

Finite Element Analysis of Steel Storage Tank Under Siesmic Load

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ABSTRACT

The objective of this article is to study the effect of various components of earthquake on sloshing response of liquid storage tanks. First, commonly used theory for unidirectional analysis of liquid behavior in cylindrical tanks was reviewed. Second, the Finite Element Modeling (FEM) strategy which was used to simulate dynamic response of the liquid tank system was described. The FEM was validated using a set of manual calculation which is used in available design guidelines. Third, a parametric study for some vertical, cylindrical ground supported tanks with different aspect ratios excited by various time series of earthquake accelerations was performed. Each tank was subjected to unidirectional and bidirectional excitations of earthquake accelerations. Fourth, the variations of maximum sloshing wave height during the above analysis were described. Fifth, the tanks under this study were analyzed in a known earth quake in India and the effects on sloshing wave height were studied. Finally, the available simplified formulation for evaluating MSWH under unidirectional excitation was extended for bidirectional excitation and a simple procedure is developed with the help of software

I. Introduction

Large capacity ground supported cylindrical tanks are used to store variety of liquids, e.g. water for drinking and fire fighting, petroleum, chemicals, and liquefied natural gas. Satisfactory performance of tanks during strong ground shaking is crucial for modern facilities. Tanks that were inadequately designed or detailed have suffered extensive damage during past earthquakes. Earthquake damage to steel storage tanks can take several forms. Large axial compressive stresses due to beamlike bending of the tank wall can cause “elephant-foot” buckling of the wall



Fig 1 Elephant-foot buckling of a tank wall
Sloshing liquid can damage the roof and the top of tank wall as



Fig 2 Sloshing damage to upper shell of tank
Base shear can overcome friction causing the tank to slide. Base uplifting in unanchored or partially anchored tanks can damage the piping connections that are incapable of accommodating vertical displacements, rupture the plate shell junction due to excessive joint stresses, and cause uneven settlement of the foundation.



Fig 3 Tanks Sliding

Initial analytical studies dealt with the hydrodynamics of liquids in rigid tanks resting on rigid foundations. Large amplitude sloshing flows considered in engineering applications are usually followed by impact of liquid at the side wall and top surface of fluid containers. Many studies for the sloshing and impact of liquid on marine industries were carried out in the 1970s and early 1980s for the design of LNG carriers and some numerical computations have been reported. Most of the studies at this time were reported for the two dimensional (2D) flows. The demand for sloshing analyses is raising again for the design of larger storage tanks, which have a greater potential for severe sloshing impact.

Some other representative works in the field of storage tank analysis, mostly focused on the application of numerical methods for modelling the sloshing phenomena have been introduced by Wu et al [5] and Faltinsen [6]. Kim [7] and Kim et al [8] also developed numerical method based on the finite difference method. This method has been extended to more complicated geometries, e.g. three dimensional prismatic tanks. Some extensions or applications of this method can be found in the works of Colagrossi and Landrini [9], Iglesias et al [10], and Oger et al [11]. Historically, mechanical models were first developed for tanks with rigid walls. Housner [16-18] was perhaps the first to propose such a simplified mechanical model for circular and rectangular rigid tanks. His simplified model is a two degree-of-freedom (DOF) system for rigid tanks, one DOF accounting for the motion of the tank liquid system, with a part of the liquid being rigidly attached (impulsive mode) and the other DOF for the motion of the sloshing liquid (convective mode). The mechanical model of Housner is still widely used with certain modifications for the analysis of rectangular and cylindrical tanks. Wozniak and Mitchell [19] generalized Housner's model for short and slender tanks. Veletsos and Yang [20] used a different approach to propose a similar type of a mechanical model for circular rigid tanks. Subsequently, Haroun and Housner [21] and Veletsos [22] developed

mechanical models for flexible tanks. Malhotra et al [23] proposed further simplifications of the mechanical model of Veletsos for flexible tanks.

II. IMPORTANCE OF MAXIMUM SLOSHING WAVE HEIGHT (M_{swh})

Liquid sloshing is associated with various engineering problems, such as the behavior of liquid in containers, the liquid oscillations in large storage tanks caused by earthquakes. The dynamic characteristic of these systems is greatly affected by the dynamics of the liquid free surface motion and it is very important regarding to the safety of transportation systems, human life and environmental issues.

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In the field of civil engineering, the sloshing phenomenon becomes important to seismic design of liquid storage tanks. Due to the requirement to remain functional after a major earthquake event, the seismic performance of liquid storage tanks has been a critical issue in municipal water supply and fire fighting systems. Water supply that is able to immediately follow critical earthquakes is vital. If a water tank collapses during destructive earthquakes the loss of public water supply can have serious consequences. Similarly, failure of tanks storing combustible materials can lead to extensive uncontrolled fires. The fluid sloshing impact causes a serious problem in liquid storage tanks subjected to earthquakes. For example the failure of the floating roof and the fire of oil-storage tanks have been frequently observed

There is a wide variety of guideline codes for earthquake resistant design of steel liquid storage tanks that use one of the above mechanical models. Notwithstanding a consensus in the codes on the treatment of several aspects of the phenomenon, various other aspects remain controversial or unresolved. One of these aspects is prediction of **Maximum Sloshing Wave Height (MSWH)** which is one of the major considerations in the design of liquid storage tanks. MSWH is used to provide sufficient freeboard between the liquid surface and the tank roof

to prevent sloshing waves from impacting the roof. If the sufficient free board is not provided, the liquid impact to the roof should be considered.

III. METHOD OF DYNAMIC ANALYSIS

The dynamic analysis of a liquid filled tank may be carried out using the concept of generalized single degree of freedom (SDOF) [2] systems representing the impulsive and convective modes of vibration of the tank liquid system. For practical applications, only the first few modes of vibration need to be considered in the analysis.

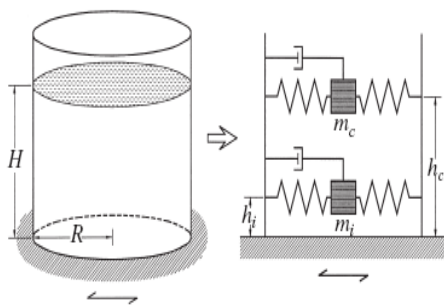


Fig 4: Liquid-filled tank modeled by generalized single-degree-of-freedom systems

The mass, height and natural period of each SDOF system are obtained by the methods described. For a given earthquake ground motion, the response of various SDOF systems may be calculated independently and then combined to give the net base shear and overturning moment.

For most tanks ($0.3 < H/r < 3$, where H is the height of water in the tank and r the tank radius), the first impulsive and first convective modes together account for 85–98% of the total liquid mass in the tank. The remaining mass of the liquid vibrates primarily in higher impulsive modes for tall tanks ($H/r > 1$), and higher convective modes for broad tanks ($H/r \leq 1$). The results obtained using only the first impulsive and first convective modes are considered satisfactory in most cases. There is, however, some merit in slightly adjusting the modal properties of these two modes to account for the entire liquid mass in the tank.

IV. SIMPLE PROCEDURE FOR SEISMIC ANALYSIS

The procedures adopted here are based on the following assumptions with certain modifications that make the procedure simple, yet accurate, and more generally applicable. Specifically, these modifications include

- Representing the tank liquid system by the first impulsive and first convective modes only

- Combining the higher impulsive modal mass with the first impulsive mode and the higher convective modal mass with the first convective mode
- Adjusting the impulsive and convective heights to account for the overturning effect of the higher modes
- Generalising the impulsive period formula so that it can be applied to steel as well as concrete tanks of various wall thicknesses.
- The impulsive and convective responses are combined by taking their numerical sum rather than their root mean-square value.

Model Properties

The natural periods of the impulsive (T_{imp}) and the convective (T_{con}) responses are,

$$T_{imp} = C_i \frac{H\sqrt{\rho}}{\sqrt{h/r}\sqrt{E}}$$

$$T_{con} = C_c \sqrt{r}$$

Where,

h = Equivalent uniform thickness of the tank wall

ρ = The mass density of liquid

E = The modulus of elasticity of the tank material

The coefficient C_i is dimensionless, while C_c is expressed in s/\sqrt{m} , where s is the seconds and m is in meters. For tanks with non-uniform wall thickness, h may be calculated by taking a weighted average over the wetted height of the tank wall, assigning the highest weight near the base of the tank where the strain is maximal.

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Seismic Responses

The total base shear is given by

$$Q = (m_i + m_w + m_r) \times S_e(T_{imp}) + m_c S_e(T_{con})$$

Where,

m_w = The mass of tank wall

m_r = The mass of tank roof

$S_e(T_{imp})$ = The impulsive spectral acceleration (obtained from a 2% damped elastic response spectrum for steel and prestressed concrete tanks, or a 5% damped elastic response spectrum for concrete tanks), and $S_e(T_{con})$ the convective spectral acceleration (obtained from a 0.5% damped elastic response spectrum).

The overturning moment above the base plate, in combination with ordinary beam theory, leads to the axial stress at the base of the tank wall. The net overturning moment immediately above the base plate (M) is given by

$$M = (m_i h_i + m_w h_w + m_r h_r) \times S_e(T_{imp}) + m_c h_c S_e(T_{con})$$

Where h_i and h_c are the heights of the centroids of the impulsive and convective hydrodynamic wall pressures and h_w and h_r are the

heights of the centers of gravity of the tank wall and roof, respectively.

The overturning moment immediately below the base plate (M') is dependent on the hydrodynamic pressure on the tank wall as well as that on the base plate. It is given by

$$M' = (m_r h_r + m_w h_w + m_c h_c) \times S_e(T_{imp}) + m_c h_c' S_e(T_{com})$$

The vertical displacement of the liquid surface due to sloshing (d) [1] is given by

$$d = R \frac{S_e(T_{com})}{g}$$

V. CASE STUDY

A steel tank with a radius r of 10 m and total height of 9.6 m is fully anchored to a concrete mat foundation. The tank is filled with water to a height H of 8 m ($H/r = 0.8$). The total mass of water in the tank (m_l) is 2.51×10^6 kg. The tank wall is made of four courses, each 2.4 m high. The lower two courses are 1 cm thick and the upper two courses 0.8 cm thick. The total mass of the tank wall (m_w) is 43×10^3 kg, and the height of its centre of gravity (h_w) is 4.53 m. The mass of the tank roof (m_r) is 25×10^3 kg and the height of its centre of gravity (h_r) is 9.6 m. The 0.5% and 2% damped elastic response spectra for the site.

Model Properties

T_{imp}	0.123s
T_{com}	4.46s

Seismic Responses

Q	11×10^6 N
M	40×10^6 Nm
M'	81 MNm

The maximum vertical displacement of the liquid surface due to sloshing, obtained
 $d = 10 \times 0.07 = 0.7$ m

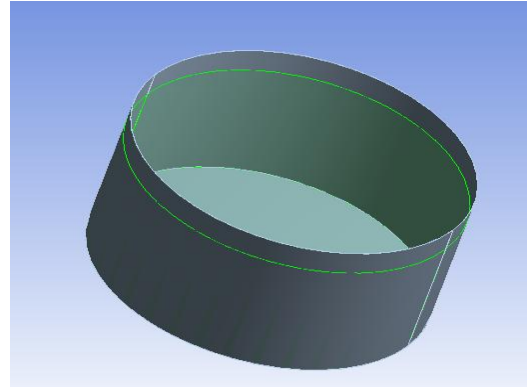
VI. COMPARISON WITH DETAILED MODAL ANALYSIS

The above steel tank was selected for comparing the results obtained from the proposed procedure with those from a detailed modal analysis. Three impulsive and three convective modes were used in the detailed analysis. The modal analysis results were calculated using a combination of root mean square and algebraic sum rules. The net impulsive and the net convective responses were calculated first, using the root-mean-square rule, then numerically added to give the overall response.

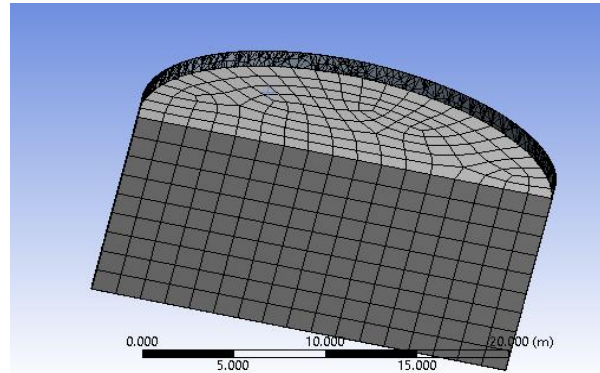
ANSYS 14.0 was used for the analysis which includes two modal analysis and one static structural and spectrum analysis. The modeling of the tank was done in modeling software called CATIA V5. Modelling software helps modeling of the tank much faster and accurate.

In this study a linear modal analysis of steel storage tank which is anchored in ground. A typical dimension of the tank was identified and the different modes are analyzed by using ANSYS 14.0. The result will give the structural deformation corresponding to the structural frequency of each mode shape.

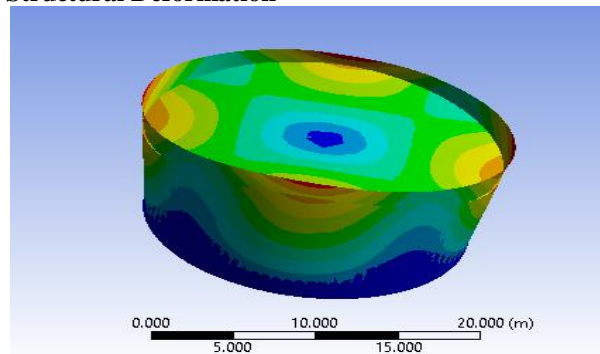
Model View



Meshed View



Structural Deformation



VII. CAMPARISON

Parameter	Analytical	Software
Maximum Sloshing Height	0.7m	0.66m and 0.73m

It was found that the result obtained from ANSYS 14.0 was matching with the analytical results. So rest of the analysis can be done in ANSYS only. This means the simulation adopted in ANSYS is correct, also the results are validated.

VIII. ANALYSIS OF TANKS UNDER DIFFERENT CONDITIONS

The tanks are analyzed under the following conditions

1. Tanks with change in height only.
2. Tanks with change in diameter only
3. Tanks with change in the level of liquid stored in it
4. Tanks with change in liquid
5. Analysis of the above tank under Bhuj Earthquake.

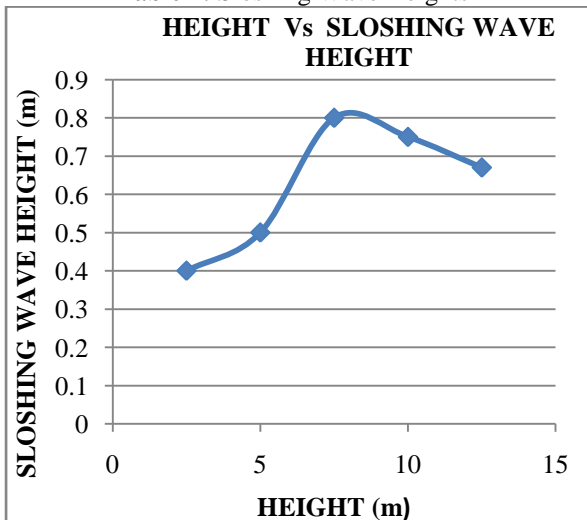
The manual designs of the tanks coming under the above three cases were done by using API 650, 11Th edition.

8.1 Tanks with Change in Height Only

The tanks analyzed in this case having diameter constant that is 20m, but the height of the tank will be changing. The liquid stored is taken as water and the free board provided is 1.5m form the top of the tank. The steel plates selected for the tanks having width of 2.5m that is jumbo plates of dimension width 2.5m and length 10m were selected this will give less joint to be welded and hence most economical. The grade of the steel plate was ASTM A36. The thickness of the base plate is providing same in all of the tanks as 10mm.

TANK NO	HEIGHT (m)	SLOSHING WAVE HEIGHT d (m)
1	12.5	0.67
2	10	0.75
3	7.5	0.8
4	5	0.5
5	2.5	0.4

Table 1: Sloshing Wave Heights



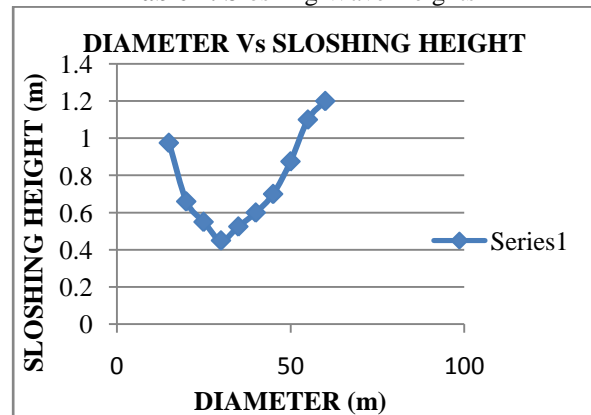
Graph 1: Height Vs Sloshing Wave Height

8.2 Tanks with change in diameter only

The tanks analyzed in this case having height constant that is 12.5m, but the diameter of the tank will be changing. The liquid stored was taken as water and the free board provided was 1.5m form the top of the tank. The steel plates selected for the tanks having width of 2.5m that is jumbo plates of dimension width 2.5m and length 10m were selected this will give less joint to be welded and hence most economical. The grade of the steel plate was ASTM A36. The thickness of the base plate is provide same in all of the tanks as 10mm

TANK NO:	DIAMETER (m)	SLOSHING WAVE HEIGHT (m)
1	60	1.2
2	55	1.1
3	50	0.875
4	45	0.7
5	40	0.6
6	35	0.525
7	30	0.45
8	25	0.55
9	20	0.66
10	15	0.975

Table 2: Sloshing Wave Heights

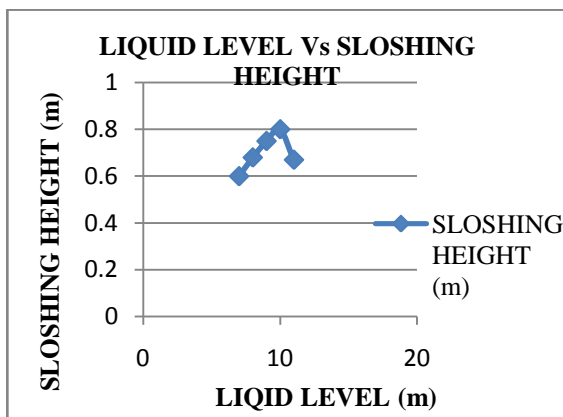


8.3 Tanks with change in the level of liquid stored in it

In this case a tank dimension of height 12.5 m and diameter 20 m was selected. Initially liquid level was at 11m then liquid level was decreased uniformly at the rate of 1 m. The other properties were similar as in above cases.

LIQUID LEVEL (m)	SLOSHING HEIGHT (m)
11	0.67
10	0.8
9	0.75
8	0.68
7	0.6

Table 3: Sloshing Wave Heights



Graph 3: Liquid Level Vs Sloshing Height

8.4 Tanks with different liquid

In this case the tanks considered in 9.1 and 9.2 were taken with changing the liquid. The considered liquid was denser than previously taken, that is water. The density considered is 1300 kg/m^3 . There was a series of liquids available with density closer to this value. So it's assumed as Liquid B.

Physical State	Clear yellow liquid
Viscosity at 25o	250-500 Cps
Solubility	Insoluble in water
Specific Gravity	1.21

8.4.1 Tanks with Change in Height Only

The tanks analyzed in this case having same details as in 8.1

TANK NO	HEIGHT (m)	SLOSHING WAVE HEIGHT d (m)
1	12.5	0.603
2	10	0.675
3	7.5	0.72
4	5	0.45
5	2.5	0.36

Table 6: Sloshing Wave Heights

Table 4: Sloshing Wave Heights

8.4.2 Tanks with Change in Diameter Only

The tanks analyzed in this case having same details as in 8.2

TANK NO:	DIAMETER (m)	SLOSHING WAVE HEIGHT (m)
1	60	1.02
2	55	0.93
3	50	0.74
4	45	0.59
5	40	0.51
6	35	0.44
7	30	0.38
8	25	0.46
9	20	0.56
10	15	0.82

Table 5: Sloshing Wave Heights
8.5 Analysis of the tanks under 2001 BHUJ Earthquake

Bhuj Earthquake was one of the recently occurred Sevier earthquakes in India. The reason behind for the selection of Bhuj Earthquake was, in the above analysis tanks are subjected to spectral acceleration taken from the design guidelines. So tanks must be analyzed in a live earthquake history. So Bhuj Earthquake was selected. The Bhuj Earthquake was happened in January 26,2001 at Ahmedabad .Earthquake happened at 08:46:42.9 I.S.T with magnitude 7.0.The earthquake having Initial Velocity of $-0.1411 \times 10^{-2} \text{ m/s}$,Initial Displacement of 3.970mm and Peak Acceleration of -1.0382 m/s^2 at 46.940sec.The accelerogram was downloaded which will give the relation of acceleration of ground with time. So these values can be directly uploaded to ANSYS by tabulating it in to MS EXAL.Since the peak acceleration of Bhuj Earthquake was less compared to the spectral acceleration provided according to taken design guidelines. The tanks will be safe under earthquake. But the maximum sloshing wave height was changed. From the analysis it's understood that all of the tanks designed can overcome the Bhuj Earthquake.

TANK NO	HEIGHT (m)	SLOSHING WAVE HEIGHT d (m)
1	12.5	0.4556
2	10	0.51
3	7.5	0.544
4	5	0.34
5	2.5	0.272

Table 7: Sloshing Wave Heights
Sloshing Wave Heights in tanks with change in height under Bhuj Earthquake

TANK NO:	DIAMETER (m)	SLOSHING WAVE HEIGHT (m)
1	60	0.81
2	55	0.74
3	50	0.59
4	45	0.47
5	40	0.40
6	35	0.35
7	30	0.30
8	25	0.37
9	20	0.44
10	15	0.66

Table 8: Sloshing Wave Heights

IX. CONCLUSION

In this paper, FEM is used to investigate the effect of bidirectional excitation on MSWH of vertical, cylindrical tanks. First, the accuracy of the FEM

strategy was validated. Then tanks were modelled and analysed. The accuracy of FEM was also checked. Finally a simplified method was developed for an accurate, simple and faster way for the design of steel storage tanks under seismic load

The main conclusions of this study may be summarized as follow:

- From the analysis of tanks with change in height only, it was observed that the sloshing wave height was increasing as the height increasing. But when reaching a maximum value the sloshing wave height got decreasing. From this it's concluded that there exists an optimum height to steel storage tanks. Using this procedure optimum height can be easily found out.
- From the analysis of tanks with change is diameter, it was observed that initially the sloshing wave height decreasing with the increasing diameter. When reaching a minimum value of sloshing wave height, then sloshing wave height increases with increase in diameter. From this its understood that there exist an optimum value of diameter which will have a minimum sloshing wave height.
- From the analysis of tanks with change in liquid levels only, It was observed that the sloshing wave height was increasing with the increase in liquid level and after reaching a particular value sloshing wave height got dreasing.From this its concluded that there exist an optimum value for liquid level for safe performance of storage tanks.
- When tanks are analysed by changing the liquid stored, it was found that the basic structural behaviour remain same as in the previous cases.
- When tanks are analysed under 2001,Bhuj Earthquake, It was fund that none of the tanks got failed. From this its concluded that the seismic design done by following the design guidelines of API: 650 and Euro code 8 were enough and can be adopted in India also.
- It was found that Finite Element Software ANSYS 14helps to reduce the lengthyprocedure of seismic design.Sotware can beused for analysis of steel storage tanks

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