Abstract—The position-sensor less direct torque and indirect flux control of brushless dc (BLDC) motor with non sinusoidal back electromotive force (EMF) has been extensively investigated. In this study, a novel and simple approach to achieve a low-frequency torque ripple-free direct torque control (DTC) with maximum efficiency based on dq reference frame is presented. The proposed sensor less method closely resembles the conventional DTC scheme used for sinusoidal ac motors such that it controls the torque directly and stator flux amplitude indirectly using d-axis current. This method does not require pulse width modulation and proportional plus integral regulators and also permits the regulation of varying signals. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. In the literature, several methods have been proposed for BLDC motor drives to obtain optimum current and torque control with minimum torque pulsations. Most methods are complicated and do not consider the stator flux linkage control, therefore, possible high-speed operations are not feasible. 

Index Terms—Brushless dc (BLDC) motor, direct torque control (DTC), fast torque response, low-frequency torque ripples, non sinusoidal back electromotive force (EMF), position-sensor less control, stator flux control, torque pulsation.

I. INTRODUCTION

The permanent-magnet synchronous motor (PMSM) and brushless dc (BLDC) motor drives are used extensively in several high-performance applications, ranging from servos to traction drives, due to several distinct advantages such as high power density, high efficiency, large torque to inertia ratio, and simplicity in their control[1]-[3].

In many applications, obtaining a low-frequency ripple-free torque and instantaneous torque and even flux control are of primary concern for BLDC motors with non sinusoidal back electro motive force (EMF).

In electromagnetic torque is calculated from the product of the instantaneous back EMF and current both in two-phase and in the commutation period. Then, the pre stored phase back EMF values are obtained using mid precision position sensor. As a result, torque pulsations due to the commutation are reduced. However, phase resistance is neglected and the torque estimation depends on parameters such as dc-link voltage and phase inductance. Moreover, instead of a simple voltage selection look-up table technique more sophisticated PWM method is used to drive the BLDC motor. Also, two phase conduction method instead of a three-phase one is used which is problem in the high speed applications.

This study presents a novel and simple position-sensor less direct torque and indirect flux control of BLDC motor that is similar to the conventional DTC scheme used for sinusoidal ac motors where both torque and flux are controlled, simultaneously. This method provides advantages of the classical DTC such as fast torque response compared to vector control, and a position-sensor less drive. As opposed to the prior two-phase conduction direct torque control methods used for BLDC motor the proposed DTC technique provides position-sensor less drive that is quite similar to the one used in conventional DTC scheme and also controls the stator flux indirectly using d-axis current.

II. PROPOSED LINE-TO-LINE PARK AND CLARKE TRANSFORMATIONS IN 2 x 2 MATRIX FORM

Since the balanced systems in dq-axes reference frame do not require a zero sequence term, first line-to-line Clarke transformation from the balanced three-phase quantities is derived and, then the line-to-line Park transformation forming a 2 x 2 matrix instead of a 2 x 3 matrix for three-phase systems can be obtained in the following. Using some algebraic manipulations, the original Clarke Transformation forming a 2x3 matrices excluding the zero-sequence term can be simplified to a 2 x 2 matrix as follows:
Which requires only two input variables \( X_{ba} \) and \( X_{ca} \) where \( X_{ba} = X_b - X_a \) and \( X_{ca} = X_c - X_a \). \( X \) represents machine variables such as currents, voltages, flux linkages, back EMFs, etc. To obtain the line-to-line Park transformation forming a \( 2 \times 2 \) matrix, the inverse of the original Clarke transformation matrix \([T_{αβ}]\) is required. Since the zero-sequence term is removed, \([T_{αβ}]\) matrix is not square anymore, but it is still singular and therefore, pseudo inverse can be found in the following

\[
[T_{αβ}]^T = [T_{αβ}]^T ([T_{αβ}]^T [T_{αβ}])^{-1}
\]

where \([T_{αβ}]^T\) and \([T_{αβ}]\) are the pseudo inverse and transpose of the original Clarke transformation matrix \([T_{αβ}]\) respectively. Here, \( abc \rightarrow \alpha \beta \) transformation can be represented as follows:

\[
[T_{αβ}]^T [T_{αβ}] \begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} = [T_{αβ}]^T [T_{LL}] \begin{bmatrix} X_{ba} \\ X_{ca} \end{bmatrix}
\]

After (3) is expanded and multiplied by the original \( 2 \times 3 \) Park Transformation matrix in both sides, algebraic manipulations lead to implications using some trigonometric equivalence. Therefore, the following \( 2 \times 2 \) line-to-line Park transformation matrix form is obtained:

\[
\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{-1}{3} & 0 \end{bmatrix}
\]

III. PROPOSED SENSORLESS DTC OF BLDC MOTORDRIVE USING THREE-PHASE CONDUCTION

A) Principles of the Proposed Method

In this study, indirect torque control method of BLDC motor explained in [14] is extended to a direct torque and indirect flux control technique, which is suitable for sensorless and flux weakening operations. The proposed method transforms \( abc \) frame quantities to \( dq \) frame ones using the new \( 2 \times 2 \) line-to-line Park transformation matrix. Rather than three measured phase back EMFs, which are used in the proposed balanced system only two electrical rotor position dependent back EMF constants \( (k_d(\theta_{re}) \) and \( k_q(\theta_{re}) \) are required in the torque estimation algorithm. Since the numbers of input variables (current and back EMF) are reduced from three to two, much simpler Park transformation can be used as given in (4). Therefore, the amount of multiplications and sine/cosine functions are minimized.

Unlike previous two-phase conduction DTC of BLDC motor drive techniques, which are proposed in [12] & [13] this method uses DTC technique with three-phase conduction, therefore, flux-weakening operation as well as a much simpler sensorless technique can easily be achieved. Compared with the two-phase conduction DTC scheme, this DTC method differs by its torque estimation and voltage vector selection table which is similar to the one used for DTC of PMSM drives explained in [15]. Although, stator flux estimation algorithm in both methods (two-phase and three-phase conduction) is the same due to the similar machine model in which the back EMF shape separates the two from each other, in two-phase conduction scheme the stator flux amplitude is uncontrollable. Since the proposed technique adopts three-phase conduction, there is a possibility to control the stator flux amplitude without commutation issue, therefore, flux-weakening and sensorless operations that involve back EMF estimation can easily be performed. Moreover, this DTC method controls the voltage vectors directly from a simple look-up table depending on the outcome of hysteresis torque and indirect flux controllers, thus the overall control is much simpler and faster torque response can be achieved compared to the conventional PWM control techniques. For machines with surface-mount magnet rotor (BLDC) stator flux linkages in rotor \( dq \) reference frame can be written as

\[
\phi_{q_s} = L_s i_{q_s} + \phi_r \sum_{n=1}^{\infty} (K_{6n} - 1 + K_{6n+1}) \sin 6n \theta_r
\]

Fig. 1. Rotor and stator flux linkages of a BLDC motor in the stationary \( \alpha\beta \) plane and synchronous \( dq \) plane
where $\phi_r$ is the peak value of the fundamental rotor magnetic flux linkage of the BLDC motor, the coefficients $K_{en-1}$ and $K_{en+1}$ represent the odd harmonics of the phase back EMF other than the third and its multiples.

$K_{en-1}$ equals $[\sin((6n - 1)\sigma)/[(6n - 1)^2]\sin \sigma]$, and $K_{en+1}$ can be depicted as $[\sin((6n + 1)\sigma)/[(6n + 1)^2]\sin \sigma]. \sigma$ is the angle between zero-crossing and phase back EMF, where it becomes flat at the top. Fundamental peak value of the rotor magnet flux linkage $\phi_r$ equals $(4k_2/\sigma \pi) \sin \sigma$, where $k_2$ is the line-to-neutral back EMF constant. Equations (5) and (6) are very close approximations of stator flux linkages in the dq reference frame for the PMSM with non-sinusoidal back EMF. It can be seen that they are not constant as in pure sinusoidal ac machines. Inductances and stator flux linkages vary by the six times of the fundamental frequency. One of the reasons to derive the equivalent inductance and then the dq frame stator flux linkages in BLDC motor is that it can be easily observable which parameters affect the amplitude of the stator flux linkages. Stator flux linkage amplitude $|\phi_{ds}| = \sqrt{\phi_d^2 + \phi_q^2}$ can be changed by varying the $d$-axis current $i_d \sin \theta$ assuming the torque is constant and it is proportional to $i_q$; therefore, an indirect flux control can be achieved in the proposed DTC of BLDC motor drive.

The $d$-axis current reference is selected zero when the motor operates in the constant torque region (below flux-weakening region). The phasor diagram for stator flux linkage vectors in BLDC motor can be drawn in the rotor dq and stationary (aBf) reference frames as shown in Fig. 1, where $L_{ds} = L_{qs} = L_c$ and $L_{dq} = L_{qs} = 0$. $L_{ds}$ and $L_{qs}$ are the mutual inductances between $d$- and $q$-axis. $L_{dq}$ and $L_{qf}$ are the mutual inductances between $dq$-axes and permanent magnet (PM), respectively, and $i$ is the equivalent current generated by PM. In Fig. 1, unlike PMSM with sinusoidal back EMF synchronous reference frame flux linkages $\phi_d$ and $\phi_q$ vary with time, therefore, stator flux linkage amplitude $|\phi_s|$ is not constant anymore. $\gamma$, $\rho$, and $\delta$

$g = \sin^{-1} \left( \frac{L_{dq} i_r q_s}{\theta q_s} \right) + \cos^{-1} \left( \frac{L_{qs} i_r q_s}{\theta q_s} \right) - \frac{\pi}{2}$ \hspace{1cm} (7)

$\rho = - \left( \theta + \gamma - \frac{\pi}{2} \right)$ \hspace{1cm} (8)

B. Electromagnetic Torque Estimation in dq Reference Frame

Because of the rotor position dependent terms in the $dq$ frame stator flux linkages in (5) and (6) and inductances, conventional torque estimation in stator reference frame used for DTC of sinusoidal ac motors is no longer valid for BLDC motor, therefore, a new torque estimation algorithm is derived in $dq$ frame consisting of actual $dq$-axes back EMF constants and currents. Instead of the actual back EMF waveforms, Fourier approximation of the back EMFs could have been adopted in torque estimation, but the results would not truly represent the reality and more complex computations are required. The torque estimation is the key factor in the proposed DTC scheme. First, two line-to-line back EMF waveforms $e_{ba}(\theta_r)$ and $e_{ca}(\theta_r)$ are obtained offline and converted to the $ba$–$ca$ frame back EMF constants $k_{ba}(\theta_r)$ and $k_{ca}(\theta_r)$, respectively. The line-to-line Park transformation matrix is used to obtain the $dq$ reference frame back EMF constants $k_{dq}(\theta_r)$ and $k_{qd}(\theta_r)$, where $\theta_r$ is the electrical rotor angular position. Then, they are stored in a look-up table for electromagnetic torque estimation.

The electromagnetic torque $T_{em}$ estimation algorithm can be derived for a balanced system in $dq$ reference frame by equating the electrical power absorbed by the motor to the mechanical power produced ($P_e = P_m = T_{em} \omega_m$) as follows

$$T_{em} = \frac{3p}{4\omega_{rs}} \left( e_q(\theta_r) i_r q_s \right) \left( e_d(\theta_r) i_r d_s \right)$$

$$- \frac{3p}{4\omega_{rs}} \left( k_q(\theta_r) i_r q_s \right) \left( k_d(\theta_r) i_r d_s \right)$$ \hspace{1cm} (11)
Where $P$ is the number of poles, $\omega_{re}$ is the electrical rotor speed, $eq(\theta e)$ and $ed(\theta e)$, $irqs$ and $irds$, $kq(\theta e)$, and $kd(\theta e)$ are the $dq$ axes back EMFs, currents, and back EMF constants according to the electrical rotor position respectively. As it can be noticed that the right-hand side equation in (11) eliminates the speed.

Table I

| Switching Table for DTIFC of BLDC Motor Using 3 Conduction |
|---------------|------------|------------|------------|------------|------------|------------|
| $\theta$     | $\tau$    | $\varphi$ |
| 0(1)          | 0(2)       | 0(3)       | 0(4)       | 0(5)       | 0(6)       |
| $\varphi = 1$ | $\tau = 1$ | $Vd(010)$  | $Vd(001)$  | $Vd(101)$  | $Vd(100)$  |
| $\varphi = -1$ | $\tau = -1$ | $Vd(010)$  | $Vd(001)$  | $Vd(101)$  | $Vd(100)$  |

Indirectly kept at its optimum level, while the motor speed is less than the base speed. The switching table for controlling both the amplitude and rotating direction of the stator flux linkage is given in Table I, where the output of the torque hysteresis comparator is denoted as $\tau$, the output of the flux hysteresis comparator as $\varphi$, and the flux linkage sector is denoted as $\theta$. The torque hysteresis comparator $\tau$ is a two valued comparator; $\tau = 1$ means that the actual value of the torque is above the reference and out of the hysteresis limit and $\tau = 1$ means that the actual value is below the reference and out of the hysteresis limit. The same logic applies to the flux related part of the control ($d$-axis current).
The one out of six voltage space vectors is selected using lookup table in every sampling time to provide fast rotation of stator flux linkage vector. Therefore, fast torque and flux responses are obtained in a predefined hysteresis bandwidth, which limits the flux amplitude.

**D. Estimation of Electrical Rotor Position**

Electrical rotor position $\theta_{re}$, which is required in the line-to-line Park transformation and torque estimation algorithm can be found by

$$\theta_{re} = \tan^{-1}\left(\frac{\varphi_{s\beta} - L_s I_{s\beta}}{\varphi_{s\alpha} - L_s I_{s\alpha}}\right)$$ (13)

To solve the common problems for integrators, a special integration algorithm for estimating the stator flux linkage proposed is used in this study. Although the [17] method in is designed for sine wave systems, the algorithm is still applicable to a BLDC motor with varying stator flux linkage amplitude as shown in Fig. 3.

**Simulation results:**

4(a) estimated electromagnetic torque

4(b) Error between reference and estimated electromagnetic torque

4(c) $q$-axis stator current and $d$-axis stator current

4(d) $ba-ca$ frame currents when $i_r^*ds = 0$ under 0.5 N·m load torque

5(a) Simulation wave form for the actual and estimated electrical rotor positions from top to bottom

5(b) error between actual and estimated electrical rotor positions under 0.5 N·m load torque.

6(a) simulation result of electrical rotor position error

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(b) estimated electromechanical torque when \( ir^*ds = 0 \) under 0.5 N·m load torque.

7) Simulation results for estimated electromechanical torque under time-varying reference when \( ir^*ds = 0 \) under 0.5 N·m load torque.

(b) Steady-state flux-weakening behavior of the simulation result actual speed and estimated electromechanical torque, respectively, (a) when \( ir^*ds = 0 \) and (b) when \( ir^*ds = -4.51 \) A under 1.1926 N·m load torque at 540 electrical rad/s desired speed (Vdc link = 115 V).

Conclusion:
This paper has successfully demonstrated application of the proposed position-sensor less three-phase conduction DTC scheme for BLDC motor drives that is similar to the conventional DTC used for sinusoidal ac motors where both torque and flux are controlled, simultaneously. This method provides advantages such as fast torque response compared to vector control, simplicity (no PWM strategies, PI controllers, and inverse Park and inverse Clarke transformations), and position-sensorless drive. It is shown that the BLDC motor could also operate in the flux-weakening region by properly selecting the \( d \)-axis current reference in the proposed DTC scheme. Then, they are used in the torque estimation algorithm. Electrical rotor position required in the torque estimation is obtained using winding inductance, stationary reference frame currents, and stator flux linkages. Since the actual back EMF waveforms are used in the torque estimation, low-frequency torque oscillations can be reduced convincingly compared to the one with the ideal trapezoidal waveforms having 120 electrical degree flat top.

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