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Multiscale Intuitionistic Fuzzy Homogeneity Measure

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ABSTRACT
In this paper, homogeneous regions formed by using multiscale intuitionistic fuzzy set is compared with homogeneous regions formed by multiscale representation of an image. The later approach does not consider the equivocal nature of the real world objects and is not accurate enough to measure the homogeneity in a real world scenario. By applying the theories of scale-space along with intuitionistic fuzzy representation for images, homogeneous regions are properly defined. Multiscale representation can tolerate the disturbance of trivial regions, and intuitionistic fuzzy representation deals with hesitancy in image boundary, therefore produces proper homogeneous regions.
Keywords - A-IFS, fuzzy image, Homogeneous region, Multiscale, Linear scale space set, allow choice of more appropriate peak and valley points so as to improve the quality of segmentation.

I. INTRODUCTION

In this paper, the advantages of A-IFS image representation and linear scale space theory is used together that deal with hesitancy in image and analyze the image at different scale which can be further used for image segmentation. Major contribution of this paper is as follows: First step is the Multiscale representation of an image which is then used to form homogeneous regions using gaussian membership function. The second step is to use Atanassov‟s IFS theory to handle hesitancy at the boundaries of an image. A-IFS image is then represented at multiple scales using which homogeneous regions are formed. The comparison of homogeneous region formed using two methods is done on quality basis.

An inherent property of real-world objects is that they only exist as meaningful entities over certain ranges of scale. This fact, that objects in the world appear in different ways depending on the scale of observation, has important implications if one aims at describing them. Multiscale representation of an image provides an option to represent an image at multiple scales. In [1] multiscale roughness measure is constructed using linear scale space and rough set to generate hierarchical roughness of color distribution under multiple scale which improves segmentation results.

Most widely used segmentation technique, the Histogram thresholding, has low computation complexity but does not consider dependence of adjacent pixels. Histogram method is also inefficient for images that blur at object boundaries. To deal with the inherent fuzziness in the digital images and to take into account both local and global information in the image, the segmentation methods based on fuzzy histogram and fuzzy homogeneity histogram is presented in [2]. The rough set-based color image segmentation techniques presented in [3] use the upper approximation called the histon, and in [4], the Atanassov intuitionistic fuzzy set [5] is used to compute (A-IFS) roughness index which is obtained from the lower and upper approximations of a rough
The rest of the paper is organized as follows; In section II, preliminary information about intuitionistic fuzzy set and linear scale space theory is presented. Section III describes the complete flow of algorithm. Experimental results are presented in section IV, followed by conclusion in section V.

II. PRELIMINARIES

In this section, we present the preliminary concepts of A-IFS theory and linear scale space theorem

2.1 Atanassov"s Intuitionistic Fuzzy Set (A-IFS) Theory

The A-IFS defined on the universe of discourse $X = \{x_1, x_2, ..., x_N\}$ is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\} \qquad (1)$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with condition $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in X$.

The numbers $\mu_A(x)$ and $\nu_A(x)$ are the degree of membership and degree of non-membership of x to A, respectively. For each A-IFS A in X, we call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ the intuitionistic index also known as degree of hesitancy of x to A and $0 \le \pi_A(x) \le 1$ for each $x \in X$.

2.2 Linear Scale Space Theory

Scale-space theory is the framework for multiscale signal representation developed by the computer vision, image processing and signal processing communities with complementary motivations from physics and biological vision. The scale-space representation is parameterized by the size of the smoothing kernel used for suppressing fine-scale structures. The mathematical expression of mostly used smoothening kernel is given as Eq.(3).The parameter *t* in this family is referred to as the scale parameter, with the interpretation that image structures of spatial size smaller than about $\sqrt{t}$ have *largely been smoothed away in the scale-space level at scale t. The main type of scale space is the linear (Gaussian) scale space.*

As the scale *t* increases, the image becomes smoother. Greater the value of *t*, wider will be the Gaussian kernel as shown in Fig 2, resulting in smoother image, which also removes noise present in an image. Gaussian kernel is the only low–pass filter which can be used for multiple scale representation, as it is of crucial importance that the smoothing filter does not introduce new spurious structures at coarse scales that do not correspond to simplifications of corresponding structures at finer scales.

III. ALGORITHM

This section describes the complete flow of the algorithm for homogeneous region formation using multiscale representation and multiscale intuitionistic fuzzy set. The block diagram representation of former approach is shown in Fig.1 (a). The block diagram for the later approach is shown in Fig.1 (b).

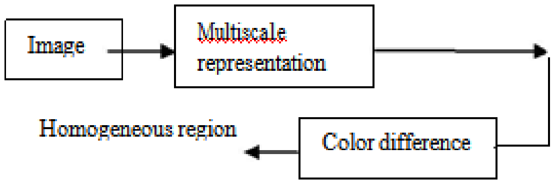

Fig 1(a): Block Diagram of algorithm based on multiscale representation of an image.

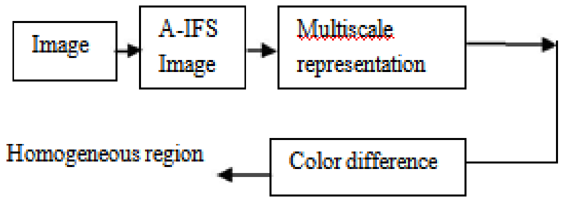

Fig 1(b): Block Diagram of Algorithm based on multiscale IFS representation of an image.

3.1 Linear scale-space representation

The linear scale-space technique will be utilized to construct the Multiscale approximation. Suppose F is an image of size $M \times N$. Given a scale parameter t and a $P \times P$ neighborhood, the linear scale-space representation of image F is the convolution of $F(m, n, i)$ with t-scale Gaussian kernel $g^t(m, n)$ (Fig. 2) covering neighborhood $P \times P$.

$$F^t = \{F^t(i) \mid i \in \{R, G, B\}\},$$
$$F^t(i) = \{L^t(m, n, i) \mid 1 \le m \le M, 1 \le n \le N\}$$

where $L^t(m, n, i) = F(m, n, i) * g^t(m, n) \qquad (2)$

$$g^t(m, n) = \frac{1}{2\pi t} e^{-(m^2+n^2)/2t} \qquad (3)$$

Fig 3 shows multiscale representation of an image „mountains" at different scale. It is clear from the Fig.3 that greater the value of scale, smoother will me the image.

3.2 Color Difference

For a pixel $F(m, n, i)$, suppose a scale t and $P \times P$ neighborhood, the color difference between $F(m, n, i)$ and its surrounding pixels in neighborhood can be defined as

$$d^t_{PxP}(m, n) = d(F(m, n, i), F^t(m, n, i)) = \sqrt{\frac{1}{2}\sum_{i=\{R,G,B\}}(F(m, n, i) - L^t(m, n, i))^2} \quad (4)$$

When the difference of a neighborhood at specific position is within a limited range, the corresponding image region will be homogeneous. Homogeneity function is defined to measure the homogeneous degree based on the color difference. In Eq. (4), ½ is used to normalize $d_{PxP}{}^t(m, n)$ between [0,1].

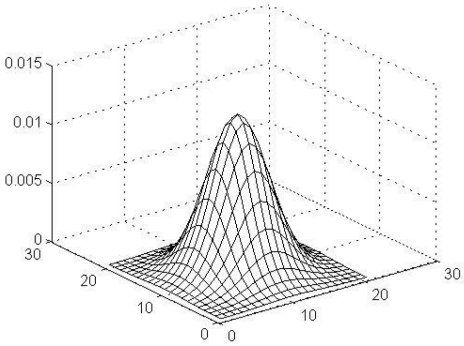

Fig 2:Gaussian kernel.

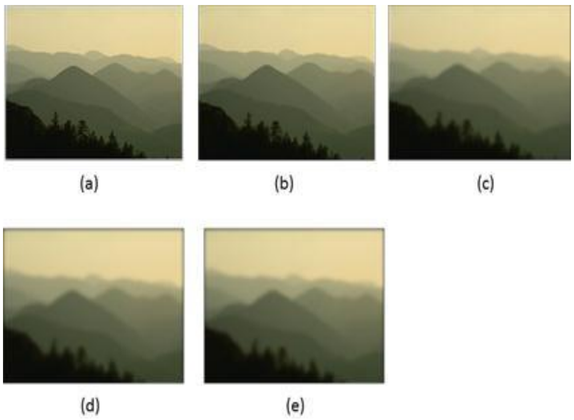

Fig 3: Multiscale representation of an image: (a) original image „mountains‟, (b-e) image at scale t=1,t=10,t=20,t=30 respectively.

3.3 Homogeneous Region

Suppose a pixel $F(m, n, i)$, under a given scale t, the homogeneous degree of $P \times P$ neighborhood relative to $F(m, n, i)$ is decided by the homogeneity function as follows.

$$s^t(m, n) = \exp\left(-0.5\left(\frac{d_T(m, n)}{\sigma}\right)^2\right) \quad (5)$$

In this paper, Gaussian membership function (MF) is used. There are several types of membership function present in the literature, but Gaussian MF has smooth transitions. The homogeneous degree decreases smoothly as the color difference increases. The relative region is believed to be heterogeneous when difference exceeds certain range. Fig. 4 shows the homogeneous regions of the „mountains image‟ under different scales. The bright regions in the images represent the homogeneous regions (the brighter, the more homogeneous). In contrast, the dark regions represent the heterogeneous regions. From

Fig. 4, we can find that the homogeneous region in the image gradually decreases with the increasing scales. At the initial scale level, the homogeneity of most objects in image can be well defined and the heterogeneous regions are regarded as the edges to segment the different homogeneous areas. As the scale increases, the information from an image is lost.

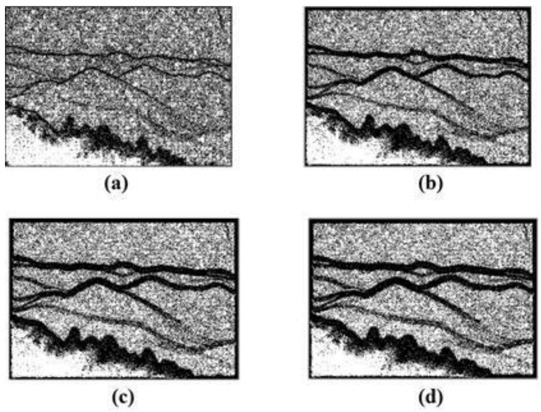

Fig 4: Homogeneous regions (a-d) at scale t=1, t=10, t=20, t=30 respectively.

3.4 A-IFS representation of an image

Given an image F of size $M \times N$, having L intensity levels g between 0 and L-1, A-IFS representation of image is given as

$F = \{\langle g_{mn}, \mu_F(g_{mn}), \nu_F(g_{mn})\rangle \mid m = 1,.. M, n = 1,..., N\}$ where $\mu_F(g_{mn})$ is the degree of membership and

$\nu_F(g_{mn})$ is the degree of the non-membership value of $(i, j)^{th}$ pixel in the image F. We consider the membership value at each pixel location to be simply the normalized intensity level as given by $\mu_F(g_{mn}) = \frac{g_{mn}}{L-1}$ for $g \in \{0,....., L-1\}$, $1 \le m \le M$ and $1 \le n \le N$.

In this paper, intuitionistic fuzzy complement is created from Sugeno fuzzy generating function [6]. The fuzzy complement function is defined as

$$c(\mu_F(x)) = g^{-1}(g(1) - g(\mu_F(x))) \qquad (6)$$

where $g(\mu_F(x)) = \frac{1}{\lambda}\log(1 + \lambda\mu_F(x))$ and the Sugeno type intuitionistic fuzzy generator is written as

$$c(\mu_F(x)) = \frac{1 - \mu_F(x)}{1 + \lambda\mu_F(x)} . \qquad (7)$$

The hesitation degree is given as

$$\pi_F(x) = 1 - \mu_F(x) - \frac{(1 - \mu_F(x))}{(1 + \lambda\mu_F(x))} \qquad (8)$$

3.5 Modified color difference formula for IFS image.

Before computing the color difference, multiscale representation of membership image is done using Eq. (2) Similarity measure between original image and IFS filtered image gets modified and is given as

$$d(F(m,n), flt(m,n)) = \left(\frac{1}{6} \sum_{i=\{R,G,B\}} \begin{bmatrix} (\mu^i(F(m,n)) - (\mu^i(flt(m,n))^2 \\ +(v^i(F(m,n)) - (v^i(flt(m,n))^2 \\ +(\pi(F(m,n)) - (\pi(flt(m,n))^2 \end{bmatrix} \right)^{0.5} \qquad (9)$$

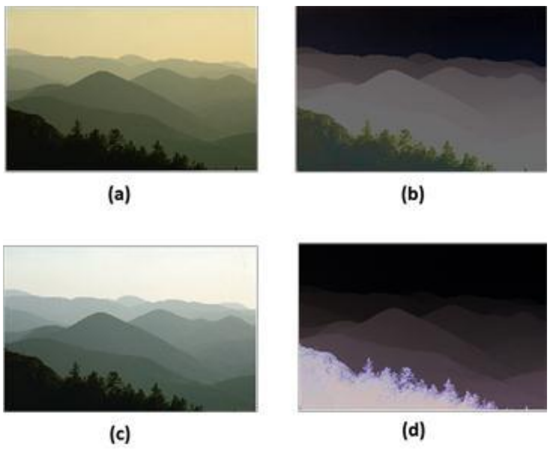

Fig 5: IFS representation of an image: (a) original image "mountains", (b) Membership image, (c) Non-membership image, (d) Hesitancy image.

3.6 Homogeneous regions.

The homogeneous regions are formed using Eq. (9). Fig. 6 shows the homogeneous region of multiscale IFS image.

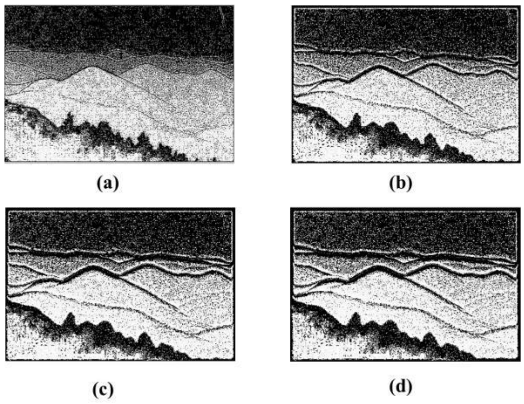

Fig 6: Homogeneous region of multiscale IFS image „mountains‟: (a-d) homogeneous region at scale t=1, t=10, t=20, t=30 respectively

IV. EXPERIMENTAL RESULTS

The algorithm is applied on random images from Berkeley database [7]. The neighborhood size of the gaussian kernel is taken as (*20* x *20)* for multiscale representation. The homogeneous regions using multiscale representation for „mountains‟ images are shown in Fig.4. And homogeneous regions using multiscale representation along with IFS representation is shown in Fig 6. The value of „λ‟ from Eq. (7) is taken as *2.5*. It is clear from Fig. 4 and Fig. 6 that homogeneous regions formed using multiscale representation doesn‟t consider the equivocal nature of real world information and it also over focus on trivial regions. The output image consist redundant data whereas homogeneous regions formed using multiscale IFS approach handles ambiguity at boundaries and thus gives proper results.

V. CONCLUSION

The paper proposed a method of homogeneity measure using A-IFS and linear scale space theory which can be used for image segmentation. Linear scale space theory is used for multiscale representation of an image. The A-IFS is constructed using sugeno fuzzy generator. This approach is been compared with the multiscale homogeneity measure approach. The qualitative evaluation shows the superiority of the approach.

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