The Haar Wavelet and The Biorthogonal Wavelet Transforms of an Image

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ABSTRACT

It is obvious that image (and signal) processing has become an exceedingly important and hot topic that comes across many areas: engineering, physical science, computer science and of course mathematics. Less obvious, but becoming increasingly well known, is that signal and image processing are getting great improvements in performance by using wavelet based methods. In an effort to achieve better understanding of the wavelet based methods, Haar and Biorthogonal wavelets are discussed on result oriented basis using Matlab environment.

Keywords – Discrete Wavelet transform, Haar,Biorthogonal,Scaling and Wavlet function.

INTRODUCTION

Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology[1]. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction[2].

Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. This idea is not new. Approximation using superposition of functions has existed since the early 1800's, when Joseph Fourier discovered that he could superpose sines and cosines to represent other functions. However, in wavelet analysis, the scale that one uses in looking at data plays a special role. Wavelet algorithms process data at different scales or resolutions. If we look at a signal with alarge "window," we would notice gross features. Similarly, if we look at a signal with a small "window," we would notice small discontinuities. The result in wavelet analysis is to "see the forest and the trees."

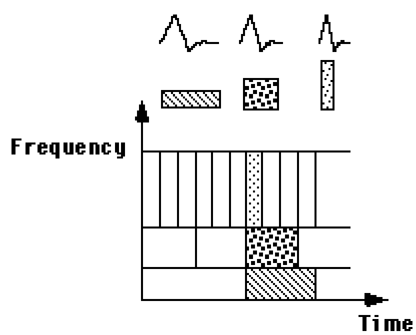

Fig:1:Multiresolution Time-Frequency Scale

The wavelet analysis procedure is to adopt a wavelet prototype function, called an "analyzing wavelet" or "mother wavelet." Temporal analysis is performed with a contracted, high-frequency version of the prototype wavelet, while frequency analysis is performed with a dilated, low-frequency version of the prototype wavelet. Because the original signal or function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data operations can be performed using just the corresponding wavelet coefficients[3]. And if you further choose the best wavelets adapted to your data, or truncate the coefficients below a threshold, your data is sparsely represented. This "sparse coding" makes wavelets an excellent tool in the field of data compression.

There are two types of wavelet transforms: Continous wavelet transform(CWT) and Discrete wavelet transform(DWT)[3]. The CWT performs a multiresolution analysis by contraction and dilatation of the wavelet functions.The discrete wavelet transform (DWT) uses filter banks for the construction of the multiresolution time-frequency plane.

For the DWT special families of wavelet functions are developed. These wavelets are compactly supported, orthogonal or biorthogonal and are characterized by low-pass and high-pass analysis and synthesis filters. Some generally used families for the DWT are discussed here[4].

I. DISCRETE WAVELET TRANSFORM

A)HAAR WAVELET TRANSFORM

Daubechies constructed the first wavelet family of scale functions that are orthogonal and have finite vanishing moments, i.e., compact support[5] [7]. This property insures that the number of non-zero coefficients in the associated filter is finite.

This is very useful for local analysis. The Haar wavelet is the simplest wavelet transform. It is also the only symmetric wavelet in the Daubechies family and the only one that has an explicit expression in discrete form. Haar wavelets are related to a mathematical operation called Haar transform, which serves as a prototype for all other wavelet transforms[6]. Like all wavelet trans-forms, the Haar transform decomposes a discrete signal into two subsignals of half its length. One subsignal is a running average or trend, the other subsignal is a running difference or fluctuation[5]. The Haar wavelet transform has the advantages of being conceptually simple, fast and memory efficient, since it can be calculated in place without a temporary array. Furthermore, it is exactly reversible without the edge effects that are a problem of other wavelet transforms. On the other hand, the Haar transform has its limitations because of its discontinuity, which can be a problem for some applications, like compression and noise removal of audio signal processing.

During computation, the analyzing wavelet is shifted over the full domain of the analyzed function. The result of WT is a set of wavelet coefficients, which measure the contribution of the wavelets at the locations and scales.

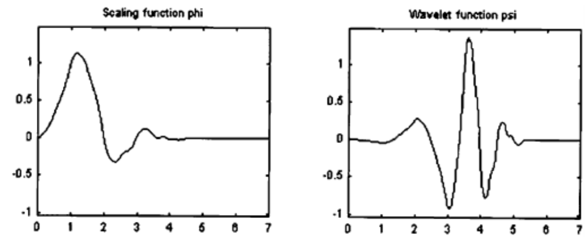

Fig 2: Scaling and Wavelet Function

The Daubechies wavelet transforms are defined in the same way as the Haar wavelet transform by computing the running averages and differences via scalar products with scaling signals and wavelets[8].
The input image and the haar transform of an input image is shown below:

Fig 3:Input Image

Fig 4:Haar Transformed Image.

B) BIORTHOGONAL WAVELET TRANSFORM

It is well known that bases that span a space do not have to be orthogonal. In order to gain greater flexibility in the construction of wavelet bases, the orthogonality condition is relaxed allowing semi-orthogonal, biorthogonal or non-orthogonal wavelet bases[9]. Biorthogonal Wavelets are families of compactly supported symmetric wavelets. The symmetry of the filter coefficients is often desirable since it results in linear phase of the transfer function[9][10]. In the biorthogonal case, rather than having one scaling and wavelet function, there are two scaling functions that may generate different multiresolution analysis, and accordingly two different wavelet functions.
The dual scaling and wavelet functions have the following properties:
1. They are zero outside of a segment.
2. The calculation algorithms are maintained, and thus very simple.
3. The associated filters are symmetrical.
4. The functions used in the calculations are easier to build numerically than those used in the Daubechies wavelets[5].

Fig 5:Biorthogonal Transform

II. CONCLUSION

Signals represented in the time domain can be evaluated for their properties in the frequency. The wavelet transform is a relatively new technique which has some attractive characteristics related to it. Two different kinds of wavelet transform can be distinguished, a continuous and a discrete wavelet transform.The Haar transform and the Biorthogonal Wavelets are explained using example input image.
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