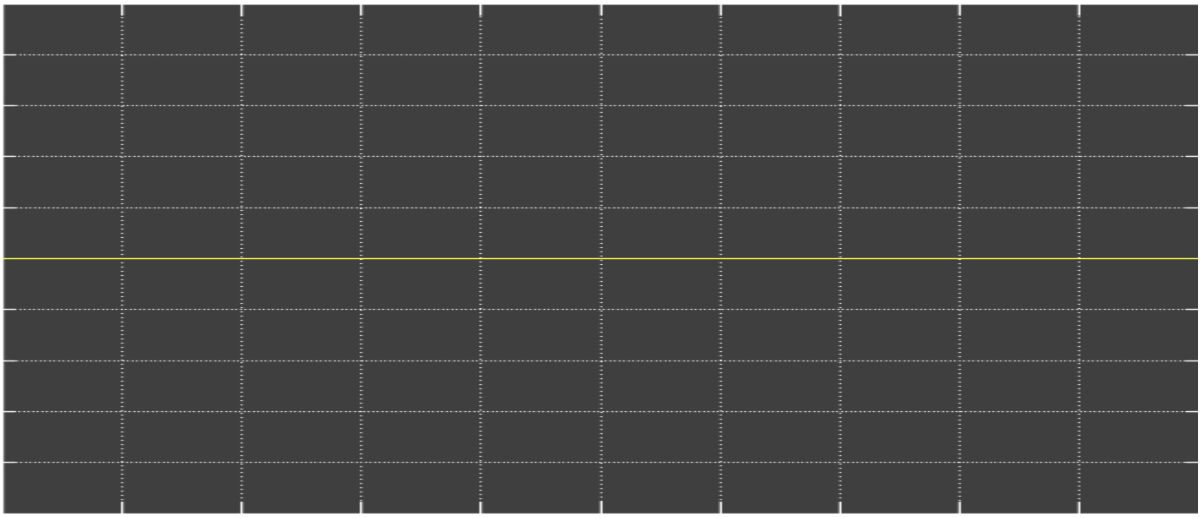


Fig13:Response of FLC when K_d = 55 K_P= 4K_I = -10.25

8.4 CONCLUSION:

Designing of PID controller both for angle and position of the cart is very difficult because the root locus does not cross the imaginary axis hence Zieglor Nichols tuning is not applicable in such case. Also two different PID controllers are used both for position of the cart and angle of the pendulum. To overcome the problem state feedback controller is used but here also steady state error exists in some state. Tto reduce the peak overshoot and settling time we applied FL controller, which gives better results Some Genetic algorithm based techniques may be applied for fine tuning of above discussed controller.

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