Efficiency Estimation Of Electric Machine Using Magnetic Flux Circuits

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Abstract:
Improving efficiency of electrical machine is of paramount importance to optimization of operational and resource management. Efficiency method which do not account for iron loss could undermine the effective efficiency estimation. This paper simulates the efficiency variations of electric machine in both transient and steady state, magnetic flux linkages are used instead of the traditional current method, though also in d-q axis. Only magnetic flux linkages are used in an arbitrary reference frame while machine efficiency in various reference frames namely; rotor, stator and the synchronous reference are investigated. Results are compared with measured quantities and excellent estimations are obtained.

Keyword: electrical machine efficiency, machine losses, magnetic circuit, generalized machine equation

1 INTRODUCTION
Estimation of electric machine performance through efficiency estimation enables electric machine engineers to device control that could lead to improvement in electric machine efficiency, for example in electric machine losses control [1]. Without a robust and good simulation method, neither good estimation of efficiency nor deep insight about loses variations could be obtained. Many researches have been carried out using magnetic circuit simulation techniques, in 1968 carpenter [2] introduced and explored the possibility of modeling eddy – current using magnetic equivalent circuits, which are analogous to electric equivalent circuits. Magnetic terminal are introduced which provide a useful means of describing the parameters in electromagnetic devices, particularly those operation depends upon induced current behavior. The virtual – work [3 – 4] principle was suggested for torque calculations and exploited in [7]. Generally, the magnetic equivalent circuit modeling method can be used to analyze many types of electrical machine in terms of physically meaningful magnetic circuits. For example, Ostovic’ evaluated the transient and steady state performance of squirrel cage induction machines by using this magnetic equivalent method [5] [6], taking into account the machine geometry, the type of windings, rotor skew, the magnetizing curve etc. In the past two decades, magnetic equivalent circuit modeling method for electrical machines has been further developed and applied to modeling many other types of machines, apart from induction machines: synchronous machines by Haydock [8], inverter–fed synchronous motors by Carpenter [10], permanent magnet motors and claw-pole alternators by Roisse [11], and 3-D Lundell alternators, which are main source of electric energy in internal combustion engine automobiles, by Ostovic’ [9].

Sewell developed Dynamic Reluctance Mesh software for simulating practical induction machines based on the previous work using magnetic circuit models for electrical machines [7]. This algorithm was developed from rigorous field theory. Combined with prior known knowledge of electrical machine behavior determined from practical experience, this model is simplified in a physically realistic manner and results in a procedure that can efficiently simulate 3D machines within an iterative design environment without requiring extensive computational resources. Its computation time was minimized by direct computation of the rate of change of flux, thus the evaluation of time and rotor position dependent inductances are avoided. Yet there is a similar dynamic mesh modeling method but with some additional developments in modeling and improvements in the solution process [12].

Many different efficiency methods have been developed with a field suitably as a goal. These estimation methods have been enlisted in a thorough study on efficiency estimation techniques performed by J. S. Hsu et al [13] and [14], which divides efficiency techniques into the following classes: nameplate method, slip method, current method, statistical method, equivalent circuit method, segregated loss method, air gap torque method and shaft torque method. In this paper efficiency is estimated through simulation of magnetic flux linkages with a generalized arbitrary reference equation transformed to all flux linkage variables which are used in formulae of loss minimization techniques [1, 13 – 14].

2 All Flux Linkages Generalized Machine Equation In An Arbitrary Reference Frame
In developing the conventional machine model for transient equation, the assumptions made were stated in [15] as thus:

1. The machine is symmetrical with a linear air-gap and magnetic circuit.
2. Saturation effect is neglected.
3. Skin-effect and temperature effect are neglected.
4. Harmonic content of the mmf wave is neglected.
5. The stator voltages are balanced.

The differential equations governing the transient performance of the induction machine can be described in several ways; they only differ in minor detail and in their suitability for use in a given application. The conventional machine model is developed using the traditional method of reducing the machine to a two-axis coil (d-q axis) model on both the stator and the rotor as described by Krause and Thomas [16].

2.1 D-Q Axis Transformation Theory

Stator variables (currents, voltages and flux linkages) of a synchronous machine were first related with variables of a fictitious windings rotating with rotor by park [17]. This method was further extended by Keyhani [18] and Lipo [19] to the dynamic analysis of induction machines. By these methods, a polyphase winding can be reduced to a set of two phase-windings with their magnetic axes aligned in quadrature as shown in Figure 1.

![Figure 1: Polyphase winding and d-q equivalent.](image)

The d-q axis transformation eliminates the mutual magnetic coupling of the phase-windings, thereby making the magnetic flux linkage of one winding independent of the current in the other winding. For three phases the transformations are follows:

\[ i_{q0} = T_{abc} i_{abc} \]  \hspace{1cm} (1)

also

\[ V_{q0} = T_{abc} V_{abc} \]  \hspace{1cm} (2)

Equations (1-2) can be applied in any reference frame by making a suitable choice for theta (θ) in equation (3).

If theta equals \( \theta_r \), then equations (1-2) lead to an expression for voltage in the rotor reference frame. Also, if \( \theta \) is equal to zero, then equations (1-2) apply to a frame of reference rigidly fixed in the stator (i.e. stationary reference frame). Otherwise, for \( \theta \) equal to \( \theta_r \) in equations (1-2), a synchronously rotating reference frame results.

2.2 Model Equation

The generalized 4\(^{th}\) order dq model equation of electrical machine [20] expressed in an arbitrary reference frame rotating with arbitrary velocity \( \omega_r \) are the following, with the stator and rotor fluxes as state variables:

\[
V_{qs} = r_s i_{qs} + p \lambda_{qs} + \omega_c \lambda_{qs} \\
V_{ds} = r_s i_{ds} + p \lambda_{ds} - \omega_c \lambda_{qs} \\
V_{qr} = r_f i_{qr} + p \lambda_{qr} \left( \omega_r - \omega_c \right) \lambda_{qr} \\
V_{dr} = r_f i_{dr} + p \lambda_{dr} \left( \omega_r - \omega_c \right) \lambda_{qr}
\]  \hspace{1cm} (4)

where flux equations

\[
\lambda_{ds} = L_s i_{ds} + L_m i_{dr} \\
\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \\
\lambda_{dr} = L_r i_{dr} + L_m i_{ds} \\
\lambda_{qr} = L_r i_{qr} + L_m i_{qs}
\]  \hspace{1cm} (5)

3 Model equations in all flux variables

When dq current variables \( i_{dq} \), \( i_{db} \), \( i_{qg} \) and \( i_{qr} \) are solved for in equation (5) and substituted in equation (4) accordingly, a generalized model equation in all flux linkages of arbitrary reference is realized as follows.

For current variables as function of linkage fluxes:

\[
i_{ds} = \frac{L_s \lambda_{qs} - L_m \lambda_{dr}}{L_s L_r - L_m^2} \\
i_{dr} = \frac{L_s \lambda_{dr} - L_m \lambda_{qs}}{L_s L_r - L_m^2} \\
i_{qs} = \frac{L_r \lambda_{qs} - L_m \lambda_{qr}}{L_s L_r - L_m^2} \\
i_{qr} = \frac{L_r \lambda_{qr} - L_m \lambda_{qs}}{L_s L_r - L_m^2}
\]  \hspace{1cm} (6)

When equation (6) is substituted in equation (4) and simplified accordingly, a generalized electric machine magnetic flux based equation is obtained as:

\[
\dot{\lambda}_{qs} + a \lambda_{qs} - c \lambda_{qr} + \omega_c \lambda_{ds} = V_{qs} \\
\dot{\lambda}_{ds} + a \lambda_{ds} - c \lambda_{qs} = V_{ds} \\
\dot{\lambda}_{qr} + b \lambda_{qr} - d \lambda_{qs} + f \lambda_{dr} = V_{qr} \\
\dot{\lambda}_{dr} + b \lambda_{dr} - d \lambda_{qs} - f \lambda_{qr} = V_{dr}
\]  \hspace{1cm} (7)

where

\[
a = \frac{r_s L_s}{L_s L_r - L_m^2}, \quad b = \frac{r_s L} {L_s L_r - L_m^2}, \\
c = \frac{r_s L_m}{L_s L_r - L_m^2}, \quad d = \frac{r_f L_r}{L_s L_r - L_m^2}, \\
f = \left( \omega_r - \omega_c \right)
\]  \hspace{1cm} (8)
4 The Efficiency Of Electric Machine In Dq – Flux Linkage Variable

Three major losses that are accounted for in this analysis are stator winding losses, rotor winding losses and iron losses. The various expressions of all these losses are given [1] as functions of dq – currents, also the stator input power is the total power. To find the efficiency in dq – flux linkage variables all the dq – current variables must be eliminated as follows

Stator power losses

\[ P_{cs} = 3r_i i_s^2 = \frac{3}{2} r_s \left( i_{ds}^2 + i_{qs}^2 \right) \]  
(9)

Rotor power losses

\[ P_{cr} = 3r_i i_r^2 = \frac{3}{2} r_i \left( i_{dr}^2 + i_{qr}^2 \right) \]  
(10)

Iron losses

\[ P_i = 3r_i i_i^2 = \frac{3}{2} r_i \left( i_{dm}^2 + i_{qm}^2 \right) \]  
(11)

Power input

\[ P_{in} = \frac{3}{2} [V_{qs} i_{qs} - V_{ds} i_{ds}] \]  
(12)

where

\[ i_s = i_{ds} + j i_{qs} \]
\[ i_r = i_{dr} + j i_{qr} \]
\[ i_m = i_{dm} + j i_{qm} \]
\[ i_{dm} = i_{ds} + i_{dr} \]
\[ i_{qm} = i_{qs} + i_{qr} \]

hence

\[ i_s^2 = i_{ds}^2 + i_{qs}^2 \]
\[ i_r^2 = i_{dr}^2 + i_{qr}^2 \]
\[ i_m^2 = i_{dm}^2 + i_{qm}^2 \]

substituting equation (6) in (9 – 12) and simplifying accordingly to get

\[ P_{cs} = 3r_i i_s^2 = \frac{3}{2} r_s \left( i_{ds}^2 + i_{qs}^2 \right) \]
\[ = \frac{3}{2} r_s \left( \frac{1}{L_r L_s - L_m} \right)^2 \times \]
\[ \left( L_r A_{ds} - L_m A_{dr} \right)^2 - \left( L_s A_{qs} - L_m A_{qr} \right)^2 \]  
\[ P_i = 3r_i i_i^2 = \frac{3}{2} r_i \left( i_{dm}^2 + i_{qm}^2 \right) \]
\[ = \frac{3}{2} r_i \left( \frac{i_m \omega_s}{r_i} \right)^2 \left( \frac{1}{L_r L_s - L_m} \right)^2 \times \]
\[ \left( L_r A_{ds} - L_m A_{dr} \right)^2 - \left( L_s A_{qs} - L_m A_{qr} \right)^2 \]  
(16)

\[ P_{cr} = 3r_i i_r^2 = \frac{3}{2} r_r \left( i_{dr}^2 + i_{qr}^2 \right) \]
\[ P_{in} = \frac{3}{2} V_{qs} i_{qs} - V_{ds} i_{ds} \]
\[ \eta = \frac{P_m - \Sigma \text{losses}}{P_{in}} \]
\[ P_m \rightarrow \text{Power input} \]
\[ \Sigma \text{losses} = P_{cs} + P_{cr} + P_i \rightarrow \text{Total Losses} \]

5 Transient Simulation Of Electric Machine Dq – Flux Linkages

In this paper transient dq – flux linkages of electric machine are simulated by transforming the arbitrary reference frame electric machine equation in its magnetic flux equivalent (7) from time domain to laplace frequency domain, and hence rational function of the flux linkages in s – domain are the result of such transformation. The inverse transforms of these flux linkages (rational functions in s – domain) are close – form transient time domain solution of electric machine flux linkages. Discretization can now be achieved at desired time point from the close – form continuous time function of flux linkages obtained above.

The transformation of the magnetic flux equivalent of the generalized electric machine (7) to laplace frequency domain may be achieved as follows
From equation (7), taking the laplace transform of (7(a)) to get

\[ \lambda_f (s) - \lambda_f (0) + a \lambda_f (s) - c \lambda_f (s) + \omega_c \lambda_f (s) = V_d (s) \]
\[ s + a \lambda_f (s) + \omega_c \lambda_f (s) - c \lambda_f (s) + 0 = V_d (s) + \lambda_f (0) \]  

Also
\[ -\omega_c \lambda_f (s) (s + a \lambda_f (s)) + 0 - \omega_c \lambda_f (s) = V_d (s) + \lambda_f (0) \]  
\[ -d (s + a \lambda_f (s)) + \omega_c (s + b \lambda_f (s)) + \lambda_f (0) \]  
\[ 0 - d (s + a \lambda_f (s)) + \omega_c (s + b \lambda_f (s)) + 0 = V_d (s) + \lambda_f (0) \]  

Equation (20 – 23) may be rearranged in compact form to get
\[ \begin{bmatrix} s + a & \omega_c & -c & 0 \omega_f (s) \\ -\omega_c & s + a & 0 & -c \omega_f (s) \\ -d & 0 & s + b & f \omega_f (s) \\ 0 & -d & -f & s + b \omega_f (s) \end{bmatrix} = \begin{bmatrix} V_d (s) + \lambda_f (0) \\ V_d (s) + \lambda_f (0) \\ V_d (s) + \lambda_f (0) \\ V_d (s) + \lambda_f (0) \end{bmatrix} \] (24)

The expression for \(a, b, c, d, e\) and \(f\) are given in equation (8)

### 6 Steady State Simulations Of Dq Flux Linkages
For steady state simulation equation (24) is modified for use in the steady state solution of the dq flux linkages. Modification is by replacing all the \(s\) in the matrix operator of equation (24) by \(j\omega\) and also transforming all the source voltages to time domain equivalent and all the initial values set to zeroes. Hence
\[ \begin{bmatrix} j\omega & a & \omega_c & -c & 0 \omega_f (t) \\ -\omega_c & j\omega + a & 0 & -c \omega_f (t) \\ -d & 0 & j\omega + b & f \omega_f (t) \\ 0 & -d & -f & j\omega + b \omega_f (t) \end{bmatrix} = \begin{bmatrix} V_d (t) \\ V_d (t) \\ V_d (t) \\ V_d (t) \end{bmatrix} \] (25)

The expression for \(a, b, c, d, e\) and \(f\) are given in equation (8)

### 7 Simulation Results
Efficiency variations of the induction motor are simulated in steady state while flux linkages in the windings are also simulated during transient. All the simulations are done with MATLAB 7.40 mathematical tools, the results of the simulation of the electrical induction motor are shown in fig. (2) thru fig. (13). Efficiency of the motor is simulated at various motor parameter variations, optimal machine parameters are obtained and compared with the designed parameters, and complete results are summarized in table (1). These results are of that of the rotor reference frame, there were no much differences observed in the results from other reference frames

The optimal slip of the motor (0.26) with maximum efficiency of (97 %) is the result of the simulation fig (4). The efficiency varies as other machine’s parameter varies with exemption of the terminal voltage which shows no significant efficiency changes at variations at no load. Though, stator and the rotor resistance variations affects the efficiency of the induction motor, their effects are not as sensitive as that of stator, rotor and the magnetizing inductances. The iron loss is still the minimum of all the losses, though estimation of the iron losses does not include the iron resistance in the generalized machine equation that calculated the flux linkages.

To the transient variation of the flux time is shown in fig (2) which shows that the stator flux is always higher than that of the rotor flux as expected. The steady state flux linkages in the stator and the rotor windings increase as the slip increase, that is reduced rotor speed. The results also shows that when the stator and the rotor flux linkages increase more than the optimum values, the operation of the motor results in reduced efficiency.

### 8 Conclusion
Simulation software has been formulated which estimates the efficiency the electric motor. The optimal parameters of the electric motor which are responsible for maximum efficiency was able to be estimated through simulation and the results agreed with the designed parametric configurations of the machine. The speed versus flux linkages of the machine are in the agreement with the expected results. The formulated simulation program has shown to be useful in accessing the optimum configuration of electrical machine parameters especially as they affect the efficiency of the machine at no load. For example, other parameters of the simulated induction motor are found to be fully optimized with only the stator winding resistance which when change from 4.85 \(\Omega\) to 5.4 \(\Omega\) improves the no load efficiency only but little figure (6).

Improvement in the method of the estimation of the machine efficiency may include the inclusion of the iron resistance \(R_i\) in the generalized machine equation that simulated the \(d – q\) flux linkages of the magnetic circuit equation.

**TABLE 1 MACHINEPARAMETER**

<table>
<thead>
<tr>
<th>Machine Parameter</th>
<th>Actual</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance</td>
<td>4.85 (\Omega)</td>
<td>5.4 (\Omega)</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>3.805 (\Omega)</td>
<td>3.8 (\Omega)</td>
</tr>
<tr>
<td>Iron loss resistance</td>
<td>500 (\Omega)</td>
<td>1000+ (\Omega)</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>0.258 (H)</td>
<td>0.259 (H)</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>0.274 (H)</td>
<td>0.272 (H)</td>
</tr>
<tr>
<td>Rotor inductance</td>
<td>0.274 (H)</td>
<td>0.272 (H)</td>
</tr>
<tr>
<td>Rotor inertia</td>
<td>0.031 Kg/m²</td>
<td>– –</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.008 Nm/s/rd</td>
<td>– –</td>
</tr>
<tr>
<td>Output power</td>
<td>1.5 Kw</td>
<td>– –</td>
</tr>
<tr>
<td>Poles</td>
<td>2x2</td>
<td>– –</td>
</tr>
<tr>
<td>Voltage</td>
<td>220/380V</td>
<td>220 V</td>
</tr>
<tr>
<td>Current</td>
<td>3.64/6.31 A</td>
<td>– –</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1420 rev/min</td>
<td>– –</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Maximum efficiency</td>
<td>– –</td>
<td>97.04 %</td>
</tr>
</tbody>
</table>
Nomenclature

$s, r$: stator and rotor indices
$x_{dq}$: components in the synchronous reference frame $dq$ axes
$R_s, R_r$: stator and rotor resistances
$R_i$: Iron loss resistances
$L_s, L_r$: stator and rotor inductances
$l_s, l_r$: stator and rotor leakage inductances
$L_m$: magnetizing flux
$\lambda_s, \lambda_r$: stator and rotor flux
$\lambda_m$: magnetizing flux
$\Omega_s$: stator pulsation (rd/s)
$\Omega_r$: rotor speed (rd/s)
$p$: number of pole pairs
$P_{cs}, P_{cr}$: stator and rotor copper losses
$P_i$: Iron losses

Steady And Transient State Flux Response Simulation Graphs

Figure 2: Transient State Stator And Rotor Time Versus Flux Response

Figure 3: Steady State Stator And Rotor Slip Versus Flux Response

Steady State Efficiency Response Simulation Graphs

Figure 4: Slip Versus Efficiency Response
Figure 5: Stator Voltage Versus Efficiency Response

Figure 6: Stator Resistance Versus Efficiency Response

Figure 7: Rotor Resistance Versus Efficiency Response

Figure 8: Iron Resistance Versus Efficiency Response

Figure 9: Magnetizing Inductance Versus Efficiency Response

Figure 10: Stator Inductance Versus Efficiency Response

Figure 11: Stator Inductance Versus Efficiency Response
Quick survey

Figure 12: Rotor Inductance Versus Efficiency Response

Optimal point survey

Figure 13: Rotor Inductance Versus Efficiency Response

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