Closed loop Identification of Four Tank Set up Using Direct Method

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ABSTRACT

System identification is the determination on the basis of input and output of the system within a specified class of systems, to which the system under test is equivalent. System identification allows you to build mathematical models of a dynamic system based on measured data. Approach to Closed loop Identification: Direct approach: Apply the basic Prediction Error Methods (PEM) in a straight forward manner: use the output, y of the process and the input, u in the same way as for open loop operation, ignoring any possible feedback, and not using the reference signal, r.The experimental setup considered in this paper is a typical four tank system which is a Multiple Input Multiple Output (MIMO) system with levels in the tanks 1 and 2 are the outputs and the voltages to the pumps are the inputs. Closed loop identification is carried out by perturbing the system with a PRBS signal which is automatically generated using a Matlab program. Experimental data is collected from the experimental set up of the four tank system. Model is developed from this data.

Keywords **– closed loop identification, direct method, four tank set up, prediction error method (PEM)**

I. INTRODUCTION

Identification in many cases, security or production reasons do not permit regulators to be removed during identification experiment. In other cases, such as economic and biological systems, the feedback effects may be inherent. Consequently, identification experiments frequently have to be performed on processes operating in closed loop.

The fundamental problem with closed – loop data is the correlation between the unmeasurable noise and the input. This is the reason why several methods that work in open loop fail when applied to close loop data. But due the above-mentioned reasons it may be necessary to perform identification experiments in closed loop.

The purpose of this paper is to apply and verify the model obtained using Direct method available for identification in closed loop derived in the prediction error (PEM) framework. The theory will be supported by real time experiments on the laboratory set up of the four tank system. The emphasis will be on comparing the methods in time domain as well as in the frequency domain, which will enable us to throw light on the issues such as the bias distribution and the variance for the various methods.

The basic method for closed loop identification is the Direct method which the simplest of all and if this fails then no other method will work.

II. LITERATURE SURVEY

System Identification deals with the problem of building mathematical models of dynamical systems based on observed data from the system [1]. In other words identification is modeling based on experiments [2].

Basic requirements to carry out system identification are:

1. A data set (persistently exciting).
2. Set of candidate models: Model Structure.
3. A rule by which candidate models can be assessed using the data.

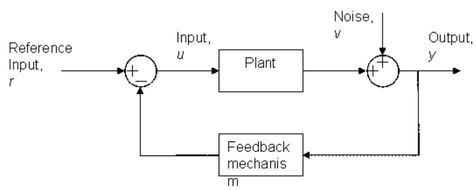

Figure 1 Closed loop System

Identification of plant models from closed-loop data is an important practical issue and may be motivated by the following reasons:

1) There are plants that contain an integrator or are unstable in open-loop operation.
2) The performance of the closed-loop system can be improved using a controller based on the identified model from the closed-loop data.
3) The research on identification for robust control shows that a plant model identified in closed-loop operation is more precise in the critical frequency zone (cross-over frequency) for the robust control design.

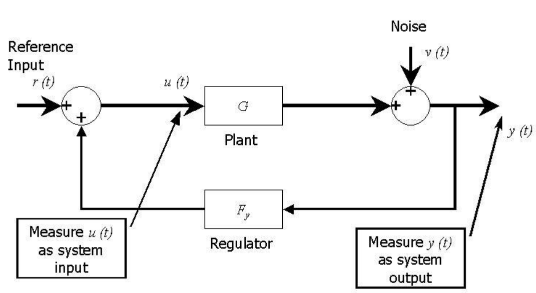

Figure 5 Direct Identification Technique

The direct approach amounts to applying a prediction error method directly to input - output data, ignoring possible feedback. This implies that this method can be applied to system with arbitrary (unknown) feedback mechanisms. In general, one works with model structures of the form,

$$y(t) = G(q,\theta)u(t) + H(q,\theta)e(t) \qquad (0.1)$$

III. INDENTATIONS AND EQUATIONS

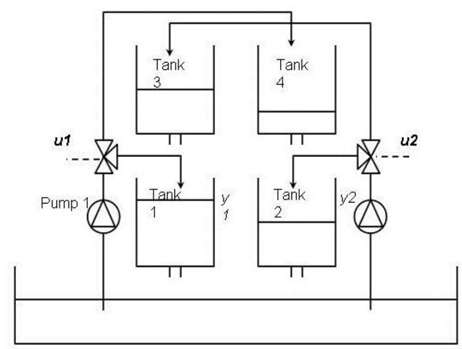

Figure 2 Schematic of Four tank Setup

The process is a typical four tank laboratory purpose system. A schematic diagram of the process is shown in Fig. 5. The target is to control the level in the lower two tanks with two pumps. The process inputs are input current to the valves and the outputs are and currents from level measurement devices. Mass balances and Bernoulli's law yield

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} i_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} i_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2) k_2}{A_3} i_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1) k_1}{A_4} i_1
\end{aligned} \qquad (0.2)$$

The voltage applied to Pump i is i_i and the corresponding flow is $k_i i_i$. The parameters γ_1, γ_2 ∈ (0,1)are determined from how the valves are set prior to an experiment. The flow to Tank 1 is $\gamma_1 k_1 i_1$ and the flow to Tank 4 is $(1-\gamma_1)k_1 i_1$ and similarly for Tank 2 and Tank 3. The acceleration of gravity is denoted g. The measured level signals are $k_c h_1$ and $k_c h_2$. The parameter values of the laboratory process are given in the following table:

A_i	(cm^2)	176.71
a_1, a_2	(cm^2)	0.7854
a_3, a_4	(cm^2)	0.6362
g	(cm/sec^2)	981

Then the linearized state-space equation is given by:

$$\frac{dx}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2) k_2}{A_3} \\ \frac{(1-\gamma_1) k_1}{A_4} & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x \qquad (0.3)$$

where the time constants are

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}, \qquad i = 1, \ldots\ldots, 4 \qquad (0.4)$$

The corresponding transfer function matrix is

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{1 + sT_1} & \frac{(1-\gamma_2) c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1) c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \qquad (0.5)$$

where $c_1 = T_1 k_1 k_c / A_1$ and $c_1 = T_2 k_2 k_c / A_2$.

The procedure for identification is as follows:

1. Apply step input to the system to get the step response of the system.
2. Calculate the time delays (if any), the gain and the time constants (or settling time) of the system.
3. Design the excitation (perturbation) input signal on the basis of the information collected.
4. Perturb the system to collect the data for identification.
5. Identify the models using the various methods and compare them.

Note: All the signals used for simulation or experimentation are deviation variables.

Here we calculate the Mean Square Error (MSE) and the Percentage Prediction Error (PPE) for comparison.

Figure 3 Actual Four tank Setup

Area of the tank	176.71	(cm^2)
Area of outlet of tank #1 & 2	0.7854	(cm^2)
Area of outlet of tank #3 & 4	0.6362	(cm^2)

The process:

The process is adapted from [4], where in it was a small setup developed for laboratory purpose. The main thing about this particular setup is that it has an adjustable zero [4], thus providing two different operating regions, namely Minimum phase (MP) and Non-minimum phase (NMP).

Equation nos. (0.2) to (0.5) describe the system. The system is linearized about an operating point. The steady state values for the system identified further are:

Level in tank #1 = 38.7887cm.

Level in tank #2 = 11.2464cm.

IV. RESULTS

Transfer Function:

$$\frac{y1}{u1} = \frac{0.004465}{z - 0.9842}$$
$$\frac{y2}{u1} = \frac{6.633e\text{-}005\ z}{z^2 - 1.912\ z + 0.9135}$$
$$\frac{y1}{u2} = \frac{7.06e\text{-}005\ z}{z^2 - 1.96\ z + 0.9604}$$
$$\frac{y2}{u2} = \frac{0.002832}{z - 0.9694}$$

$$a = \begin{bmatrix} 0.9842 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.912 & -0.9135 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.96 & -0.9604 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.9694 \end{bmatrix}$$
$$b = \begin{bmatrix} 0.0625 & 0 \\ 0.007813 & 0 \\ 0 & 0 \\ 0 & 0.007813 \\ 0 & 0 \\ 0 & 0.0625 \end{bmatrix}$$
$$c = \begin{bmatrix} 0.07145 & 0 & 0 & 0.009037 & 0 & 0 \\ 0 & 0.00849 & 0 & 0 & 0 & 0.04531 \end{bmatrix}$$
$$d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The cross validation results for both the outputs are good with a PPE of 4.5038 10.3869 for output #1 and #2 respecively.

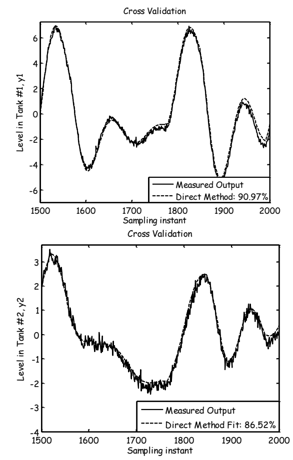

Figure 4 Cross Validation

The Step, Bode and Nyquist plots are indicating estimation close to the open loop system for the output #1.

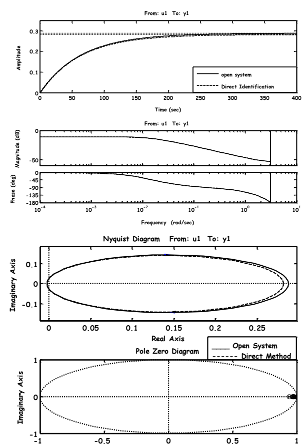

Figure 5 Step, Bode & Nyquist Plot for Output #1

The Step, Bode and Nyquist plots are indicating estimation close to the open loop system for the output #2.

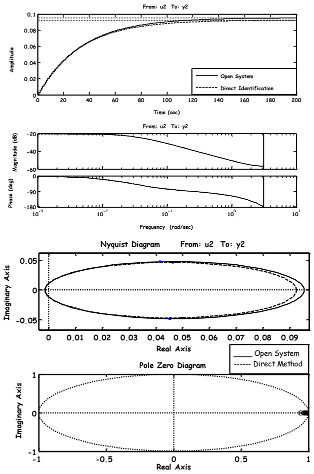

Figure 6 Step, Bode & Nyquist Plot for Output #2

The correlations plots are indicating that the residuals are nearly white and that the effect of feedback is also very less for both the outputs.

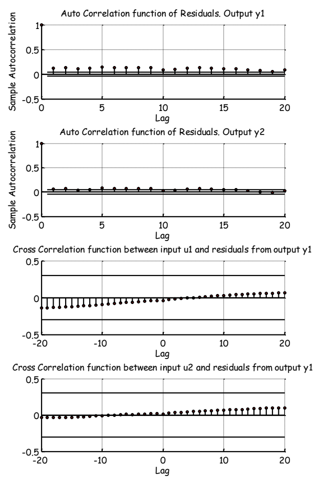

Figure 7 Cross correlation Plots

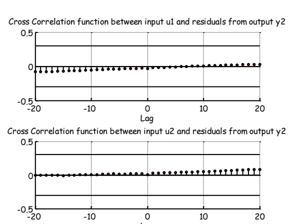

Figure 8 Correlation Plot

Output	MSE	PPE (%)
#1	0.0230	4.5038
#2	0.0249	10.3869

V. Conclusion

The Direct method gives more consistent estimates. The models are compared on the basis of Mean Square Error (MSE) and the Percentage Prediction Error (PPE) as well as they are compared on the basis of their correlation plots. The best models are considered in the result.

The orders of the models are restricted here to a lesser number but if the orders were further increased chances of better estimation cannot be neglected.

There are other two methods for which it can be done and that is going to be done in future. The methods are namely Indirect method and Joint Input Output method.

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