Computation of Robust PI Controller for Systems with Parametric Uncertainty

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Abstract-This paper describes a new technique of PI controller for systems with parametric uncertainty. A PI controller is designed using necessary and sufficient condition for robust Hurwitz polynomial. The method is illustrated through a typical numerical example available in the literature

Index Terms *— Kharitonov's theorem, Hurwitz polynomial, parametric uncertainty, PI controller.*

I.INTRODUCTION

There has been a great amount of research work on the tuning of PI, PID and lag/lead controllers since these types of controllers have been widely used in industries for several decades [1-5]. However, many important results have been recently reported on computation of all stabilizing P, PI and PID controllers after the publication of work by Ho *et al.* [6-9].Robust stability analysis with uncertain parameters has been very important research topic. Since control systems operate under large uncertainty present in the control system causes degradation of system performance and destabilization. An important approach to this subject via expressing the characteristic polynomial by an interval polynomial i.e, a polynomial by whose coefficient each varies independently in a prescribed interval. The stability analysis of polynomials subjected to parameter uncertainty have received considerable attention after the celebrated theorem of Kharitonov [11] , which asses robust stability under the condition that four specially constructed extreme polynomials, called Kharitionov [11], polynomials are Hurwitz.

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The problem of robust stability of interval polynomial is also dealt in [11]-[17]. In this regard few entrance point results that are pure gain compensator c(s) = K stabilizes entire interval plant family if and only if it stabilizes a distinguished set of eight of the extreme plants. Hollot and Fang [19] considered that same setup as Ghosh but allow the controller to be first order. They prove that to robustly stabilize the extreme plants which are obtained by taking all possible combinations of extreme values of the plant numerator has degree m and the plant denominator is monotonic with degree n, the number of extreme plants can be high as N_{ext} = 2m-n+1 in [20], barmish proved that, it is necessary and sufficient to stabilize only sixteen of the extreme plants. A complete survey of these extreme points is given in [21] .In [21], a necessary and sufficient condition for interval polynomials is proposed using the results of Nie [22] for fixed polynomials.

In this paper the PI controller is designed for higher order parametric uncertain system. The necessary and sufficient conditions are applied to the for higher order parametric uncertain system. This method is illustrated through a typical numerical example available in the literature. The following section illustrates the procedural steps of the proposed method.

II.PROBLEM STATEMENT

Consider the set of real polynomials of degree n of the form

$$A(s) = a_0 + a_1 s + a_2 s^2 + \cdots\cdots a_n s^n \qquad \cdots\cdots(1)$$

Where the coefficients lie within given region

$$a_0 \in [x_0, y_0], a_1 \in [x_1, y_1] \cdots\cdots a_n \in [x_n, y_n] \quad \cdots(2)$$

We assume that the degree remains invariant over the family, so that a $\notin$ $[x_n,y_n]$ such a set of polynomial called a real interval family and is referred as an interval polynomials. The set of polynomials given by is stable if and only if each and every element of the set is a Hurwitz polynomial. A necessary and sufficient condition for robust stability of interval polynomial is proposed using the algebraic stability criterion for fixed polynomial due to Nie which is stated in the following Lemmas

*Lemma1:*The interval polynomial A(s) defined in (1) is Hurwitz for all $a_i \in [x_i , y_i]$ where i=0,1,2,………n. If the following necessary conditions are satisfied

$$y_i \geq x_i > 0,\ i = 0,1,2\cdots\cdots n$$

$$x_i x_{i+1} > y_{i-1} y_{i+2},\ i = 0,1,2,\cdots\cdots n-2 \qquad \cdots(3)$$

Lemma2: the interval polynomial A(s) defined in (1) is Hurwitz for all $a_i \in [x_i, y_i]$ where i=0,1,2,………n. If the following sufficient conditions are satisfied

$$y_i \geq x_i > 0,\ i = 0,1,2\cdots\cdots n$$

$$0.4655 x_i x_{i+1} > y_{i-1} y_{i+2},\ i = 0,1,2,\cdots\cdots n-2 \qquad \cdots(4)$$

Consider a system whose transfer function with parametric uncertainty is given by

$$G(s,b,a) = \frac{N(s,b)}{D(s,a)} \qquad \cdots(5)$$

Where the numerator and denominator polynomials are of the form

$$N(s,b) = b_0 + b_1 s + \cdots\cdots b_m s^m$$

$$D(s,a) = a_0 + a_1 s + \cdots\cdots a_n s^n$$

Where vectors **b** and **a** lie in given rectangles B and A respectively.

$$a \in A : \{a : a_i^- \leq a_i \leq a_i^+\} \quad \text{for } i = 0,1,\cdots\cdots n$$

$$b \in B : \{b : b_i^- \leq b_i \leq b_i^+\} \quad \text{for } i = 0,1,\cdots\cdots m$$

Where a_i € [1,1] and the bound on $a_i^-, a_i^+, b_i^-, b_i^+$ are specified a priori

Let PI controller transfer function in parametric uncertainty form is given by

$$C(s) = \frac{N_c(s)}{D_c(s)} = K_p + \frac{K_I}{s} \qquad \cdots(6)$$

$$\text{where } K_P \in \left[K_{p\min}, K_{p\max}\right]$$

$$K_I \in \left[K_{I\min}, K_{I\max}\right]$$

The characteristic equation of closed loop system of reduced model with PI controller is given as

$$N_c(s)N\ (s) + D_c(s)D\ (s) = 0 \qquad \cdots(7)$$

The values of K_P and K_I in parametric uncertain form are obtained by solving the characteristic equation with Routh's criterion. Closed loop control of the system with PI controller is shown in Fig.1.

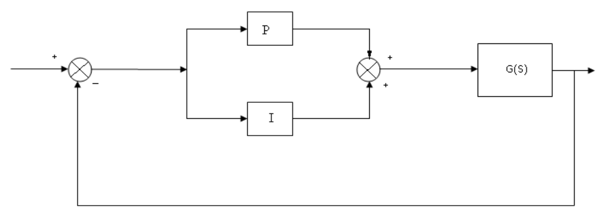

Fig 1: Closed loop system of higher order model with PI controller

III.NUMERICAL EXAMPLE

Consider a higher order system whose transfer function with uncertainty is given by[24].

$$G(s) = \frac{[28.5,30.5]s^2 + [6.935,8.935]s + [18.2,20.2]}{[1,1]s^6 + [17.47,19.47]s^5 + [46.78,48.78]s^4 + [67.52,69.52]s^3 + [64.86,66.86]s^2 + [43.3,45.3]s + [14.16,16.16]}$$

Let the PI Controller transfer function in parametric is given by

$$C(s) = [k_{p\min}, k_{p\max}] + \frac{[k_{\mathrm{Im}in}, k_{\mathrm{Im}ax}]}{s}$$

By applying necessary and sufficient conditions the values of K_P and K_I in parametric uncertainty are obtained by using the equations (3), (4) and (7)

$$k_{p\min} = 0.28384$$

$$k_{p\max} = 0.4070$$

$$k_{\mathrm{Im}\,in} = 0.00164$$

$$k_{i\max} = 0.07969$$

The PI controller transfer function in parametric uncertainty is given by

$$C(s) = [0.28384, 0.4070] + \frac{[0.00164, 0.07969]}{s}$$

The closed loop step response of the system with PI controller is shown in Fig2 and Fig 3

It is observed from the Fig 2 and Fig 3 that the designed PI controller obtained from the proposed method stabilizes the higher order uncertain system.

IV.SIMULATIONRESULTS

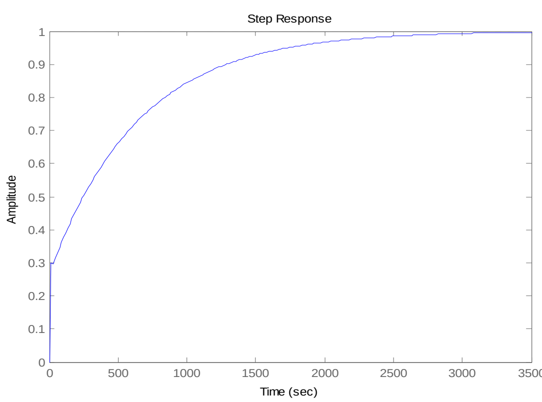

Fig 2: Closed loop step response with PI controller (lower bound)

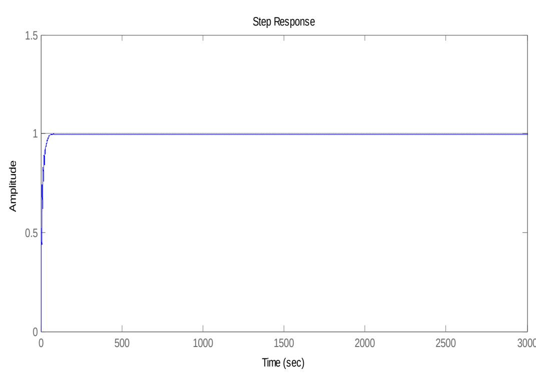

Fig 3: Closed loop step response with PI controller (higher bound)

V.CONCLUSION

A PI controller is designed for higher order uncertain systems from robust Hurwitz polynomial. The proposed PI controller procedure is illustrated through a typical numerical example available in the literature.

It is observed from the simulation results of Fig 2 and Fig 3, that the designed PI controller obtained from the proposed method stabilizes the higher order uncertain system.

VI.REFERENCES

[1] Ziegler, J. G. and N. B. Nichols, "Optimum settings for automatic controllers," *Trans. ASME,* vol. 64, pp. 759-768, 1942.

[2] Astrom, K. J., T. Hagglund, C. C. Hang and W.K. Ho, "Automatic tuning and adaptation for PID controllers- a survey," *Control Eng. Pract.*, vol. 1, pp. 699-714, 1993.

[3] Astrom, K. J. and T. Hagglund, *PID Controllers: Theory, Design, and Tuning.* Instrument Society ofAmerica, 1995.

[4] Zhuang, M. and D. P. Atherton, "Automatic tuning of optimum PID controllers*," IEE Proc. Part D,* vol. 140, pp. 216-224, 1993.

[5] Ho, W. K., C. C. Hang and L. S. Cao, "Tuning of PID controllers based on gain and phase margin specifications," *Automatica*, vol. 31, pp. 497-502, 1995.

[6] Ho, M. T., A. Datta and S. P. Bhattacharyya, "A new approach to feedback stabilization," *Proc. of the35th CDC,* pp. 4643-4648, 1996.

[7] Ho, M. T., A. Datta and S. P. Bhattacharyya, "A linear programming characterization of all stabilizing PID controllers," *Proc. of Amer. Contr. Conf.*, 1997.

[8] Ho, M. T., A. Datta and S. P. Bhattacharyya, "A new approach to feedback design Part I: Generalized interlacing and proportional control," *Dept. of Electrical Eng., Texas A&M Univ., College Station, TX, Tech. Report TAMU-ECE97-001-A*, 1997.

[9] Ho, M. T., A. Datta and S. P. Bhattacharyya, "A new approach to feedback design Part II: PI and PIDcontrollers," *Dept. of Electrical Eng., Texas A&M Univ., College Station, TX, Tech. Report TAMUECE97- 001-B*, 1997.

[10] M.Siva Kumar,B.J.J.Raju,A.sarada Devi and P.Naga Manasa,"A novel approach for the design of stabilizing PI Controllers" Dept. of Electrical Eng., *proceeding of International Conference MS,07,India,December 3-5,2007*. 1988.

[11] B. R. Barmish, A generalization of Kharitonov's four polynomial concept for robust stability problems with linearly *Dependent coefficient perturbations*, IEEE Trans. Automatic Control, Vol. 34, pp. 157- 165, 1989.

[12] A. C. Bartlett, C. V. Hollot and L. Huang, *Root location of an entire polytope of polynomials: it suffices to check edges*, Mathematics of Control, Signals and Systems, Vol. 1, pp.

61-71, 19

[13] S.P.Bhattachrya, *Robust stabilization against structured perturbations*, Lect. Notes in Control and Information Sciences,Vol. 99, Springer Verlag, Berlin, 1987.

[14] S. P. Bhattacharyya, H. Chapelett and L. H. Keel, *Robust Control: The Parametric Approach,* Prentice Hall Inc., NJ. 1995.

[15] P. Dorato (Ed), *Robust Control*, IEEE Press, NY, 1987.

[16] P. Dorato and R. K. Yedavalli (Eds.), *Recent Advances in Robust Control*, IEEE Press, NY, 1990.

[17] B. M. Patre and P. J. Deore, *Robust Stabilization of Interval Plants*, European Control Conference ECC-03, University of Cambridge, 01-04 Sept.2003.

[18] B. K. Ghosh, *Some new results on the simultaneous stabilization of a family of single input single output systems*, Systems and Control Letters, Vol. 06, pp. 39-45, 1985.

[19] C. V. Hollot and F. Fang, *Robust stabilization of interval plants using lead or lag compensators*, Systems and Control Letters, Vol. 14, pp. 9-12, 1990.

[20] B. R. Barmish, C. V. Hollot, F. J. Kraus and R. Tempo, *"Extreme point results for robust stabilization of interval plants with first order compensators,"* IEEE Trans, on Automatic Control, vol. AC-37, pp. 707- 714, 1992. **380**

[21] B. R. Barmish and H. I. Kang, *A survey of extreme point results for robustness of control systems*, Automatic a, Vol. 29, No. 1, pp. 13-35, 1993.

[22] *A new class of criterion for the stability of the polynomial* "Act Mechnica sinica , pp. 110-116,1976

[23] B. Bandyopadhyay," Control of higher order system via its reduced model" IEEE symposium

[24] M.SivaKumar,D.SrinivasaRao,Ch.Anu Radha,D.Koteswara Raju,"A novel approach for design of robust PI controller for higher order systems with parametric uncertainty " proc . of International Conference on Control Instrumentation System Confernce (CISCON) ,MIT , Manipal, India, Nov 3-6, 2011.

VII.BIOGRAPHIES

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