Static Force Analysis of Mini Hydraulic Backhoe Excavator And Evaluation Of Bucket Capacity, Digging Force Calculations

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Abstract
Rapidly growing rate of industry of earth moving machines is assured through the high performance construction machineries with complex mechanism and automation of construction activity. Design of backhoe link mechanism is critical task in context of digging force developed through actuators during the digging operation. The digging forces developed by actuators must be greater than that of the resistive forces offered by the terrain to be excavated. This paper focuses on the evaluation method of bucket capacity and digging forces required to dig the terrain for light duty construction work. This method provides the prediction of digging forces and can be applied for autonomous operation of excavation task. The evaluated digging forces can be used as boundary condition and loading conditions to carry out Finite Element Analysis of the backhoe mechanism for strength and stress analysis. A generalized breakout force and digging force model also developed using the fundamentals of kinematics of backhoe mechanism in context of robotics. An analytical approach provided for static force analysis of mini hydraulic backhoe excavator attachment.

Key words: Digging Forces, Autonomous Excavation, Resistive forces, Heaped capacity

I. INTRODUCTION

Applications for backhoe excavator in India include use as a utility machine at large construction sites (roads and dams for example) and urban infrastructure projects as well as the loading of hoppers and trucks, trenching, the cleaning of canals and ditches, general excavation, solid waste management and even demolition and mining work. However, the backhoe loader, with over 70 per cent of its usage being in excavating tasks, is most frequently used as a production machine as opposed to a utility machine in other parts of the world [3]. An excavator is an engineering vehicle consisting of a backhoe with cabin for the operator and wheeled or tracked system for movement and engine is used for power generation. Hydraulic system is used for operation of the machine while digging or moving the material. Excavation is of prime importance in mining, earth removal and general earthworks. Hydraulic cylinders apply forces to boom, arm and the bucket to actuate the mechanism. Depending on the mechanism position, working pressure and diameter of the hydraulic cylinders, the amount of excavation force changes. In practice, the arm cylinders are used for adjusting the bucket position not for digging. They may be used for lifting purpose. While arm and bucket cylinder is used for excavation. Thus, calculation of breakout or digging force must be carried out separately when arm or bucket cylinder is the active cylinder [2]. The maximum digging forces are the digging forces that can be exerted at the outermost cutting point. These forces are calculated by applying working circuit pressure to the cylinder(s) providing the digging force without exceeding holding circuit pressure in any other circuit. Weight of components and friction are to be excluded from these force calculations [6].

II. PROBLEM FORMULATION

In the era of globalization and tough competition, the use of machines is increasing for the earth moving works; considerable attention has been focused on designing of the earth moving equipments. Thus, it is very much necessary for the designers to provide not only a equipment of maximum reliability but also of minimum weight and cost, keeping design safe under all loading conditions [2]. Although excavation is ubiquitous in the construction industry, most day-to-day operations proceed on technology that is decades old—technology that has not kept pace with other industries. A recent trend towards greater automation of excavation machines reflects a larger movement in the construction industry to improve efficiency. Currently, human operators require ten to fifteen years of experience before they can be considered experts. Their work is often dirty, strenuous and repetitive [5]. Autonomous excavation has attracted interest because of the potential for increased productivity and lower labor costs. This research concerns the problem of automating a hydraulic excavator for mass excavation, where tons of earth
are excavated and loaded into trucks. This application is commonly found in many construction and mining scenarios. In such applications, fast operational speed of these machines is desired, because it directly translates to increased productivity. Much of the prior research in autonomous excavation has focused on digging and related topics such as soil modeling and bucket-soil force interactions. Only a few researchers have looked into the free motion planning problem within the context of the mass excavation task. Also, much of the autonomous excavation research has concentrated on functionality, where simply digging a full bucket of material is good enough [4]. To perform an excavation task it is necessary that the digging forces produces by actuators must be higher than that of the resistive forces offered by the terrain. For autonomous excavation task it is very important to evaluate the digging forces. The presented research work is on the evaluation of digging forces, which are according to standards of SAE. In addition, a generalized digging force model developed based on fundamentals of kinematics of backhoe excavator attachment in context of robotics which can be use as a boundary condition (time varying or dynamic) to carry out the dynamic finite element analysis of the proposed backhoe excavator. Moreover; static force analysis carried out by considering the maximum breakout force condition and static force analysis done for the different parts of the backhoe excavator and can be taken as boundary conditions for static FEA.

III. BUCKET CAPACITY CALCULATION

Bucket capacity is a measure of the maximum volume of the material that can be accommodated inside the bucket of the backhoe excavator. Bucket capacity can be either measured in struck capacity or heaped capacity. Globally two standards used to determine the heaped capacity, are: (i) SAE J296: “Mini excavator and backhoe bucket volumetric rating”, an American standard (ii) CECE (Committee of European Construction Equipment) section VI, a European standard [2]. The struck capacity directly measured from the 3D model of the backhoe bucket excavator for our case as shown in Fig.1 by following the SAE J296 standards [2]. As can be seen from the left side of the Fig. 1, \(P_{\text{area}}\) is the area bounded by struck plane (blue line) and side protector (red curve), and it is 66836 mm\(^2\).

As can be seen from Fig. 1 the heaped capacity can be given as:

\[ v_h = v_s + v_e \]

Where, \(v_s\) is the struck capacity, and \(v_e\) is the excess material capacity heaped. Struck capacity \(V\) can be calculated from Fig. 1 as:

\[ V_s = P_{\text{area}} (w_r + w_r/2) = 0.02072 \text{ m}^3 \]

Excess material capacity \(V\) for angle of repose 1:1 can be calculated from Fig. 1 as:

\[ V_e = (l_{w_r}^2/4 - w_r^2/12) = 0.00709 \text{ m}^3 \]

By using equations (1), (2), and (3) the bucket capacity for the proposed 3D backhoe bucket model comes out to be 0.02781 m\(^3\) = 0.028 m\(^3\).

IV. DIGGING FORCES

Bucket penetration into a material is achieved by the bucket curling force \(F_B\) and arm crowd force \(F_S\). The rating of these digging forces is set by SAE J1179 standard “Hydraulic Excavator and Backhoe: Digging Forces” [6]. These rated digging forces are the forces that can be exerted at the outermost cutting point (that is the tip of the bucket teeth). These forces can be calculated by applying working relief hydraulic pressure to the cylinders providing the digging force.
Fig. 2 shows the measurement of bucket curling force \( F_B \), arm crowd force \( F_S \), the other terms in the figure \( d_A, d_B, d_C, d_D, d_E, \) and \( d_F \) shows the distances as shown in Fig. 2. According to SAE J1179: Maximum radial tooth force due to bucket cylinder (bucket curling force) \( F_B \) is the digging force generated by the bucket cylinder and tangent to the arc of radius \( d_D \).

\[
F_B = \frac{\pi (\theta/4) \rho}{d_d} \left( \frac{d_A \times d_C}{d_E} \right)
\]

(4)

Where \( D_B \) is the end diameter of the bucket cylinder in (mm) and the working pressure is \( p \) in MPa or N/mm\(^2\) and other distances are in mm and remains constant. Equation (4) determines the value of the bucket curl or breakout force in N. Now let us determine the maximum radial tooth force due to arm cylinder \( F_S \). Maximum tooth force due to arm cylinder is the digging force generated by the arm cylinder and tangent to arc of radius \( d_F \).

\[
F_S = \frac{\pi (\theta/4) D_A^2 \times d_E}{d_F}
\]

(5)

V. FORCE CALCULATION WHEN THE BUCKET CYLINDER IS ACTIVE

Force created by the bucket cylinder \( A_9A_{10} \) (length of the arm cylinder) \( F \) can be found by using its end cylinder diameter and working pressure as described in the previous section.

\[
F_{a7,a8} = \rho \left( \frac{\pi}{4} \right) D_a^2
\]

(15)

As can be seen from the Fig. 3 the breakout force from the bucket cylinder \( F_{Bucket} \) (acting on the teeth of the bucket in the tangential direction of \( A_3A_4 \) radius) will be the moment created by the bucket cylinder \( M_{Bucket} \) divided by the distance \( A_3A_4 \). This leads to;

\[
F_{arm} = \frac{M_{arm}}{d_a \phi}
\]

In equation (16) the length \( A_3A_4 \) is fixed from the geometry of the bucket and thus known to us.
Here, only the bucket cylinder is active and the bucket is made curling inward from the point $A_3$ to point $A_4$ for the excavation operation to be carried out by bucket cylinder.

Now moment created on bucket $M_{\text{Bucket}}$ will be the product of the force created by the bucket cylinder $F$ and the perpendicular distance to the cylinder, so $M_{\text{Bucket}}$ can be given by:

$$M_{\text{arm}} = a_2 \sin (a_7 a_8 a_2) F_{a_7 a_8} (17)$$

From the equation (18) the angle can be determined either from the sensors (in case of autonomous backhoe operations) or from the joint 4 angle $\theta$. If the joint 4 angle $\theta$ is known then by following the reverse procedure of the end of the section in which the arm cylinder is active, from equations (14), (13) and (12) the length of the bucket actuator $A_A$ can be determined. Thus by using the equations (17), and (18) the breakout force or bucket digging force can be determined in the generalized form from equation (16). In this section, both the breakout force of bucket cylinder $F_{\text{Bucket}}$ and the digging force of the arm cylinder $F_{\text{Arm}}$ have been determined in the generalized form. These two forces are the function of the respective joint angles, and these joint angles are the function of time while excavating the earth. So equation(7) and (16) provides the generalized digging and the breakout forces as a function of time (dynamic), and thus can be used as a boundary condition for the dynamic FEA of the backhoe excavator, but the dynamic FEA of the backhoe excavator is not the part of the research reported in this paper. MATLAB code also developed for this generalized digging force model.

**VI. STATIC FORCE ANALYSIS**

In this section, calculation for the static force analysis of the backhoe excavator for the condition in which the mechanism produces the maximum breakout force has been explained. Unlike the previous section’s flexibility where the force analysis could be done for any of the position and orientation (collectively known as the configuration) of the mechanism from the available breakout and digging forces, in static analysis one configuration of the mechanism has to be decided first for which the analysis is to be carried out. From all the configurations, the maximum breakout force condition is the most critical one as it produces the highest breakout force, and thus for this condition the force analysis is done, and will be used as a boundary
condition for static FEA. The free body diagram of bucket, arm, and boom, directions and magnitudes of the forces are explained in the next section. Fig. 4 shows the configuration in which the mechanism is producing the maximum breakout force.

6.1. Bucket static force analysis

Fig. 5 shows the free body diagram of the bucket. As can be seen the reaction force on the bucket teeth at point A4 due to the breakout force 7.626 KN acts at the angle 38.23º for configuration of the maximum breakout force condition. Static forces on joints can be calculated by considering the summation of forces must be equal to zero and summation of moments equal to zero for equilibrium condition of the bucket, arm and boom respectively. All the forces in the Fig. 5, Fig. 6, Fig. 7 and Fig. 8 are in Kilo Newton (KN). Firstly the reaction force acting on the bucket teeth (at point A4) is resolved in the horizontal (X) and the vertical (Y) directions by using the following equations (19) and (20).

\[
F_{4X} = F_B \cdot \cos(\rho) \\
F_{4V} = F_B \cdot \sin(\rho)
\]

Where, \( \rho \) is the angle between the breakout force of bucket and the ground level as horizontal reference surface of 38.23º as shown in Fig. 5. Now considering the bucket in equilibrium \( \Sigma M = 0 \), taking moment about the bucket hinge point \( A_3 \) leads to;

\[
F_4 \cdot l_4 = F_{eb} \cdot l_{eb} + F_{11} \cdot l_{11}
\]

(21)

Where, \( F_4 \) is the force acting at bucket tool tip when the bucket approaches to the earth in the maximum breakout force condition as shown in Fig. 4 and Fig. 5, which is equivalent to the bucket breakout force \( F_B \).

\( l_4 \) is the distance of the tool tip of the bucket from the bucket hinge point (547 mm), \( l_{eb} \) is the distance between the C.G. of the bucket to the bucket hinge point (220 mm), \( l_{11} \) is the distance of the bucket hinge point to the idler link hinge point on bucket (181 mm), \( F_{eb} \) is the gravitational force acting on bucket (0.235 KN) and \( F_{11} \) is the force acting on hinge point of the idler link on bucket which can be found by using equation (21) and acting at an angle \( \beta \) of 64º as shown in Fig. 5. The force \( F_{11} \) can be resolved in horizontal (X) and the vertical (Y) directions by using the following equations (22) and (23).

\[
F_{11x} = F_{11} \cdot \cos(\beta_1) \\
F_{11v} = F_{11} \cdot \sin(\beta_1)
\]

(22)

(23)

Considering \( \Sigma F = 0 \), force on the bucket hinge point \( A_3 \) can be found out as shown in Fig. 5.

<table>
<thead>
<tr>
<th>Joint of the bucket</th>
<th>Forces (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal (X)</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>-5.933</td>
</tr>
<tr>
<td>( A_{11} )</td>
<td>-9.977</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>15.97</td>
</tr>
</tbody>
</table>

The negative sign shows the force acting in the leftward direction for horizontal component of the force and downward direction for vertical component of the force. The forces on each of the joints of the bucket are shown in Table 1.

6.2. Arm static force analysis

In Fig. 6 (a) shows the important dimensions and angles for the moments and the resolution of forces respectively. Fig. 6 (b) shows the static forces acting at the different points on the arm. The Force \( F_{11} \) is the force acting on the intermediate link \( A_{10}A_{12} \) from the idler link \( A_{11}A_{10} \) at an angle of 70.5º as shown in Fig. 6(a).

Considering the arm in equilibrium \( \Sigma M = 0 \) and taking moment about the arm to boom hinge point \( A_2 \) leads to;

Fig. 5. Free body diagram of bucket
Fig. 6. (a) Geometrical dimensions of the arm  
(b) Free body diagram of the arm

Where, $F_8$ is the force acting at arm cylinder front end hinge point ($A_8$) which can be determined using the equation (20). Here, $l_8$ is the distance between the arm hinge point ($A_2$) and arm cylinder front end hinge point ($A_8$) in maximum breakout force condition of 285 mm as shown in Fig. 4, $F$ is the vertical force component acts on bucket hinge point ($A_3$) of 15.74 KN as shown in Fig. 6(b), $l_{3H}$ is the horizontal distance between the bucket hinge point ($A_3$) and arm hinge point ($A_2$) of 466 mm as shown in Fig. 6(a), $F_{ga}$ is the gravitational force on arm of 0.289 as shown in Fig. 6(b), $l_{ga}$ is the distance between the C.G. of arm and arm hinge point ($A_2$) of 194 mm as shown in Fig. 6(a), $F$ is the horizontal force component acts on bucket hinge point ($A_3$) of 15.97 KN as shown in Fig. 6(b), $l_{3Y}$ is the vertical distance between the bucket hinge point ($A_3$) and arm hinge point ($A_2$) of 551 mm as shown in Fig. 6(a), $F_{12}$ is the force acting on intermediate link due to idler link of 7.784 KN as shown in Fig. 6(b), $l_{12}$ is the distance between arm hinge point ($A_2$) and intermediate link hinge point on arm ($A_{12}$) of 591 mm as shown in Fig. 4, $F_9$ is the force acting on arm through bucket cylinder of 22.405 KN as shown in Fig. 6(b), and $l_9$ is the distance between arm hinge point ($A_2$) and the bucket cylinder end hinge point ($A_9$) of 294 mm as shown in Fig. 4. Considering $\Sigma F = 0$, force on the arm to boom hinge point $A_2$ can be found out as shown in Fig. 6(b). The forces on each of the joints of the arm are shown in Table 2.

Table 2. Static forces on arm joints

<table>
<thead>
<tr>
<th>Joint of the bucket</th>
<th>Horizontally (X) component</th>
<th>Vertically (Y) component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>-15.97</td>
<td>-15.74</td>
</tr>
<tr>
<td>$A_2$</td>
<td>-5.358</td>
<td>5.646</td>
</tr>
<tr>
<td>$A_3$</td>
<td>13.264</td>
<td>18.057</td>
</tr>
<tr>
<td>$A_4$</td>
<td>-44.196</td>
<td>0</td>
</tr>
<tr>
<td>$A_5$</td>
<td>52.26</td>
<td>-7.949</td>
</tr>
</tbody>
</table>

6.3. Boom static force analysis

Fig. 7 shows the free body diagram of the boom, in which Fig. 7 (a) shows the important dimensions and angles for the moments and the resolution of
forces respectively. The Fig. 7 (b) shows the static forces acting at the different points on the boom. The force $F_7$ is the force acts by arm at point $A_7$ through arm cylinder which is same as the force $F_8$ but direction is opposite.

The force $F_7$ can be resolved in horizontal $(X)$ and the vertical $(Y)$ directions by using the following equations (31) and (32). Here, $\beta$ is the angle made by force on boom through arm cylinder with horizontal reference at point $A_7$ of $0^\circ$ as shown in Fig. 7(b).

\[
F_{7H} = F_7 \cdot \cos(\beta_7) \\
F_{7V} = F_7 \cdot \cos(\beta_7)
\]

Considering the boom in equilibrium $\Sigma F = 0$ and taking moment about the arm to boom hinge point ($A_1$) leads to:

\[
F_5 \cdot l_5 = (F_{2H} \cdot l_{2V}) + (F_{gbo} \cdot l_{gbo}) - (F_{2V} \cdot l_{2H}) - (F_7 \cdot l_7)
\]

Where, force $F_5$ is the acting at point $A_5$ through boom cylinder which is acting at angle $\beta$ at point $A_5$ of $45.58^\circ$ as shown in Fig. 7(b). $l_5$ is the distance between boom hinge point and boom cylinder end hinge point on swing link of 218 mm as shown in Fig. 4. $F_{2H}$ and $F_{2V}$ are the horizontal and vertical components of the force acting at point $A_2$ of 52.26 KN and 7.963 KN respectively as shown in Fig. 7(a). $l_{2H}$ and $l_{2V}$ are the horizontal and vertical distances of point $A_2$ form boom hinge point $A_4$ of 1301 mm and 348 mm respectively as shown in Fig. 7(a). $F_{gbo}$ is the gravitational force acts on boom of 0.432 KN as shown in Fig. 7(b). $l_{gbo}$ is the horizontal distance between C.G. of boom and boom hinge point $A_1$ of 524 mm as shown in Fig. 7(a). $l_7$ is the vertical distance between arm cylinder end hinge point $A_7$ and boom hinge point $A_1$ of 633 mm as shown in Fig. 4. The force $F_5$ can be resolved in horizontal $(X)$ and the vertical $(Y)$ directions by using the following equations (34) and (35).

\[
F_{5H} = F_5 \cdot \cos(\beta_5) \\
F_{5V} = F_5 \cdot \cos(\beta_5)
\]

Considering $\Sigma F = 0$, force on the bucket hinge point $A_1$ can be found out as shown in Fig. 7(b). The forces on each of the joints of the boom are shown in Table 3.

<table>
<thead>
<tr>
<th>Joint of the</th>
<th>Forces (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal (X) component</td>
</tr>
<tr>
<td>$A_2$</td>
<td>-52.26</td>
</tr>
<tr>
<td>$A_7$</td>
<td>44.196</td>
</tr>
<tr>
<td>$A_5$</td>
<td>-69.033</td>
</tr>
<tr>
<td>$A_1$</td>
<td>-77.033</td>
</tr>
</tbody>
</table>

6.4. Swing link static force analysis

Fig. 8 shows the free body diagram of the swing link, it shows the resolved forces in horizontal and vertical directions at each joint of the swing link. The force $F_6$ is acting at point $A_6$ of boom cylinder end hinge point through the boom cylinder which is equal to the force $F_5$ but opposite in direction. $F_{01}$ and $F_{02}$ are the forces acts on swing cylinder front end hinge points of $A_{01}$ and $A_{02}$ respectively through swing cylinders 1 and 2 of 30.827 KN. These forces can be finding out by using the equation

\[
W \text{ here, } D \text{ is the swing cylinder end diameter of 50 m m. } \text{ and } p \text{ is the working pressure of the hydraulic circuit of 15.7 MPa. The forces on each joint of the swing link are shown in Table 4.}
\]

<table>
<thead>
<tr>
<th>Joint of the</th>
<th>Forces (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swing link</td>
<td>Horizontal (X) component</td>
</tr>
<tr>
<td>$A_1$</td>
<td>77.033</td>
</tr>
<tr>
<td>$A_5$</td>
<td>69.033</td>
</tr>
<tr>
<td>$A_{01}$</td>
<td>-30.827</td>
</tr>
<tr>
<td>$A_{02}$</td>
<td>-30.827</td>
</tr>
</tbody>
</table>

7. COMPARISON OF BACKHOE EXCAVATOR MODELS

Table 5 shows the comparison of physical dimensions, bucket specifications and digging forces of the design ed pro posed backhoe excavator with the standard excavators.
VIII. CONCLUSION

The capacity of the bucket has been calculated according to the standard SAE J296 and comes out to be 0.028 m$^3$. This bucket specification is the most superior when compared to all other standard mini hydraulic excavator models available in the market. The breakout force calculation is done by following the standard SAE J1179 and comes out to be 7626 N. The SAE standards only provide the breakout and digging forces for maximum breakout force condition but for autonomous application it is important to understand and to know or predict the digging forces for all position of bucket configuration, which is presented here by development of the generalized breakout force model. A generalized breakout force (when the bucket cylinder is active), and the digging force (when the arm cylinder is active) models are developed as a function of time and can be used as a boundary condition for the dynamic FEA of the backhoe excavator. The static force analysis performed by considering the maximum breakout force configuration and can be used as a boundary condition for static FEA of the backhoe parts. The comparison of the different backhoe excavator models in context of physical dimensions, bucket specifications and digging forces shows that by kipping slightly less or same link dimensions the required digging force of proposed backhoe attachment is reduced to 7626 N, which are enough and more than resistive forces offered by ground 3916.7 N [1] for light duty construction work, which requires less pressure and power to actuate the backhoe mechanism for digging task and fuel consumption is less, ultimately the operating cost gets reduced.

<table>
<thead>
<tr>
<th>Name of Manufacturer</th>
<th>Bucket Specifications</th>
<th>Nominal Force</th>
<th>Proposed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Name</td>
<td>Description</td>
<td>Capacity (m$^3$)</td>
<td>Weight (kg)</td>
</tr>
<tr>
<td></td>
<td>Arm length</td>
<td>0.025</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Boom length</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Overall height</td>
<td>210</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Table 5. Comparison of physical dimensions, bucket specifications and digging forces

REFERENCES