

High Resolution Direction of Arrival Estimation (Coherent Signal Source DOA Estimation)

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ABSTRACT

High-resolution signal parameter estimation is a significant problem in many signal processing applications. Such applications include direction of arrival (DOA) estimation for narrow band signals and wideband signal emitted by multiple sources and received by sensor arrays. It is well known that MUSIC algorithm outperforms any other method existing in the literature. In this article a modified MUSIC algorithm is proposed using the conjugate data. Strong consistency of the modified method is established. It is observed that the modified MUSIC works significantly better than the ordinary MUSIC at different SNR in terms of the mean squared error and for coherently sources. However, an important disadvantage of the ULA geometry in DOA estimation is that it can only estimate the azimuth angle. To estimate the elevation angle, Uniform circular arrays UCA geometries and planar array are employed in many applications. As the concentric circular arrays (CCA) that are discussed in this paper have better angle resolutions compared to ULAs,

1. Introduction

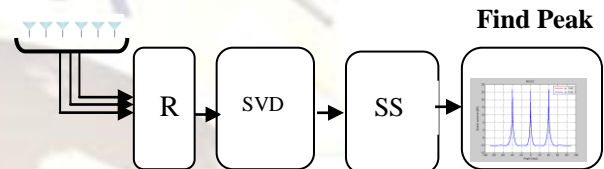
Many subspace decomposition methods divide the observation space into the signal and noise subspaces. The first step in these techniques is to estimate the signal and noise subspaces by decomposing the array correlation matrix into its Eigen-structure form. The subspace spanned by the eigenvectors of Covariance matrix corresponding to dominant Eigen-values is termed as Signal subspace. The detection methods use the fact that signal Eigen-values are larger than the noise Eigen-values. One of the most popular method is MUSIC (Multiple Signal Classification)

2. MUSIC Algorithm

MUSIC is an acronym which stands for Multiple Signal classification. It is high resolution technique based on exploiting the Eigen-structure of input covariance matrix. This approach was first posed by Schmidt. It is a simple, popular high resolution and efficient Method. It promises to provide unbiased estimates of the number of signals, the angles of arrival and the strengths of the waveforms[1].

A- Mathematical Model

The word MUSIC is an abbreviation of (Multiple Signal Classification),



Fig(1). MUSIC method block diagram.

A uniform linear array (ULA) composed of M sensors and d narrowband signals of the different DOAs $[\mathbf{a}(\theta_1) \mathbf{a}(\theta_2) \cdots \mathbf{a}(\theta_d)]$ was considered. Then, an observed snapshot from the M array elements was modeled as

$$x_k(t) = A(\theta)s(t) + v(t) \quad (1)$$

$\mathbf{x}(t)$: is the signal vectors at the array elements output.

$\mathbf{s}(t)$ is the signal vectors of the source.

$\mathbf{v}(t)$ is the noise vector at the array elements output.

$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1) \mathbf{a}(\theta_2) \cdots \mathbf{a}(\theta_d)]$ is the steering matrix .

$\mathbf{a}(\theta_d)$ is the array steering vector corresponding to the DOA of the d^{th} signal.

The array covariance matrix R of the received signal vector in the forward direction can be written as

$$R_{xx} = E[X(t)X^H(x)] = \frac{1}{L} \sum_{l=1}^L X(t)X^H(x) \quad (2)$$

Where K is the number of snapshot.

The MUSIC algorithm, a block diagram of which can be found in Fig. , can be summarized in as follows[2][3]:

1- First, N samples from each receiver channel must be collected to form $M \times N$ array. For simulation purposes, this array can be generated according to equation (2). Next, the covariance matrix \mathbf{R}_{xx} must be estimated from received data

2- Perform Eigen-value decomposition on \mathbf{R}_{xx}

$$\mathbf{R}_{xx} \mathbf{E} = \mathbf{E} \mathbf{\Lambda} \quad (3)$$

where $\mathbf{\Lambda} = \text{diag}\{\lambda_0, \lambda_1, \dots, \lambda_{M-1}\}$,

$\lambda_0 \leq \lambda_1 \leq \dots \leq \lambda_{M-1}$ are the Eigen-value

and $\mathbf{E} = [e_0 \ e_1 \ \dots \ e_{M-1}]$ are the corresponding eigenvectors of \mathbf{R}_{xx} .

3- From the multiplicity \hat{D} of the smallest Eigen-value λ_{\min} , estimate the number of signals \hat{K} :

$$\hat{K} = M - \hat{D} \quad (4)$$

4- Form the noise subspace \mathbf{E}_n from the

eigenvectors corresponding to the \hat{D} smallest Eigen-values. Determine the MUSIC spatial spectrum

$$P_{MUSIC}(\theta) = \frac{1}{\mathbf{a}(\theta)^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta)} \quad (5)$$

where $\mathbf{a}(\theta)$ is a search vector which is function of θ and is given by

$$\mathbf{a}(\theta) = [1, e^{j\theta}, e^{j2\theta}, \dots, e^{j(M-1)\theta}]^H \quad (6)$$

$$\mathbf{E}_n = [e_{d+1} \ e_{d+2} \ \dots \ e_{M-1}] \quad (7)$$

5- Find the \hat{K} largest peaks of $P_{MUSIC}(\theta)$. These correspond to the DOA of the received SOI(s).

B- Simulation Result

Uncorrelated sources

A coded simulation has been carried out to evaluate this method. In the simulation, we assume that three uncorrelated narrow band signals impinging a linear array of number of elements ($M=11$) at angles of arrivals 40° , 0° and -30° . The three signals are assumed to have equal signal to noise ratio (SNR=0 dB), the number of snapshots taken from the array is $N=128$, the detected signals computed using MUSIC algorithm against angles are shown in Fig(2). It is obvious that the detected peaks indicating the DOAs of the three signals which typically agree with the incident signals.

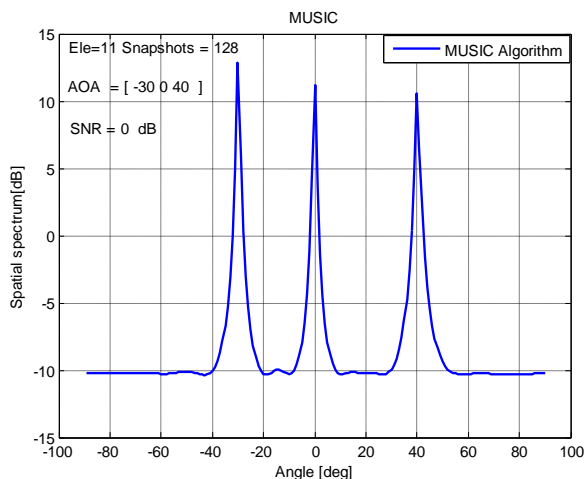


Fig (2). The spatial spectrum of the Bartlett estimator *Coherent sources*

A simulation program has been carried out to verify the capability of the MUSIC algorithm estimator.

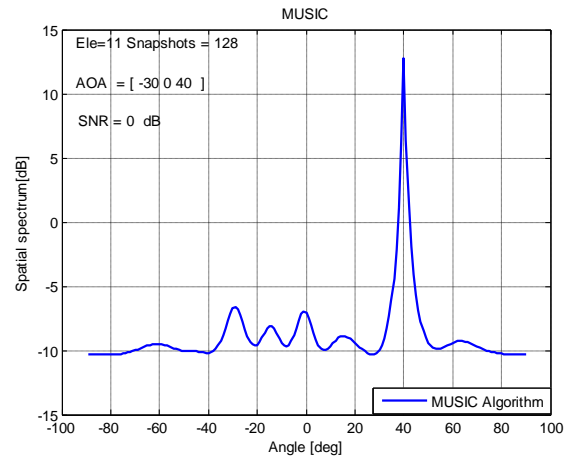
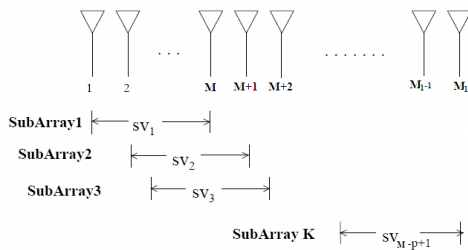


Fig (3). The spatial spectrum of the Bartlett estimator

In the simulation we considered the same assumptions that we have taken in the previous simulation program of the MUSIC except that the two signals completely coherent wide band $s_1(t)$ and $s_2(t)$ respectively coming from the at angles of arrival -30 and 0 degrees. Fig (3) shows the spatial spectral function obtained by the MUSIC algorithm in the case of correlated signal. It is seen that the direction of both the correlated signals at -30 and 0 degrees could not be estimated clearly, this means that, MUSIC algorithm cannot estimate the corresponding DOA of each signal. It is seen that the MUSIC method spectrum a search vector $\mathbf{a}(\theta)$ is needed to obtain the DOA of the received signals. Thus, to get better DOA resolution the scanning rate by the search vector should be very small, which inevitably results in a high computational complexity. To alleviate the computational overhead, the Root-MUSIC method technique is presented in next subsections.

2. Spatial Smoothing Technique

The MUSIC algorithm failed to estimate the DOA for correlated signals. To overcome this problem, the spatial Smooth technique is used for enhance MUSIC algorithm in the case of correlated signals. Correlated (coherent) signal can arise from multipath propagation where, due to reflections, the same signals arrives to an array multipath directions. Smart jammers can also create coherent signals. The spatial smoothing technique used to decorrelate correlated signal. By dividing the M elements into $(Q=M-P+1)$ overlapped sub arrays each of them contains P elements ($P < M$), as Fig (4)[4],[5].



Fig(4) Sub arrays for spatial smooth technique

The Spatial Smoothing Technique, a block diagram of which can be found in Fig. (4), can be summarized in as follows:

1- L elements (The antenna array $L > M$) are subdivided into P overlapping sub arrays that are translated versions of one another, with each sub array having $Q_{sm} = Q - P + 1$ sensors as Fig (4). The received signal by the K^{th} sub arrays is therefore given by ,

$$x_k(t) = A(\theta)B^{k-1}s(t) + n_k(t) \quad (8)$$

Where B is a $(D \times D)$ diagonal matrix defined as $B = \text{diag}[e^{-j\theta}, \dots, e^{-jD\theta}]$.

2- Find the correlation matrix of the K^{th} sub array from equation (). Then the K^{th} correlation matrix of the K^{th} sub arrays is given by

$$R_{xx} = A(\theta)B^{k-1}R_s(B^H)^{k-1}A^H(\theta) + \sigma^2I \quad (9)$$

The total spatially smoothing correlation matrix is defined as the sample means of the sub arrays correlation.

$$\bar{R} = \frac{1}{P} \sum_{k=1}^P R_k \quad (10)$$

Then using equation (10) we can rewrite as

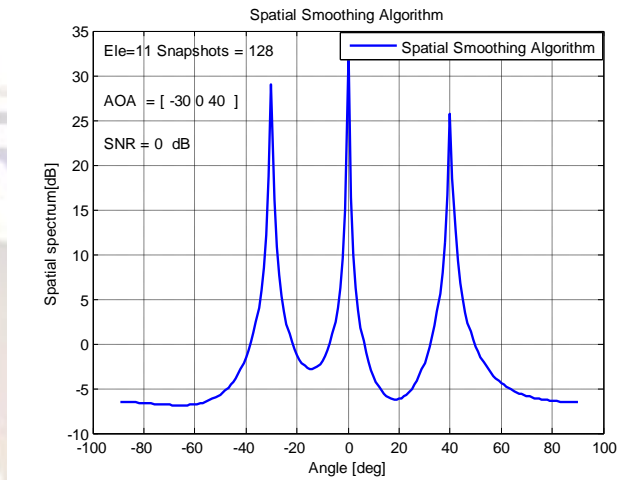
$$\bar{R} = A(\theta) \left(\frac{1}{P} \sum_{k=1}^P B^{k-1} R_s (B^H)^{k-1} \right) A^H(\theta) + \sigma^2 I \quad (11)$$

When $P > D$, the modified signal covariance matrix will be nonsingular (full rank) regardless of the correlated of the signals. Then the average of these correlation matrices $1 \leq K \leq Q$ will results in non-singular correlation matrix regardless of the correlation of the incident signals. Then the spatial smoothing correlation matrix has the same form as the un-correlation matrix case if we apply the MUSIC algorithm [4],[5].

3- Then applying the MUSIC algorithm.

A- Simulation Result

A simulation program has been carried out to verify the capability of the Spatial Smoothing Technique algorithm estimator. In the simulation we considered the same assumptions that we have taken in the previous simulation program of the MUSIC except that the two signals completely coherent wide band $s_1(t)$ and $s_2(t)$ respectively coming from the array from -30 and 40 degrees.



4. Amendment MUSIC Algorithm Coherent Signal Source DOA Estimation

Fig(5) shows the spatial spectral function obtained by the Spatial Smoothing Technique in the case of correlated signal. It is seen that the direction of both the correlated signals at -30 and 40 degrees could be estimated clearly, this means that, Spatial Smoothing Technique estimate the corresponding DOA of each signals.

A. Introduction

MUSIC algorithm is an effective method for direction of arrival(DOA), but it can only do with uncorrelated signals. There are coherent signal and related signal in the actual communication environment. If the condition does not meet, there will be bias occurred and even failure in the use of MUSIC algorithm for signal DOA estimation. In order to solve the problem of the DOA estimation of coherent signals, an improved algorithm is presented in this section. By processing the covariance matrix of the array output signal, which can effectively estimate the signal DOA and identify the coherent signal source. Simulation results show that the DOA can be correctly estimation when the signal interval is relatively small and the signal to noise ratio is small. This indicates that the algorithm is effective and has practical value in engineering.

B. Theory

A uniform linear array (ULA) composed of M sensors and d narrowband signals of the different DOAs $[\mathbf{a}(\theta_1) \mathbf{a}(\theta_2) \dots \mathbf{a}(\theta_d)]$ was

considered. Then, an observed snapshot from the M array elements was modeled as

$$x_k(t) = A(\theta)s(t) + v(t) \quad (12)$$

The array covariance matrix R of the received signal vector in the forward direction can be written as

$$R_{xx} = E[X(t)X^H(x)] = \frac{1}{L} \sum_{L=1}^K X(t)X^H(x) \quad (13)$$

The matrix R is called *centrohermitian* if the following condition is satisfied [6] as

$$R = JR_{xx}J^H \quad (14)$$

where J represents the exchange matrix, i.e., 1's on the antidiagonal and 0's elsewhere as

$$J = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

The FB averaging can be written as [6]

$$R_{FB} = \frac{1}{2}(R_{xx} + JR_{xx}J^H) \quad (16)$$

The proposed method employs the FB averaging for the covariance matrix of the observation data. Here, the forward covariance matrix is obtained from equation (2) and the backward covariance matrix is $JR_{xx}J^H$. This FB averaging has two advantages:

- (1)- it is equivalent to doubling the number of snapshots of the data.
- (2)- correlated sources can be decorrelated. Then the DOAs of the multiple incident signals can be estimated by locating the peaks of the MUSIC spectrum given by

$$P_{MUSIC}(\theta) = \frac{1}{a(\theta)^H E_n E_n^H a(\theta)} \quad (17)$$

where $E_n = [e_{d+1} \ e_{d+2} \ \dots \ e_{M-1}]$ is subspace noise. Or alternatively [4] from matrix S divided the diagonal into two array as:

$$P_{MUSIC}(\theta) = \frac{a^H(\theta) * RA * a(\theta)}{a(\theta)^H E_n E_n^H a(\theta)} \quad (18)$$

from decomposition of correlation matrix R_{xx} as

$$SVD(R_{xx}) = U.S.V^H$$

where RA can be calculated as

$$RA = E_s B E_s^H \text{ and}$$

$E_s = [e_1 \ e_{d+2} \ \dots \ e_{d-1}]$ is signal subspace

where $B =$ diagonal $(1/SS\text{-sigma} * I)$

$$SS = \text{diagonal}(S_s) \quad SN = \text{diagonal}(S_n)$$

$$\sigma = \text{tr}(S_n) / (M - D)$$

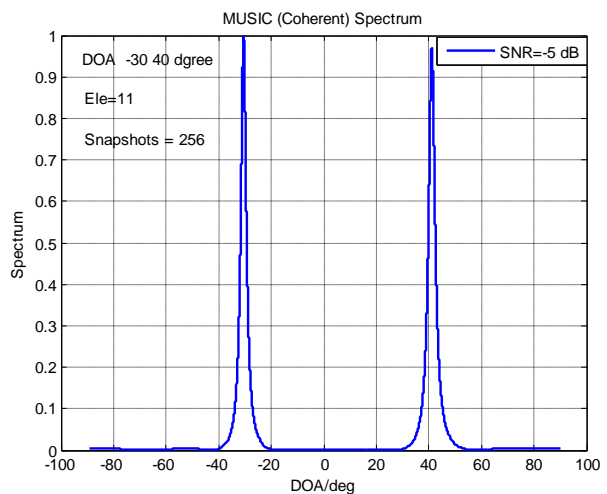
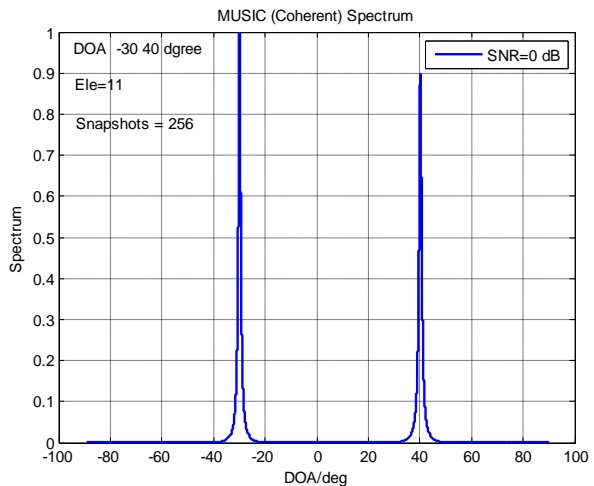
$$f_{music}(\theta) = \frac{a^H(\theta) * RA * a(\theta)}{a^H(\theta) E_n * E_n^H a(\theta)} \quad (19)$$

As noticed from the denominator, the orthogonality between $a(\theta)$ and UO will make it minimum, and hence will increase $f_{music}(\theta)$. Hence the \hat{D} largest peaks of the MUSIC spectrum correspond to the DOAs of the signals impinging on the array.

C. Simulation Results

Coherent sources

A coded simulation has been carried out to evaluate this method. In the simulation, we assume that three the two signals completely coherent wide band $s_1(t)$ and $s_2(t)$ respectively coming from the at angles of arrival -30 and 40 degrees impinging a linear array of number of elements ($M=11$).



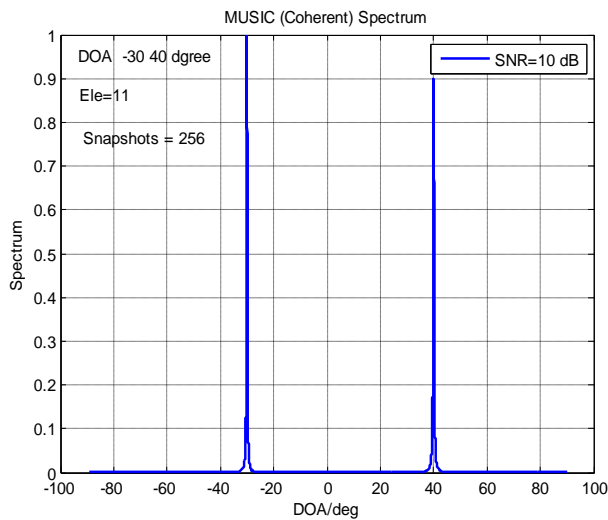


Fig (6) The spatial spectrum of the Amendment MUSIC algorithm

Fig(6). It is obvious that the detected peaks indicating the DOAs of the two signals which typically agree with the incident signal.

5. Behavior analysis of proposed beamformer

A. Effect The Number of Elements of The Antenna Array on DOA's Estimation

The effect of the number of array elements on the DOA estimation is shown in following figure. From Fig (7) it is clear, that as number of array elements increase as Amendment MUSIC technique can accurately estimate the DOA of incident signals.

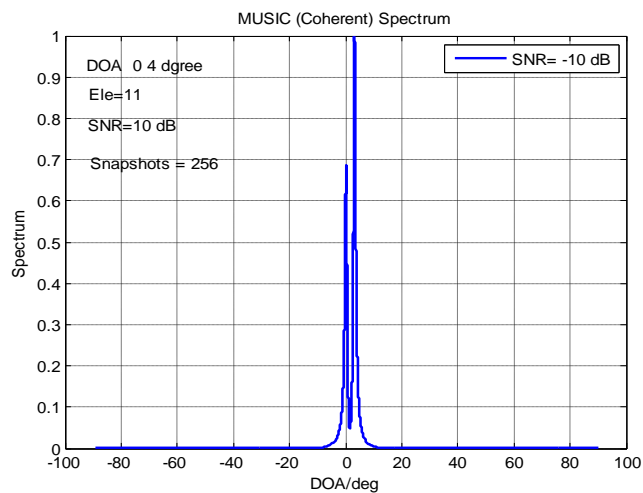


Fig (7). The Effect The Number of Elements of The Antenna Array on DOA's Estimation

Works properly even when. The angle separation between incident signals is not wide enough. This is shown in Fig (8). It is assumed that there are three uncorrelated signals at the input of the array also, it is that the SNR=0dB and snapshots N=256 samples. The performance of the DOA's estimation is evaluated for different value of number of array elements. The Amendment MUSIC algorithm can resolve angle separation up to 4° for AOA's 0° and 4° as shown Fig (8). The resolution will be improved by increase the number of array elements and decrease the beam width (makes narrower beam) then good separation.

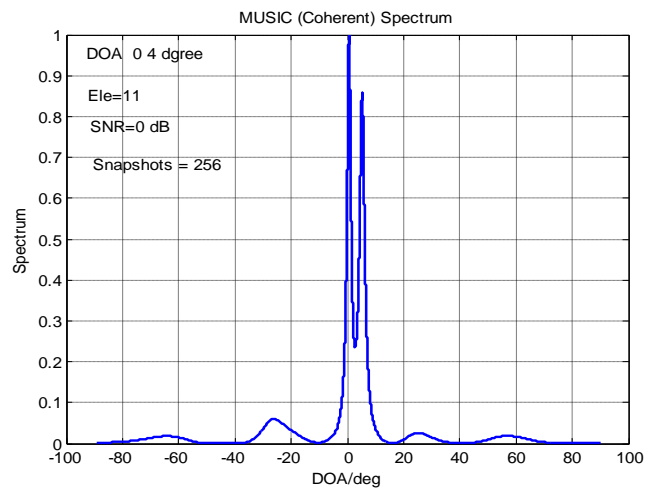


Fig (8) The spatial spectrum of the Amendment MUSIC algorithm (separation 4°)

B. Effect of The Signal to Noise ratio (SNR) on DOA's Estimation

The effect of the SNR on the DOA estimation is shown in following figures. It is clear from Fig (9). that as SNR increase as Amendment MUSIC and Delay and Sum Method can accurately estimate the DOA of incident signals. This is because increase the SNR, will makes narrower beam width around incident signals directions.

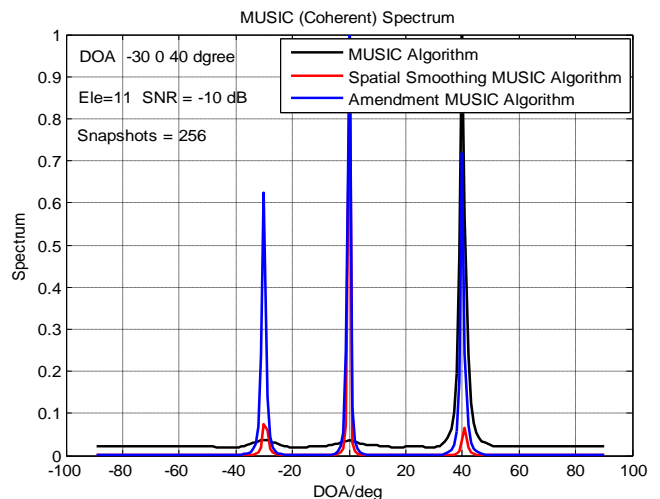


Fig (9). The Effect The SNR of DOA's Estimation

Works properly even when. The angle separation between incident signals is not wide enough. This is shown in Fig (10).

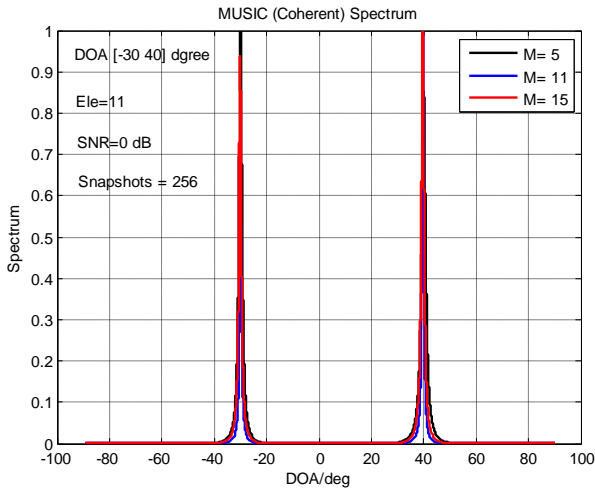


Fig (10) The spatial spectrum of the Amendment MUSIC algorithm (separation 4°)

It is assumed that there are three uncorrelated signals at the input of the array also, it is that the SNR=-10dB and snap shots N=256 samples. The performance of the DOA's estimation is evaluated for different value of number of array elements. The Amendment MUSIC algorithm can resolve angle separation up to 4° for AOA's 0° and 4° as shown Fig (11). The resolution will be improved by increase the SNR and decrease the beam width (makes narrower beam) then good separation.

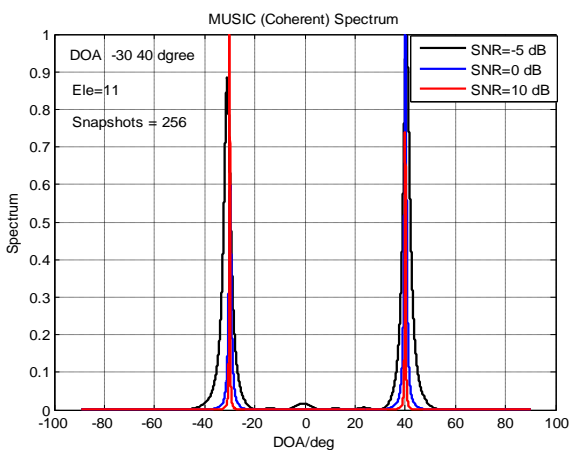


Fig (11) Comparison the MUSIC , Spatial MUSIC and Amendment MUSIC algorithms for coherent sources

Here in Fig (11). the proposed algorithm gives accurate DOAs estimations compared with Spatial

Smoothing MUSIC, and all the sources are detected even if the DOAs of the sources are coherent, whereas in the same Figure the MUSIC algorithm with fails to detect the three sources. The estimation of direction-of-arrival (DOA) of source signals has been well-studied research area in both military and civil applications. The MUSIC (Multiple Signal Classification) algorithm based on the signal subspace separation idea has already been investigated and employed for uniform linear arrays (ULA). However, an important disadvantage of the ULA geometry in DOA estimation is that it can only estimate the azimuth angle. To estimate the elevation angle, Uniform circular arrays UCA geometries and planar array are employed in many applications. As the concentric circular arrays (CCA) that are discussed in this paper have better angle resolutions compared to ULAs, and as they require smaller physical area compared to UCAs with the same number of array elements, CCAs are proposed to be used in mobile applications as a preferable alternative. The DOA estimation problem for CCAs is defined and the steering matrix is expressed in this paper, moreover the performance analysis is investigated by simulations[7],[8].

6- 2D Direction of Arrival Estimation

A- Direction of Arrival Estimation using Circular Arrays

Circular array - antenna elements arranged around a circular ring. In the last two decades, adaptive antenna has been widely used in many applications such as radar, sonar and wireless communication systems. Considerable research efforts have been made to estimate the direction of arrival (DOA) and various array signal process techniques for DOA estimation have been proposed. In particular, the DOA estimation for uniform circular arrays (UCAs) has been developed in these scenarios, which desired all-azimuth angle coverage. By the virtue of their geometry, UCAs are able to provide 360° of coverage in azimuth plane. Moreover, they are known to be isotropic. That is, they can estimate the DOA of incident signal with uniform resolution in the azimuth plane. In addition, direction patterns synthesized with UCAs can be electronically rotated in the plane of the array without significant change of beam shape [7-9]. The most commonly used DOA estimation techniques is spectrum based method, such as subspace-based algorithm solutions, such as multiple signal classification (MUSIC) [7]. The techniques mentioned were mainly developed for uniform linear arrays (ULAs). Some of these algorithm, such as UCA-MUSIC [9], and their variations have been expanded for UCAs. UCA-MUSIC techniques as the MUSIC for ULAs, the DOAs are determined by finding the directions for which their antenna response vectors lead to peaks

in the MUSIC spectrum formed by the eigenvectors of the noise subspace. Thus, the capacity of DOA estimation using UCA-MUSIC is bounded by the number of antenna elements. The application of the above techniques is limited to cases where the number of signal sources is less than the number of antenna elements. These techniques require subspace estimation and Eigen decomposition leading to high computational complexity, there by limited their use to applications where fast DOA estimation is not required. Further, in the presence of interference, these techniques need to estimated the DOAs of all target signals and interference, which decreases the accuracy of the DOA estimation.

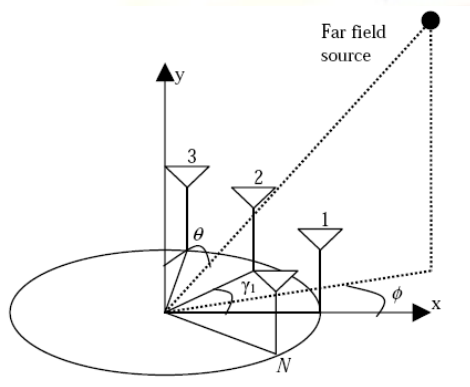


Fig.(12) Uniform circular array geometry

The MUSIC algorithm estimates the DOAs from the D deepest nulls of the UCA - MUSIC function [7-8-9]:

$$f_{UCA-MUSIC}(\phi, \theta) = a^H(\phi, \theta) E_n E_n^H a(\phi, \theta) \quad (20)$$

where E_n is called the noise subspace which of matrix R_{xx} that correspond to the smallest Eigen-values from the noise subspace and composed of $E_n = [e_{D+1}, \dots, e_{D+2}, \dots, e_{M-1}]$ and $a(\phi, \theta) = [a_1(\phi, \theta), a_2(\phi, \theta), \dots, a_d(\phi, \theta)]^T$ is a steering vector with dimension d . The peak values of Eq. (20) represent the estimated values of 2-D angles of arrival.

Simulation Result: A UCA of radius $r = 2/\lambda$ for (uniform circular array can be determined by: $d = 2r \sin(\pi/M)$). being the maximum phase mode excited, was employed for the simulations. The number of array elements was chosen to be $M = 11$. In each of the simulation examples outlined below, two equal powered sources were located at $\phi = 50^\circ, 300^\circ, \theta = 50^\circ, 30^\circ$, 256 samples of data were taken from the array, and algorithm performance for each case was analyzed based on an average over 1000 independent trials. the detected signals computed using MUSIC Method against angles are shown in Figure. Fig.(13). shows

the spatial spectrum of the received signal. There is peak indicating the DOAs of the mentioned signals

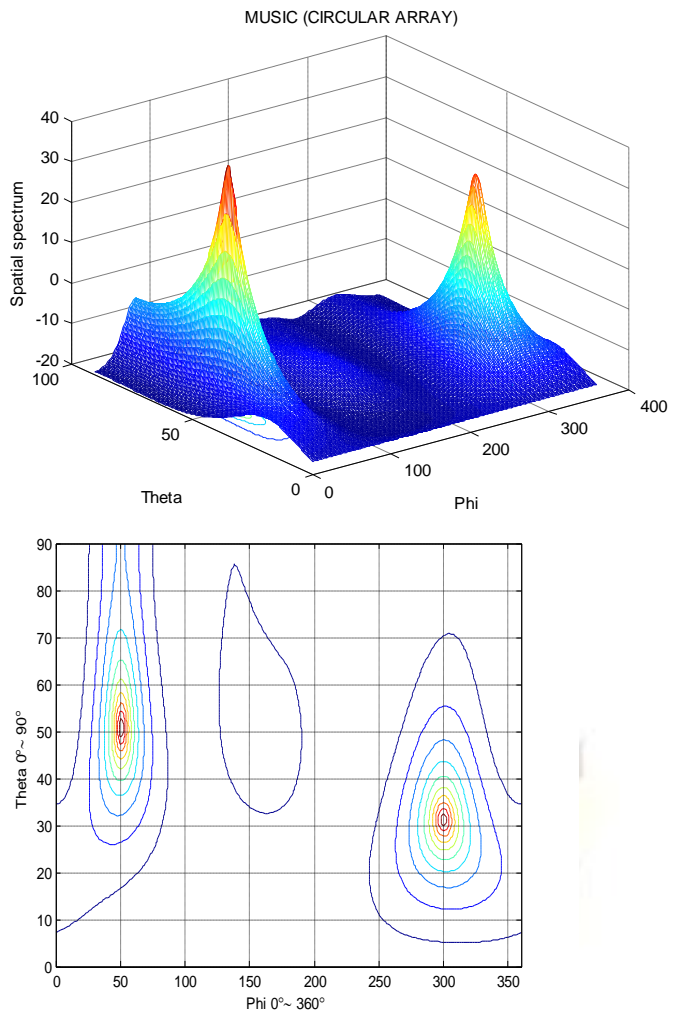


Fig (13). The spatial spectrum of the MUSIC algorithm for Circular array antenna

B- Direction of Arrival Estimation using Planar arrays

Planar arrays provide directional beams, symmetrical patterns with low side lobes, much higher directivity (narrow main beam) than that of their individual element. In principle, they can point the main beam toward any direction. Applications – tracking radars, remote sensing, communications, etc.

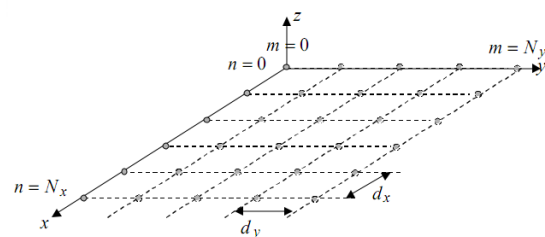


Fig.(14) rectangular planar array

The MUSIC algorithm estimates the DOAs from the D deepest nulls of the UCA - MUSIC function:

$$f_{URA-MUSIC}(\phi) = a^H(\phi, \theta) E_n E_n^H a(\phi, \theta) \quad (21)$$

Simulation Result: A coded simulation has been carried out to evaluate this method. In the simulation, we assume that narrow band signals impinging a rectangular array of number of elements ($M=11, N=11$) at angles of arrivals $\phi = 20^\circ, -20^\circ, \theta = -30^\circ, 30^\circ$. SNR=0 dB, the number of snapshots taken from the array is $N=128$, the detected signals computed using MUSIC Method against angles are shown in Figure. Fig(14). shows the spatial spectrum of the received signal. There is peak indicating the DOA, of the mentioned signal.

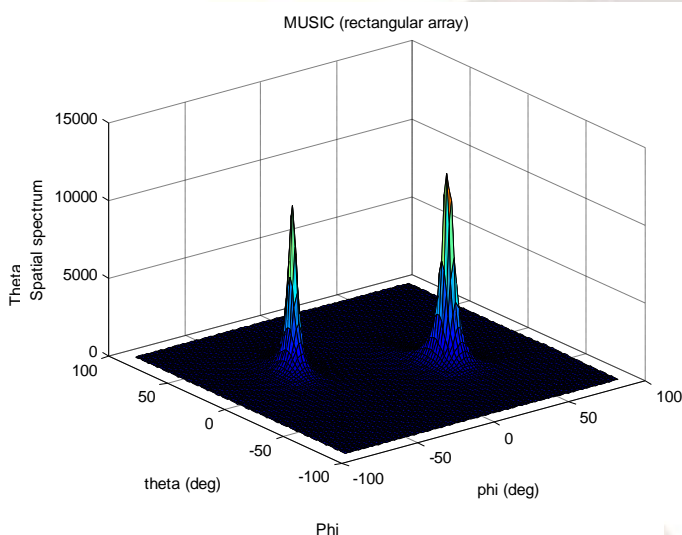


Fig (15). The spatial spectrum of the MUSIC algorithm for Rectangular array antenna

The simulations show that the MUSIC method doubles the maximum estimation number of standard MUSIC. Using uniform-circular signals, the performance of URAs is improved remarkably while the improvement of UCAs is not so significantly. Moreover, the influence of arrays structures on the MUSIC method is discussed. With the direction of the incoming signals known or estimated, the next step is to use spatial processing techniques to improve the reception performance of the receiving antenna array based on this information. the MUSIC algorithm modified for work on coherent DOA's shown from figure It is obvious that the detected peaks indicating the DOAs of the two signals which typically agree with the incident signal.

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