

## Data Generation of Structural Configurations

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### ABSTRACT

Engineers and architects have always held a special interest for structural systems that enable them to cover large spans with minimal interference from internal supports. It is perhaps no surprise then that dome structures, capable of encompassing maximum volume with minimum surface area, are one of the oldest structural forms and has been used in architecture since the earliest times. The processing of geodesic forms has always been an extremely difficult task. The constraint of the processing difficulties, however, did not allow the designers to take full advantage of the whole spectrum of possibilities and their scope remained rather limited. In contrast, the conceptual methodology that is presented in this paper combined with suitable computer software such as Formian, provides a means for dealing with the processing of any kind of polyhedral configuration and geodesic with relative ease and elegance. In this paper the generation of geodesic forms is solved in two stages.

**Keywords** – Formex algebra, Formian, Geodesic Forms, Polyhedral forms, retronorms

### I. INTRODUCTION

Formex configuration processing uses the concepts of formex algebra through the programming language Formian to generate and process configurations. Being modeled on formex algebra, the language allows statements to be written in a concise and yet readily understood manner. It also has simple to use graphics facilities and a built-in editor, enabling problems of data generation to be accomplished in one programming environment. Also, in Formian the generated data may be stored in the form of a rule thus enabling information about a structural system to be represented in just a few lines of formulation [1], [2]. This allows the data to be modified easily and provides a convenient means of keeping the information for future reference. In dealing with formex formulation of a configuration, it is usual to begin by formulating a topological description of the configuration using formex functions. The next stage involves the employment of a transformation for associating geometric coordinates with nodes of the configuration. A transformation of this kind is referred to as a retronorm. Two categories of retronorms are employed in the Formian. Firstly, there are the standard retronorms that are incorporated in the Formian Interpreter. The second category of

retronorms is referred to as supplementary retronorms [3]. A supplementary retronormic function is introduced through a program segment which is supplied by the user in order to create a nonstandard retronorm. The program segment is linked to the body of the Formian Interpreter. This paper deals with the implementation of a supplementary retronormic function called the "traction retronorm". The traction retronorm enables a configuration to be projected on different types of surfaces such as spheres or ellipsoids, paraboloids, cylinders, hyperbolic paraboloids or planes. The term traction is used to imply projection of a configuration on a surface or surfaces. Traction is derived from the Latin word "tractus" meaning "drawing". The projection may be central, axial parallel or radial and these types of projection will be discussed in due course [4], [5].

Furthermore, this paper deals with the establishment of the concepts and constructs through which polyhedral and geodesic configurations may be created with the polyhedron function. The polyhedron function provides a basis for the configuration processing of geodesic configurations in a compact and readily understood manner and it allows one to work with the same set of tools in all data generation problems eliminating the need for the employment of an assortment of programs which are dealing with specific problems. Hence, the emphasis is on the manner in which the concepts of formex algebra are employed for the generation of polyhedral and geodesic domes rather than the details of the formulations and no prior knowledge of formex algebra is necessary for following the material. The preliminary concepts and ideas of formex configuration processing were evolved during the last four decades [6]. The paper does contain some formex formulations but these are included to give a feel for the appearance of formex formulations rather than their details being essential for understanding of the material.

### II. ARCHIMEDEAN POLYHEDRA

The first stage in the generation of a geodesic form is the creation of the polyhedral configuration. The approach presented in this paper provides a methodology that allows polyhedral configuration of all kinds to be generated in a convenient manner. However, polyhedral configurations based on semi-regular (Archimedean) polyhedral are used in this paper to demonstrate the concepts and constructs through which polyhedral

configurations may be created. There are thirteen semi-regular polyhedra, and are as shown in Tables 1-3. These polyhedra consist of various combinations of triangles, squares, pentagons, hexagons, octagons

and decagons [7], [8], [9], [10]. Ten of the Archimedean polyhedra utilize only two kinds of polygons and the remaining three utilize three kinds of polygons, as

**Table 1.** The Archimedean Polyhedra and the polyhedron code P for each one

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Table 2. The Archimedean Polyhedra and the polyhedron code P for each one

<p><b>Icosidodecahedron P=12</b></p>	<p><b>Truncated Dodecahedron P=13</b></p>
<p><b>Truncated Icosahedron P=14</b></p>	<p><b>Snub Cube P=15</b></p>

shown in Table 4. In these Tables each polyhedron is shown together with the global Cartesian x-y-z coordinate system. The origin of the coordinate system is at the centre of the polyhedron and is indicated with a large dot. The point where the positive side of the x-axis intersects the polyhedron is indicated by a little circle with an enclosed x. This point is referred to as the “x-point”. The positions of the positive directions of y- and z- axes are indicated by arrows with z being always vertical. Each face of the polyhedron is identified with a “face code” which is given at one corner of the face.

A face code consists of a number followed by a letter and possibly followed by an asterisk. The number in a face code is the identification number of the face. The letter in a face code determines the points A, B, C, ..., etc of the configuration that is to be placed on the indicated corner of the face. These letters for configurations corresponding to different shapes of polyhedral faces are as shown in Fig. 1. If a face code has an asterisk, it implies that the configuration which

is to be placed on the face is the reflection, with respect the x-y plane, of the given configuration.

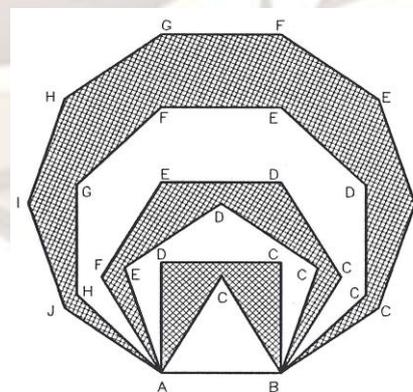


Fig. 1. Types of polygons in Archimedean Polyhedra

Table 3. The Archimedean Polyhedra and the polyhedron code P for each one

<p><b>Small Rhombicosidodecahedron P=16</b></p>
<p><b>Great Rhombicosidodecahedron P=17</b></p>
<p><b>Snub Dodecahedron P=18</b></p>

**Table 4** The type of polygons in each of the Archimedean polyhedral

Polyhedron	Type and Number of Faces
Truncated Tetrahedron	4  4 
Cuboctahedron	8  6 
Truncated Cube	8  6 
Truncated Octahedron	 6  8
Small Rhombicuboctahedron	 8  18
Great Rhombicuboctahedron	 12  8  6
Icosidodecahedron	20  12 
Truncated Dodecahedron	20  12 
Truncated Icosahedron	12  20 
Snub Cube	32  6 
Small Rhombi Icosidodecahedron	 20  30  12
Great Rhombi Icosidodecahedron	 30  20  12
Snub Dodecahedron	 80  12

### III. POLYHEDRON FUNCTION

Consider the triangular and square configurations of Figs 2, 3. In this figures the configurations are shown together with their normal U1-U2-U3 for the formex formulation and may be represented in terms of the formex variables P5 and P6 which are given as,

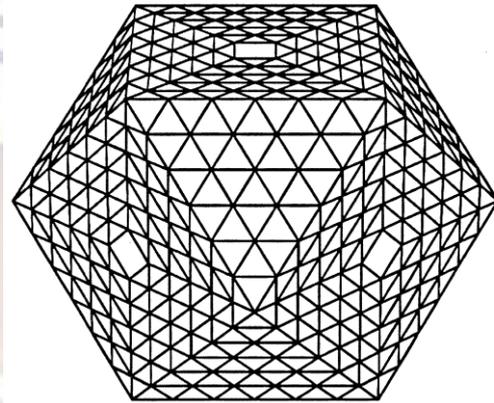
$$P5 = \text{GENID}(6,6,2,\text{SQRT}|3,1, -1) | \text{ROSAD}(1,\text{SQRT}|3/3,3, 120) | [0,0; 2,0]$$

and

$$P6 = \text{LUX}([6,6]) | \text{ROSAD}(6,6) | \text{BB}(1,1/\text{SQRT}|3) | P5$$

A "formex" (singular) is a mathematical entity which may be used to represent a configuration. A reader encountering the above formices (plural) for the first time is bound to find it a little confusing but once the basic principles of formex algebra are understood, the formex approach is found to be a simple way of describing a configuration.

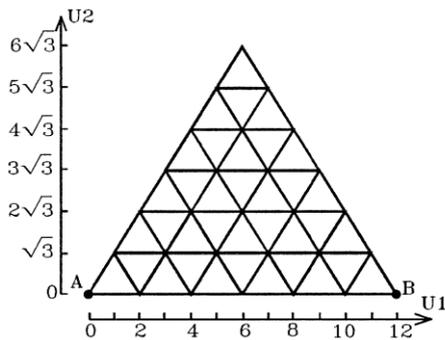
To explain the above formulations, one may begin by describing formex P5 which represents only a part of the configuration of Fig. 4.



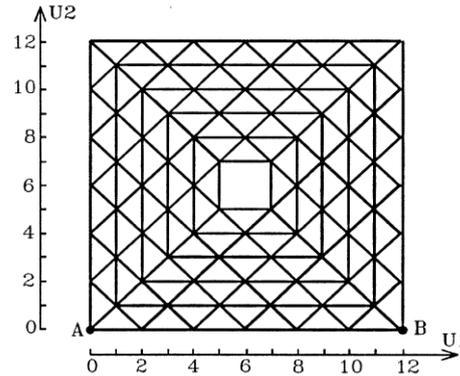
**Fig. 4.** A graphical representation of the cuboctahedron

The triangular configuration is formulated with respect to a suitable reference system as shown in Fig 2. The construct  $[0,0; 2,0]$  represents the first horizontal member of the triangular configuration and the construct  $\text{ROSAD}[1,\text{SQRT}|3/3, 3, 120]$  represents the small triangle near the origin of the reference system. The "function"  $\text{ROSAD}$  implies rotational replication. This function effectively, creates a rotational replication of the horizontal member with the centre of rotation given by  $(1,\text{SQRT}|3/3)$ . The other two parameters  $(3, 120)$  in the  $\text{ROSAD}$  function imply number of members to be created at an angle of  $120^\circ$ . The entire configuration is generated by repeating the triangular pattern using another formex function. The function  $\text{GENID}(6,6,2,\text{SQRT}|3,1,-1)$  is used to imply 6 replicational translations in the first direction and 6 in the second, with a step of 2 and  $\text{SQRT}|3$  units in first and second direction, respectively. The parameters 1, -1 demonstrate the step and the amount of triangles eliminated in each row.

Moreover, the square configuration is formulated with respect to a suitable reference system shown in Fig. 3. The function  $\text{BB}(1, 1/\text{SQRT}|3)$  is referred to as a basibifect retronorm effecting scaling in the first and second directions by factors 1 and  $1/\text{SQRT}|3$ , respectively. The formex function  $\text{ROSAD}(6,6)$  is used to imply rotational replication, with the centre of rotation given by  $(6,6)$ . The function  $\text{LUX}$  stands for "luxum" which is a latin word



**Fig. 2.** A triangular configuration



**Fig. 3.** A square configuration

used in formex algebra to imply “disconnected parts” and is used here to imply the disconnection of the four elements whose apex is at point (6,6).

Finally, let it be required to map these configurations onto all the faces of a cubotetrahedron. A Formian statement describing this operation may be given as

$$C = \text{PEX} \mid (\text{POL}(7,10,[0,0; 12,0], [1; 8] \mid \text{P5} \# \text{POL}(7,10, \dots [0,0; 12,0], [9; 14]) \mid \text{P6})$$

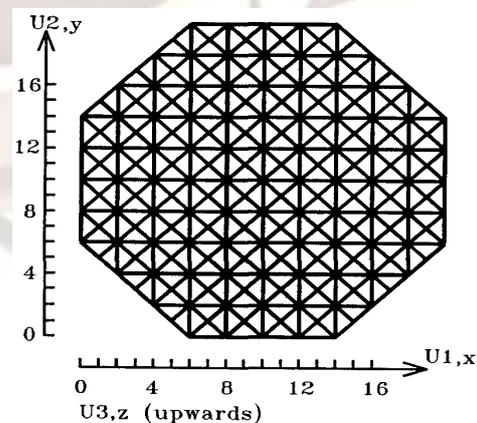
As it is shown in Table 1 the cuboctahedron has a top x-y square face, eight triangular, four square non x-y faces and a bottom x-y square face. The constructs  $\text{POL}(7,10,[0,0; 12,0], [1; 8])$  and  $\text{POL}(7,10, [0,0; 12,0], [9; 14])$  are formex functions representing a rule for transformation of given formices P5 and P6 into a formex C. The parameters 7, 10, [0,0; 12,0], [1; 8] and [9; 14] are parts of the rule defining the particulars of the transformation and are referred to as canonic parameters. The above function is referred to as a “polyhedron function”. The polyhedron function can be used to create single layer or multi layer polyhedral configurations. The general form of the polyhedron function for single layer polyhedral configurations may be written as  $\text{POL}(P,R,[A1,A2; B1,B2] \ll \{F1, F2, \dots, Fn\} \gg)$  where the first canonic parameter P is referred to as the “polyhedron code”. The polyhedron code specifies the type of polyhedron which is to be used as the basis for the operation. Tables 1 to 3 list the code numbers for the Archimedean polyhedra. The integer 7 given as the polyhedron code in the above polyhedron function specifies a cuboctahedron. The “radius specifier” determines the size of the polyhedron by specifying the radius of the circumsphere, that is, the sphere that contains all the vertices of the polyhedron. This parameter is given as 10 units of length. The “locator” specifies the manner in which a given configuration is to be mapped onto a face of the polyhedron. To elaborate, consider the configurations shown in Figs 2&3. Two corners of the configurations are denoted by the letters A and B. The configurations are intended to be placed on the triangular and the square faces of the cuboctahedron

in such a way that AB fits an edge of the cuboctahedron. This convention is conveyed by including the U1-U2 coordinates of A and B in the locator.

The last canonic parameter is referred to as the “face list”. The role of the face list is to specify those faces of the polyhedron onto which the configuration is to be mapped. The face list is enclosed in special brackets which are referred to as option brackets. Absence of the face list in the polyhedron function implies that the configuration is to be mapped onto all the faces of that polyhedron. However, one has the option of generating only a part of the polyhedral configuration by specifying the required face numbers through the face list. The face numbers for the Archimedean polyhedra are given in Tables 1 to 3. For instance, the polyhedral configuration of Fig. 4 is based on a cuboctahedron and is obtained by mapping the configuration of Figs 2 and 3 onto the triangular and square faces, respectively.

#### IV. THE TRACTATION RETRONORM

The next stage in the creation of a geodesic form evolves the projection of the polyhedral configuration on one or more specified surfaces.



**Fig. 5.** A single layer grid pattern together with the global coordinate system x-y-z and the normal U1-U2-U3

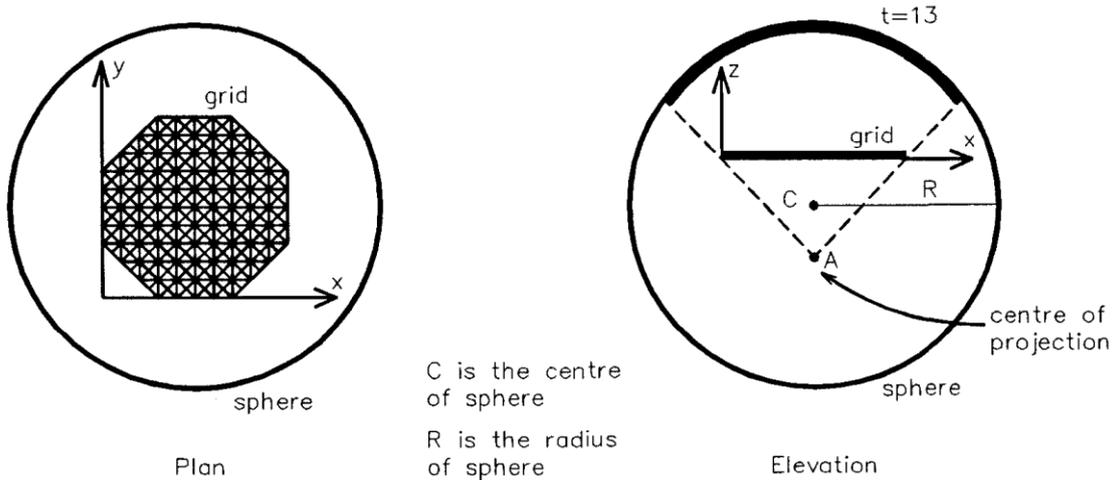


Fig. 6. The plan view and elevation of the grid, together with the sphere on which the central projection is to be carried out.

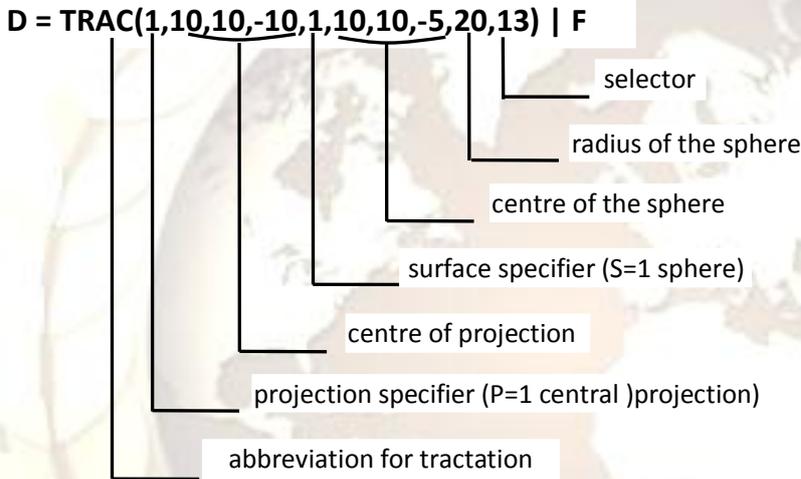


Fig.7. A formex variable D that implies central projection

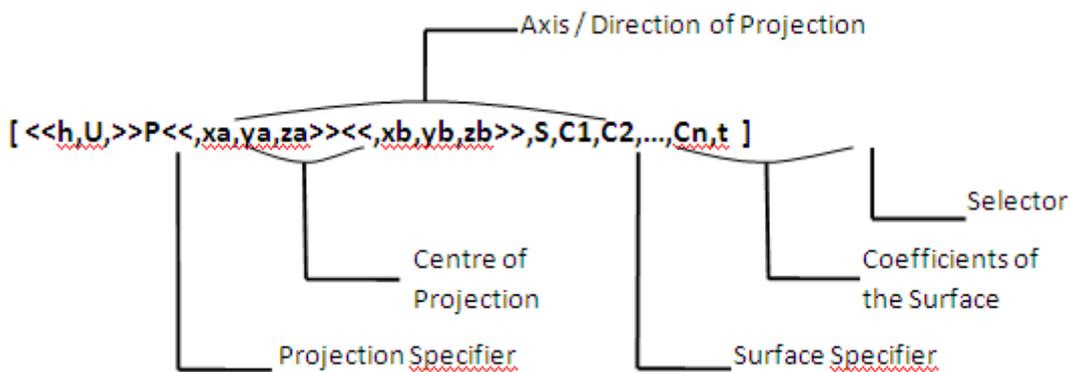


Fig. 8. A general descriptor of the tractation retronorm

To obtain the projection of the polyhedric configuration the tractation retronorm is employed. Consider a single layer grid pattern which is to be projected on a sphere. The grid together with the global coordinate system x-y-z and the normal U1-U2-U3 for the formex formulation are shown in Fig.

5. The grid may be represent in terms of a formex F where  
 $F = \text{PEX} \mid \text{LUX}(\text{ROSAD}(10,10) \mid \text{GENID}(3,3,2,0,-1) \mid [0,0] \dots \text{RINID}(10,10,2,2) \mid \text{ROSAD}(1,1) \mid \{[0,0;2,0],[0,0;0,1]\})$

Fig. 6 shows the plan view and elevation of the grid, together with the sphere on which the projection is to be carried out. For the specification of the sphere, the centre  $C(x_c, y_c, z_c)$  and the radius  $R$ , have been defined. The centre of projection is specified as the point  $A(x_a, y_a, z_a)$ . The above specifications, together with another three parameters may be given as shown in Fig. 7.

These three parameters may be the type of projection, the type of surface and the required solution. The first parameter is referred to as "projection specifier" and the second parameter is referred to as "surface specifier". The last parameter is referred to as the "selector" and will be described in detail below. TRAC in Fig. 7 is an abbreviation for "tractation" and is followed by a sequence of parameters enclosed in parentheses. The construct that consists of TRAC together with the ensuing parameter list is referred to as the "tractation retronorm" and what is enclosed in parentheses is called the "descriptor". A general descriptor is of the form given in Fig. 8. The significance of the values of the projection specifier  $P$ , the centre of projection, the axis or direction of projection, the selector and the coefficients of the surface are discussed in detail in the sequel. However, as far as the significance of the values of  $h$  and  $U$  these will be discussed in detail below. A graphical representation of formex  $D$ , projected on the sphere is shown in Fig. 9.

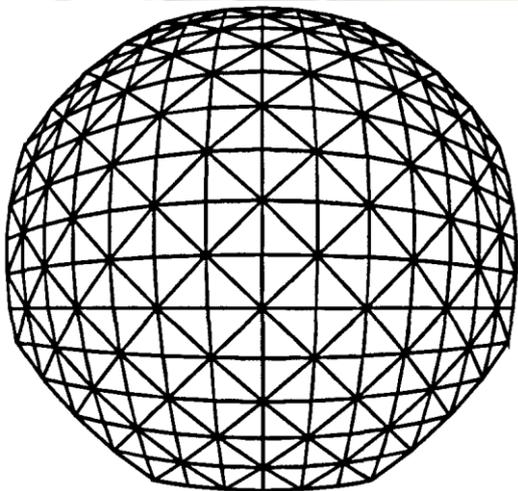


Fig. 9. A graphical representation of formex variable  $D$ , projected on the sphere using central projection.

#### 4.1 Projection specifier

The projection specifier  $P$  in Fig. 7 is given as 1. The centre of projection  $A(x_a, y_a, z_a)$  is specified by the coordinates 10,10 and -10 relative to the global coordinate system. The projection specifier  $P$  is a nonzero positive integer defining the type of projection. The value of the projection specifier  $P$  may be from 1 to 4, as given in Table 5.

#### 4.2. Surface Specifier

The surface specifier  $S$  in Fig. 7 is given as 1, where 1 stands for a sphere. The surface specifier  $S$  is an integer expression whose value is a nonzero positive integer defining the surface of projection. The value of the surface specifier  $S$  may be from 1 to 9 representing the surfaces shown in Table 6. In this paper the first two surface specifiers will be discussed [5]. The description of a surface requires a number of coefficients that should follow the surface specifier. These, for different surfaces, are also given in Table 6.

#### 4.3. Selector

The projection of a point of a surface usually involves a quadratic equation [11]. The value of the parameter  $t$  specifies the course of action to be taken when the projection of a point cannot be determined uniquely and is referred to as "selector". To elaborate, in obtaining the projection  $P$  of a point  $T$  on a surface  $S$ , the following situations may arise:

- i.  $P$  is determinable uniquely,
- ii.  $P$  is nonexistent,
- iii. There is more than one solution for  $P$

Various possible courses of action in the case of nonexistent or multiple solutions for  $P$ , together with the corresponding section codes are listed in Table 7.

### V. PROJECTION ON AN ELLIPSOID

The configurations generated in this section are the results of projecting a grid on an ellipsoid. To begin with, consider the grid shown in Fig. 10. The interconnection pattern of the configuration is given by

GRID = PEX | RINID(10,10,2,2) | ROSAD(1,1) |  
{[0,0; 2,0], [0,0; 0,1]}

where GRID is a formex that represents the grid pattern of Fig. 10.

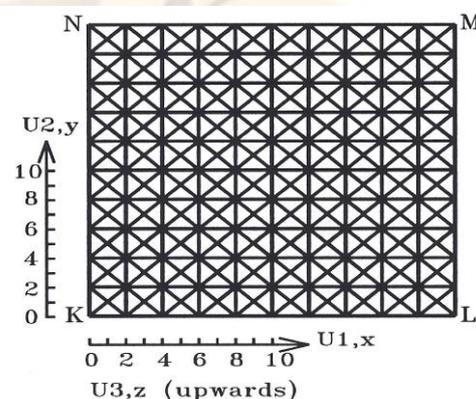


Fig. 10 The grid configuration

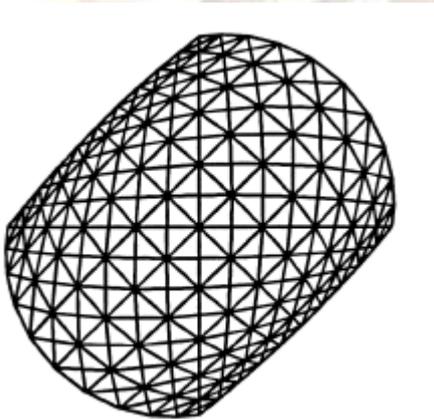
Suppose that, this grid is to be projected on an ellipsoid using the tractation retronorm and the descriptor for tractation may be defined as

$[P, x_a, y_a, z_a, S, x_c, y_c, z_c, a, b, c, t]$

where P is the projection specifier,  $x_a$ ,  $y_a$  and  $z_a$  are the coordinates of the centre of projection with respect to the global coordinate system. S is the surface specifier, where  $S=2$  implies an ellipsoid, with its centre given by the coordinates  $x_c$ ,  $y_c$  and  $z_c$  with respect to the global coordinate system and a, b and c are the semi-axes of the ellipsoid. Suppose that, the descriptor is specified as

$[1, 10, 10, -10, 2, 10, 10, -5, 30, 13, 15, 13]$

where the first parameter  $P=1$  implies central projection. A graphical representation of formex GRID projected on the ellipsoid is as shown in Fig. 11.

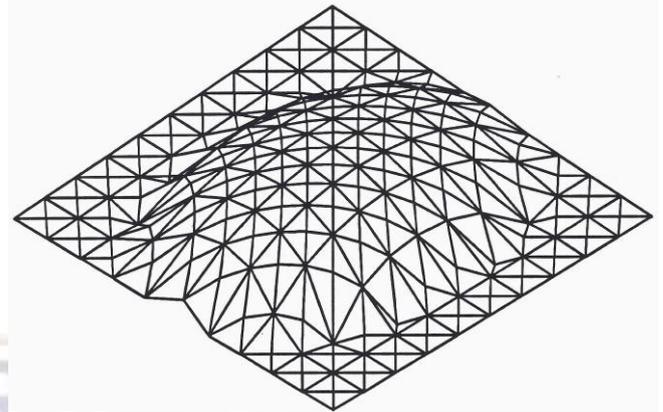


**Fig. 11.** A graphical representation of formex GRID projected on an ellipsoid using central projection

As, a further example of central projection consider the grid represented by the formex GRID and let this be projected onto an ellipsoid using a different radius and centre of projection. The other parameters in the descriptor remain as before. The descriptor in this case may be given by

$[1, 10, 10, 20, 2, 10, 10, -5, 9, 13, 10, 13]$

A graphical representation of formex GRID projected on the ellipsoid is shown in Fig. 12. In the descriptor the semi-axes of the ellipsoid has been chosen as  $a=9$ ,  $b=13$ ,  $c=10$ .



**Fig. 12.** A graphical representation of formex variable D, projected on an ellipsoid using central projection. Since the axis of the ellipsoid is smaller compared to the grid, only a part of the grid projected onto the ellipsoid.

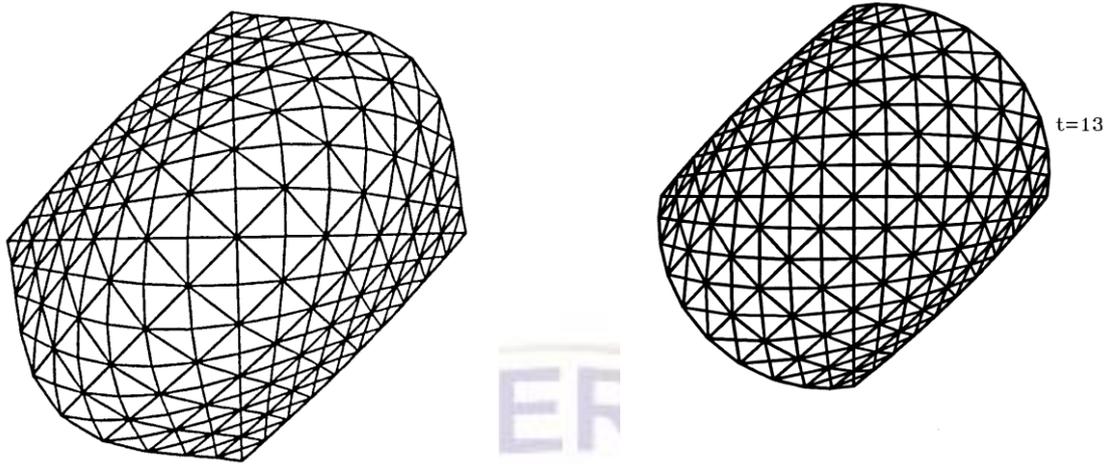
The length of the grid is equal to 20, since the ellipsoid is smaller as compared to the grid, only a part of the grid is projected and remains as before. As mentioned before in the tractation retronorm the user is able to choose between different types of projection, such as axial, parallel or radial. The radial projection is a special case of central projection. Radial projection may be used only for surfaces such as spheres, ellipsoids. The descriptor in this case may take the form

$[P, S, x_c, y_c, z_c, a, b, c, t]$

where the value of the projection specifier P for radial projection is equal 4. Also,  $x_c$ ,  $y_c$ ,  $z_c$  are the coordinates of the centre of ellipsoid which coincide with the centre of projection, relative to the global coordinate system. If the descriptor is specified as

$[4, 2, 10, 10, -5, 30, 13, 15, 13]$

then a graphical representation of formex GRID projected on the ellipsoid using radial projection is as shown in Fig. 13.



**Fig 13.** A graphical representation of formex GRID projected on the ellipsoid using radial projection.

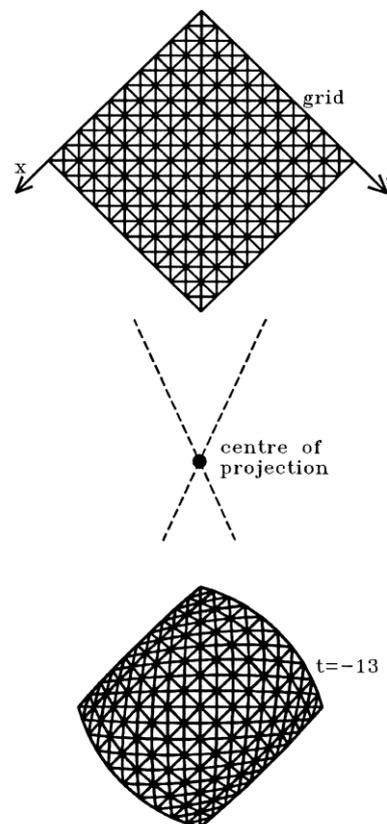
As a projection of the grid on the ellipsoid involves a quadratic equation, there may be two solutions. So far, in the examples given above the value of the selector has been specified as  $t=13$ , which stands for the solution with the larger  $z$ . Now, suppose that the grid is projected on the ellipsoid using central projection and the descriptor is given as

$$[1,10,10,-10,2,10,10,-5,30,13,15,-13]$$

where  $t=-13$  stands for the solution with the smaller  $z$ , a graphical representation is as shown in Fig. 14. It can be seen that each one of the values of the parameter  $t$  gives rise to a different configuration. The choice of the configuration is entirely dependent on the application and the requirement of the user.

## VI. FURTHER INTERCONNECTION PATTERNS

The polyhedral configuration shown in Fig 15 is obtained by mapping the configurations of Figs 16 (a) and (b) on different parts of a snub cube. In order to obtain the geodesic forms of Figs 17 and 18 the polyhedral configuration of Fig 15 is projecting on an ellipsoid and a sphere, respectively, using central projection. The geodesic form of Fig 19 is obtained by first mapping the patterns of Figs 20 (a) and (b) on the top part of a truncated



**Fig. 14.** A graphical representation of the formex GRID projected on an ellipsoid using as selector the values of  $t=13$  and  $t=-13$ .

dodecahedron and then projecting this on an ellipsoid. In the case of the polyhedral form of Fig 21 the patterns of Figs 22(a), 22(b), 22(c) and 22(d) are mapped onto the square, hexagonal and decagonal faces, respectively. The geodesic form of Fig 23 is obtained by projecting the polyhedral form of Fig 21 on a sphere using central projection.

## VII. CONCLUSION

For large and complex structural forms, the shear volume of information to be handled can

make data generation a time consuming and error prone task. To overcome this problem, suitable systems have been developed by which computer graphics and data generation for any type of structure can be done conveniently.

a variety of polyhedral and geodesic configurations. An important aspect of this paper is the establishment of the concepts and constructs through which polyhedral and geodesic configurations may be created with the polyhedron function. The polyhedron function provides a basis for the configuration processing of structural configurations in a compact and readily understood manner and it allows one to work with the same set of tools in all data generation problems eliminating the need for the employment of an assortment of programs which are dealing with specific problems.

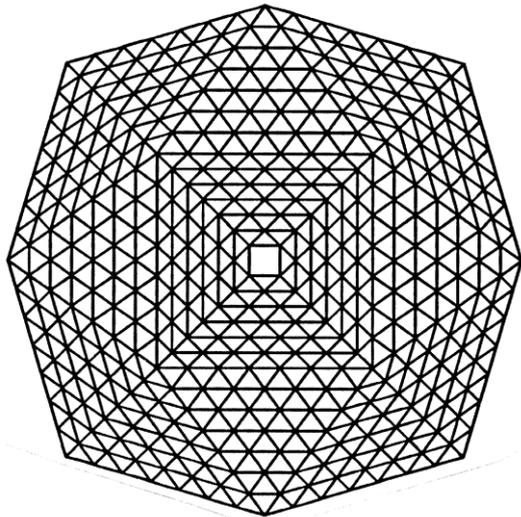


Fig. 15. A graphical representation of a snub cube

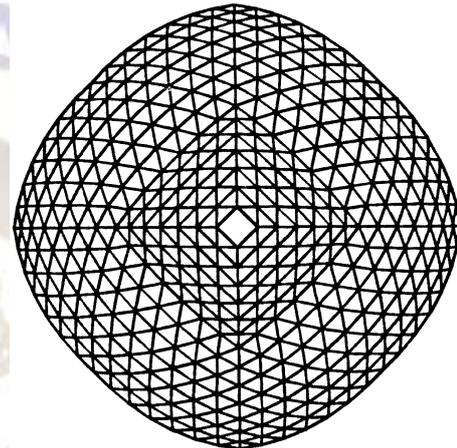


Fig. 18. A snub cube projected on a sphere

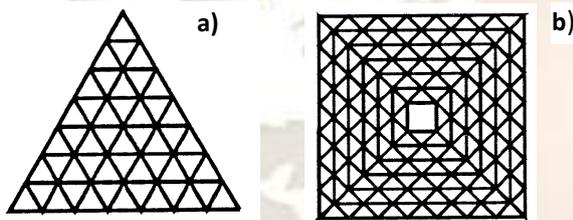


Fig. 16. The configurations of the triangular and square faces

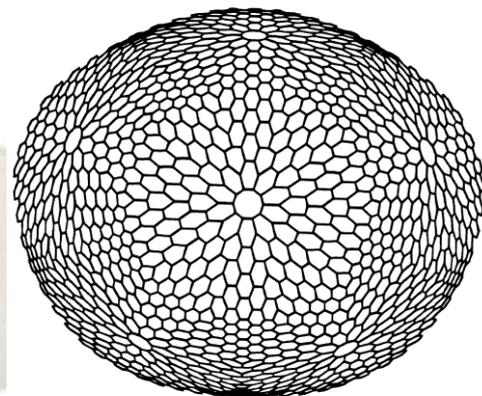


Fig. 19. A graphical representation of a truncated dodecahedron projected on an ellipsoid.

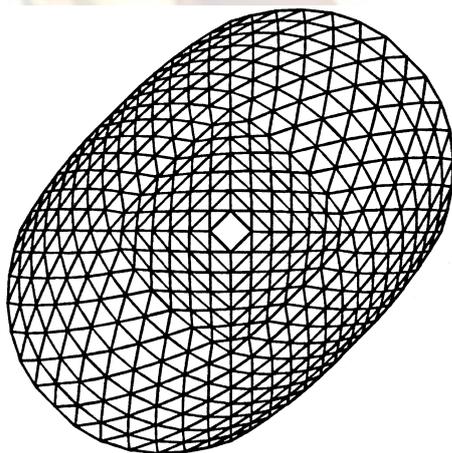


Fig. 17. A snub cube projected on an ellipsoid  
Formex algebra is one such mathematical system. The ideas of formex algebra can be applied to many branches of science and technology. In this paper the concepts of formex algebra and its programming language Formian have been described in relation to

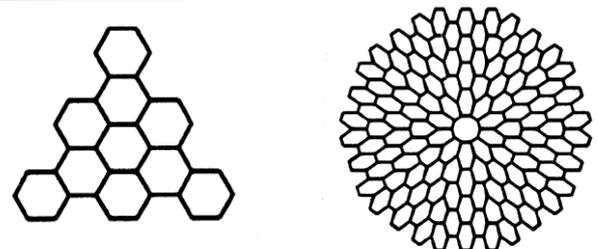
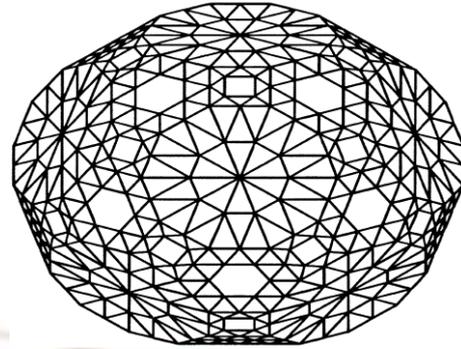


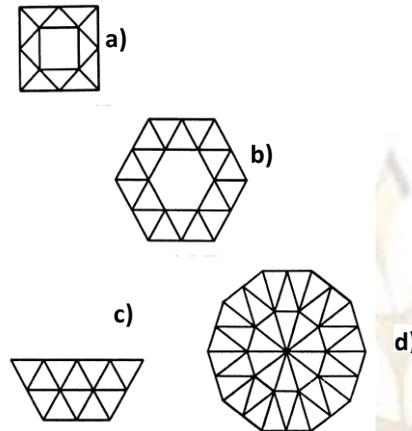
Fig. 20. The configurations that is to be mapped onto the faces of the truncated dodecahedron

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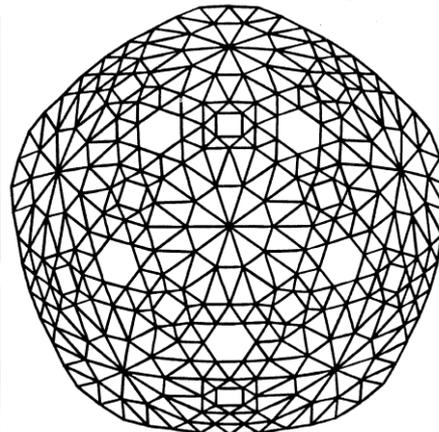
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**Fig. 21.** A graphical representation of a great rhombicosidodecahedron



**Fig. 22.** The configurations that is to be mapped onto the faces of the great rhombicosidodecahedron



**Fig. 23.** A great rhombicosidodecahedron projected on a sphere