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Comparative study Of Interpolated DFT algorithm

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Abstract—
Discrete Fourier Transform (DFT) is probably the most popular signal processing tool. Wide DFT use is partly dedicated to Fast Fourier Transform (FFT) algorithms (Cooley & Tukey, 1965, Oppenheim et al., 1999, Lyons, 2004). DFT may also be efficiently computed by recursive algorithms in the window sliding by one sample (Jacobsen & Lyons, 2003, Duda, 2010). Unfortunately, DFT has two main drawbacks that deteriorate signal analysis which are (Harris, 1978, Oppenheim et al., 1999): 1) spectral leakage, and 2) sampling of the continuous spectrum of the discrete signal. Spectral leakage is reduced by proper time windows, and the frequency bins between DFT bins are computed by interpolated DFT(IpDFT) algorithms, thoroughly presented in this paper.
Keywords —DFT, IpDFT, FFT, DFT.

I. INTRODUCTION

Discrete Fourier Transform (DFT) is probably the most popular signal processing tool. Wide DFT use is partly dedicated to Fast Fourier Transform (FFT) algorithm. Unfortunately, DFT has two main drawbacks that deteriorate signal analysis which are 1) spectral leakage, and 2) sampling of the continuous spectrum of the discrete signal. Spectral leakage is reduced by proper time windows, and the frequency bins between DFT bins are computed by interpolated DFT (IpDFT) algorithms.

II. SIGNAL MODEL

IpDFT algorithms may be derived for discrete sinusoidal or damped sinusoidal signals. Discrete sinusoidal signal is defined as

Vn = *A*cos(ω0*n* +ϕ), *n* = 0,1,2,...,*N* − 1 , (1)

and discrete damped sinusoidal signal is defined as cos(0) , 0, 0,1,2,..., 1 *dn*

Vn A ω *n* ϕ *e d n N* = + − ≥ = − , (2)

sample, *N* is the number of samples, and *d* is damping factor. If discrete signals (1) and (2) result from sampling analog counterparts then

ω0 = 2π (*F*0 /*Fs*) (3)

where *F*0 is the frequency of analog signal, *V*(*t*)=*A*cos(2*πF*0*t*+*φ*) or (*t*)=*A*cos(2*πF*0*t*+*φ*)*edFst*, in hertz, *Fs* is sampling frequency in hertz, and *t* is continuous time in seconds.

III. DFT ANALYSIS

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Math Type Equation). "Float over text" should *not* be where *A*>0 is signal's amplitude, 0<*ω*0<*π* is signal's frequency in radians or radians per sample also referred as angular frequency or pulsation, and *ω*0=*π* rad corresponds to the half of the sampling rate *Fs* in hertz, -*π*<*φ*≤*π* is the phase angle in radians, *n* is the index of the selected.Fourier transform (FT) of infinite length discrete time signal X*n* is defined as

( ) ∑ ω (4)

where *n* is integer sample index that goes from minus to plus infinity and *ω* is continuous frequency in radians (angular frequency, pulsation). Continuous spectrum *X*(*ej*) defined by (4) is periodic with the period 2*π*. The notation *X*(*ej*) instead of *X*( ), stresses up the connection between FT and *Z* transform. For finite length discrete time signal *vn* containing *N* samples DFT is defined as

∑ ( ) .(5)

From it is seen, that by DFT the FT spectrum is computed only for frequencies $k=(2\pi/N)k$, that is DFT samples continuous spectrum of the discrete signal. Finite length signal vn, $n=0,1,2,...,N-1$ is obtained from infinite length signal xn, $n=...-2, -1,0,1,2,...$ by windowing, that is by multiplication with discrete signal wn, called window, with nonzero values only on positions $n=0,1,2,...,N-1$

$Vn =$. (6)

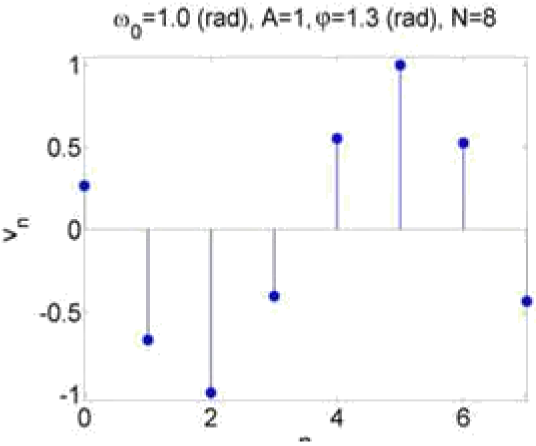

Fig. 1 illustrates equations given above for sinusoidal signal (1) analyzed with rectangular window for A=1, =1 rad, = 1.3 rad, and N =8.

IV. TIME WINDOWS

Window properties are determined by its spectrum $W(ej\omega)$ that consists from the main lobe which is the highest peak in the spectrum and side lobes. The main lobe of the window should be as narrow as possible, and side lobes should be as low as possible. Narrow main lobe improves frequency resolution of DFT analysis, while low side lobes reduce spectral leakage. Rectangular window is the one with the narrowest main lobe, which is an advantage and the highest side lobes which is disadvantage. All the other time windows reduce side lobes, and thus spectral leakage, by the cost of widening main lobe i.e. reducing frequency resolution. It is also known that rectangular window has the best noise immunity although systematic errors caused by leakage may be dominant for signal containing small number of cycles. Time windows are defined as cosine windows or non cosine windows.

V. INTERPOLATED DFT ALGORITHMS

The IpDFT problem for sinusoidal and damped sinusoidal signals is depicted and may be formulated as follows. Based on the DFT spectrum Vk (5) of the signal xn analyzed with the known window wn ,find the frequency correction δ so to satisfy the equation

( ) (7)

where $\omega 0$ is signal's frequency, N is the number of samples and k is the index of DFT bin with the highest magnitude. If $|Vk+1|>|Vk-1|$, as in Fig. then there is '+'.

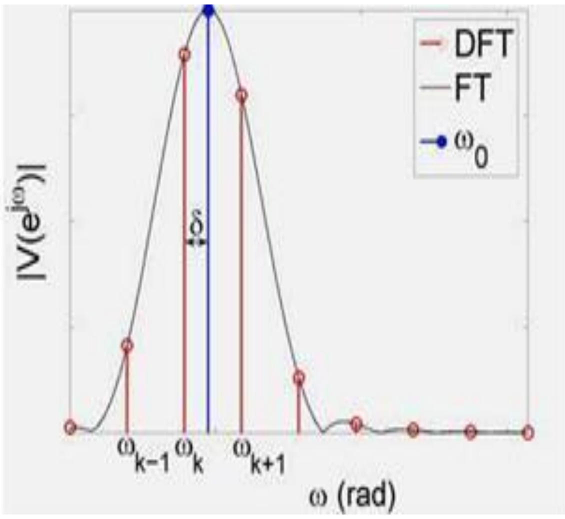

Fig.2. Illustration of IpDFT problem; $_{-1}$, frequencies of DFT bins - signal's frequency, - frequency correction.

VI. SOME PROPERTIES OF IPDFT ALGORITHMS

In this section we present results of simulations that describe systematic errors and noise immunity of IpDFT methods. Because of space constrains, only the results of frequency or frequency and damping estimation are presented. Including results for amplitude and phase estimation would multiply the number of figures. Furthermore, in practice estimation of frequency and damping is of primary importance, and once having frequency and damping the amplitude and phase may be estimated by LS or FT. It is also true that amplitude and phase estimation errors, not shown in this section, behave similarly to frequency estimation errors.

First, systematic errors of IpDFT algorithms for sinusoidal and damped sinusoidal signals are presented, and then robustness against additive, zero-mean, Gaussian noise is shown. In all simulations number of samples N=512 was chosen. Simulations were conducted in Matlab 64-bit floating point precision. Accuracy of this precision determined by the function *eps* (Matlab) is on the level 10– 15–10–16, and estimation errors cannot be lower than this accuracy.There are two types of error which are removes by using IpDFT.

VII. CONCLUSION

This paper describes DFT interpolation algorithms for parameters estimation of sinusoidal and damped sinusoidal signals. IpDFT algorithms have two main advantages:

1. Low computational complexity attributed to fast algorithms of DFT computation.
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4No need for the signal model (as opposed to parametric methods). IpDFT methods may be used as fully functional estimators, especially when noise disturbance is not very strong. If the signal model is known IpDFT may be used for providing starting point for LS optimization that is optimal for Gaussian zero-mean noise disturbance. For the signals with disturbances not possible to include in the signal model, as e.g. unknown drift, IpDFT with adequate time window may offer better performance than optimization.

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