

## Comparative Analysis of Different PID Tuning Techniques for Coupled Tanks System

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### Abstract

In this paper we have comparatively analyse the performance of different PID tuning Techniques for inherently nonlinear system. The ability of Proportional Integral Derivative (PID) controllers to compensate many practical industrial processes has led to their wide acceptance in industrial applications. There are several methods for tuning a PID controller. This paper takes a qualitative look at six PID tuning methods, with comparison of accuracy and effectiveness. The coupled Tanks system is consider in this paper which is inherently nonlinear system. Simulation results demonstrate the performance of different PID tuning techniques and it is analysed and compared on the basis of time domain specifications.

**Keywords-** PID Controller, Tuning Methods, Coupled Tanks System, Time Domain Specification.

### I. INTRODUCTION

Controlling liquid levels and flow in tanks of a coupled tank system is considered as an important benchmark control platform due to their wide spread applications in the process control industries. The control objective in the coupled tank system is to maintain the liquid level at a desired level. The coupled tank system dynamics has interred dependent as it is a MIMO system. This dynamics is nonlinear due to valve characteristics and also exhibits non-minimum phase behaviour [1] [12]. The nonlinearity and the non-minimum phase behaviour make the associated control problem very challenging. A number of PID controllers have been extensively used in process control industries. These PID controllers exploit several tuning methods for obtaining appropriate control parameters [6]. The different tuning methods are explained in this paper and the performance of individual method is analysing on the basis of time domain specification.

The paper is organized as follows. Section II provides development of a linear simplified mathematical model of coupled tank system (CTS). Section III proposes the different PID tuning methods for controlling the water level of the tanks which is coupled with each other. In section IV gives the simulation results with discussion. Finally conclusions are made in section V.

### II. DYNAMIC MODEL OF COUPLED TANKS SYSTEM

For applying the different PID tuning method we must require attached at the base of each tank in order to measure the water level of the corresponding

tank.

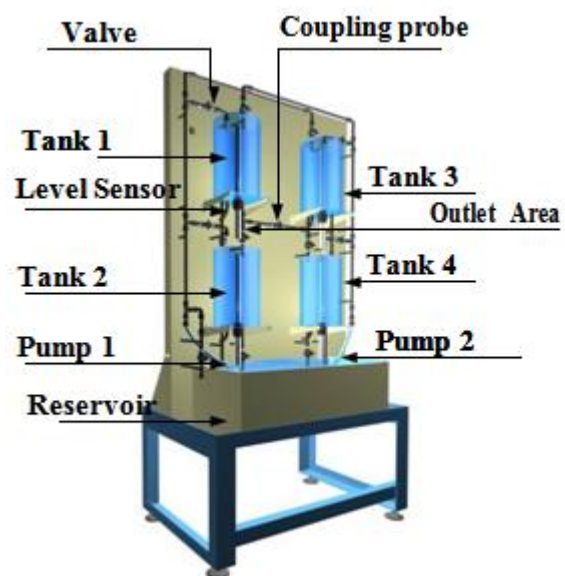


Fig.1 Schematic Diagram of a Coupled Tank System

There are two pumps installed in the reservoir in order to drive the water from bottom to top of the tank. A scale is attached in front of all individual tanks for the purpose of monitoring the water level. Figure.2 shows the experimental setup of coupled tanks system consists of four tanks but in this paper we have consider only two tanks which is tank 1 and tank 2.

The simplest nonlinear model of the coupled

tank system can be obtained by considering the mass balance equation, which is relating the water level  $h_1$ ,  $h_2$  and the applied voltage „ $u$ “ to the pump[12].

$$\frac{dh_1}{dt} = \frac{a}{A} \frac{1}{2gh_1(t)} u(t) \quad (1)$$

$$\frac{dh_2}{dt} = \frac{a_1}{A} \frac{1}{2gh_1(t)} - \frac{a_2}{A} \frac{1}{2gh_2(t)} \quad (2)$$

dynamic model of coupled tanks system. Figure.1 shows the schematic diagram of the coupled tank system. From figure.1 it is seen that it consists four translucent tanks and each tank is fitted with an outlet pipe in order to transmit the over flow water to reservoir. In this process, fifth tank is used for water storage purposes i.e. as a reservoir. A level sensor is also  
 Where,

$h_1$  = water level in tank 1  $h_2$  = water level in tank 2  
 $a_1$ = outlet area of tank 1  $a_2$ = outlet area of tank 2

$A$  = cross-sectional area of tanks  $g$  = gravitational constant  
 $\eta$  =constant relating to the control voltage with the water flow from the pump.

For applying different PID tuning method a linear model is considered for the controller design.



Fig.2. Experimental set up of Coupled the Tank System

Hence, the above nonlinear model can be converted into a linear model by using Taylor series expansion using two working points which are

$$h_{10} = \frac{1}{2g} \left( \frac{\eta u_0 A}{a_1} \right), \quad h_{20} = \frac{(a_1)^2}{(a_2)^2} h_{10} \quad (3)$$

After linearizing equation land 2 at operating point, we get

$$\Delta h_1(t) = \frac{(a_1)^2}{A} \frac{g}{\eta u_0} \Delta h_1(t) + \frac{1}{A} \Delta u(t) \quad (4)$$

$$\Delta h_2(t) = \frac{(a_1)^2}{A} \frac{g}{\eta u_0} \Delta h_1(t) - \frac{(a_1)^2}{A} \frac{g}{\eta u_0} \Delta h_2(t) \quad (5)$$

### III. PID TUNING TECHNIQUE

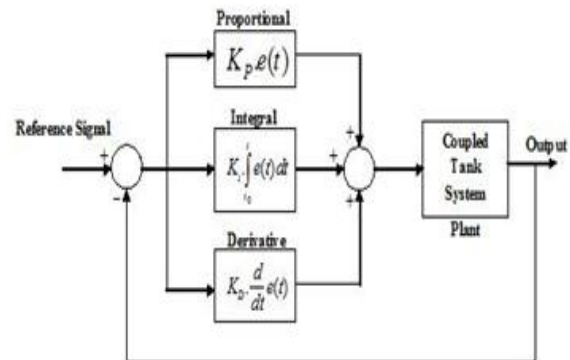


Fig.3. Basic control system configuration

In the process control, more than 95% of the control loops are of the Proportional-Integral-Derivative (PID) type. This method is more popular among all control methods. The determination of proportional ( $K_p$ ), derivative ( $K_d$ ) and integral ( $K_i$ ) constants are known as tuning of PID controller. The basic control structure of PID controller is shown in figure.3. The transfer function of PID controller is

$$G_c(s) = K_c \left( 1 + \frac{1}{T_i s} + T_d s \right) \quad (6)$$

Where,  $K_c$  is proportional gain,  $T_i$  is integral time constant and  $T_d$  is derivative time constant. For getting the value proportional gain, integral and derivative gain and its tuning there are different methods of tuning PID controller from which some of the tuning methods are explain are as follows.

#### A. Ziegler-Nichols Method

The Ziegler-Nichols design methods [9] are the most popular methods used in process control to determine the parameters of a PID controller. The

step response method is used for getting the parameter of Ziegler-Nichols method. The step response is based on an open-loop system of the process [10]. Hence requiring the process to be stable, the unit step response of the process is characterized by two parameters L and T. These are determined by drawing a tangent line at the inflexion point, where the slope of the step response has its maximum value. The intersections of the tangent and the coordinate axes give the process parameters as shown in Figure 4, and these are used in calculating the controller parameters. The parameters for PID controllers obtained from

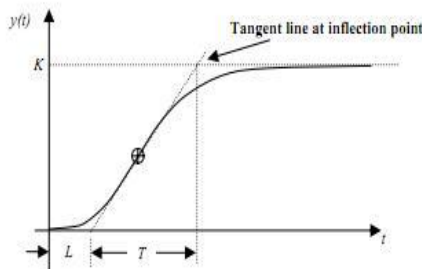


Fig.4. Step response of open loop system

The Ziegler-Nichols step response method are shown in Table 1.

Controller	$K_p$	$T_i$	$T_d$
PID Controller	$1.2T/L$	$2L$	$0.5L$

Table.1. PID controller parameters in the Ziegler-Nichols step Response method

### B. Fine-tuned PID controller

The individual effect of  $K_p$ ,  $K_i$  and  $K_d$  summarized in Table 2 can be very useful in fine tuning of PID controller in Table 2 can be very useful in fine tuning of PID controller [8]. Beginning with the values of  $K_p$ ,  $K_i$  and  $K_d$  obtained from Z-N step response method, unit step response for different combination of  $K_p$ ,  $K_i$  and  $K_d$  were observed

Parameter	Rise Time	Overshoot	Settling Time	Steady-state Error
Increasing $K_p$	Decrease	Increase	Small Change	Decrease
Increasing $K_i$	Decrease	Increase	Increase	Large Decrease
Increasing $K_d$	Small Change	Decrease	Decrease	Small Change

The Cohen-Coon method is a more complex version of the Ziegler-Nichols method [7]

.This method is more sensitive than the Ziegler-Nichols method. The controller parameters with this method are given by equation (7), (8) and (9).

$$K_p = \frac{1}{KL} \left( \frac{16\tau + 30}{12\tau} \right) \quad (7)$$

$$T_i = \frac{L(32 + 6\bar{T})}{13 + 8T} \quad (8)$$

$$T_d = 4 \frac{L}{11 + 2\frac{L}{T}} \quad (9)$$

### D. Chien, Hrones and Reswick method (CHR)

Chien- Hrones- Reswick (CHR) method is the modified version of the Ziegler-Nichols method [7].

This method was developed in 1952 by Chien-Hrones-Reswick know the which provides a better way to select a compensator for process control applications. In process industry, controller parameter are often tuned according to CHR Recommendation. The controller parameters from Chien, Hrones and Reswick set point response method are summarized in Table 3.

Overshoot	0%			20%		
	$K_p$	$T_i$	$T_d$	$K_p$	$T_i$	$T_d$
P	$0.3/a$	--	--	$0.7/a$	--	--
PI	$0.35/a$	$1.2T$	--	$0.6/a$	$T$	--
PID	$0.6/a$	$T$	$0.5L$	$0.95/a$	$1.4T$	$0.47L$

Table 3.Controller parameter

### E. ITAE Tuning Method

ITAE method is described in [4]. Integral of Time multiplied by Absolute Errors (ITAE) Criterion is given by [8]

$$J_{ITAE} = \int_0^{\infty} t |e(t)| dt \quad (10)$$

Where t is the time and e (t) is the error which is calculated as the difference between the set point and the output.

### F. AMIGO Tuning Method

In Astrom and Hagglund method is proposed that accomplishes this goal in a simple way. The Method which is known as AMIGO (Approximate M constrained integral gain Optimisation), which consist in applying a set of equation to calculate the parameter of the controller in a similar way to the procedure used in Ziegler- Nichols

method. The suggested AMIGO Tuning Rule for PID Controller is

$$K_p = \frac{1}{K} \left( \frac{T}{L} \right)^{0.2+0.45} \left( \frac{T}{L} \right)$$

$$T_i = 0.4L + 0.8T$$

$$T_d = L + 0.1T$$

$$T_i = \frac{0.5LT}{0.3L + T}$$

#### IV. RESULTS AND DISCUSSION

For applying above tuning method, we should know the value of T and L. the value of L and T is obtained from plotting the step response shown in Fig. 4, T & L are obtained as T= 3.5, L = 56.5. As per Table 1, the value of controller parameter for Ziegler-Nichols methods are  $K_P = 19.371$ ,  $K_i = 2.767$  and  $K_d = 17.4339$ , Desired height of tank 1 is 20cm. After fine tuning, PID controller parameters obtained are  $K_P = 25$ ,  $K_i = 1.55$  and  $K_d = 15$ .

From equation 7, 8 and 9 we can calculate the tuning parameter for Cohen coon method. The value for  $K_P = 3.706$ ,  $K_i = 0.4414$  and  $K_d = 4.662$  and for calculating the tuning parameter of CHR method we need the value of a. The value of a is the distance between origin to the distance tangent cuts negative vertical axis by extending tangent toward downward side. From step response the value of a = 0.37 and from table 3 we calculate the value  $K_P = 1.621$ ,  $K_i = 0.0287$  and  $K_d = 2.8378$  for 0% overshoot.

From equation 11, 12 and 13 we can calculate the tuning From equation 11, 12 and 13 we can calculate the tuning  $K_i = 71.80$  and  $K_d = 1.3137$  and for AMIGO tuning method the value of tuning parameter calculated by equation 14, 15 and 16. The value of  $K_P = 1.244$ ,  $K_i = 17.825$  and  $K_d = 21.73$

By using the tuning parameter of all the method of PID tuning we get the responses which are given in figure 6, 7 and 8. In figure 5 we plot the desired water level of tank1 and actual response of coupled tank system without PID tuning parameter. It is shown that it is not reaching the desired height of water level so we need PID tuning for achieving desired height of water level. In figure 6 we plot response of Ziegler-Nichols verses fine-tuned method. From figure 6 it is seen that fine-tuned method gives better response than the Ziegler-Nichols method. In figure 7 we plot response of Cohen Coon verses CHR method. In figure 8 we plot response of ITAE verses AMIGO method. The performance of each and every method is analysing with the table 4.

PID Tuning Method	Rise Time ( $t_r$ )	Maximum Overshoot ( $M_p$ )	Settling time ( $t_s$ )	Steady state error ( $e_{ss}$ )
Untuned PID	8.45	0	15	1.4
Z-N tuning Method	0.695	1.98	1.63	0
Fine Tune Method	0.583	0.0084	1.7	0
Cohen Coon Method	2.97	5.6	19	0
CHR Method	8.08	0	16	0
ITAE Tuning Method	0.385	74.4	147	0
AMIGO Tuning Method	1.03	14.5	72.3	0

Table.4. Comparative analysis of all the six methods of PID tuning

#### V. CONCLUSIONS

The paper describes design of PID controller for a coupled tanks system which is inherently nonlinear system. Total Six PID tuning techniques were implemented and their performances analysed. The system exhibits a smallest rise time but having largest peak overshoot and settling time with ITAE tuning technique. Fine-tuned exhibits smallest maximum overshoot and settling time with acceptable amount of rise time. Among the Six PID tuning techniques, the fine-tuned PID controller gives the best results for a coupled tanks system.

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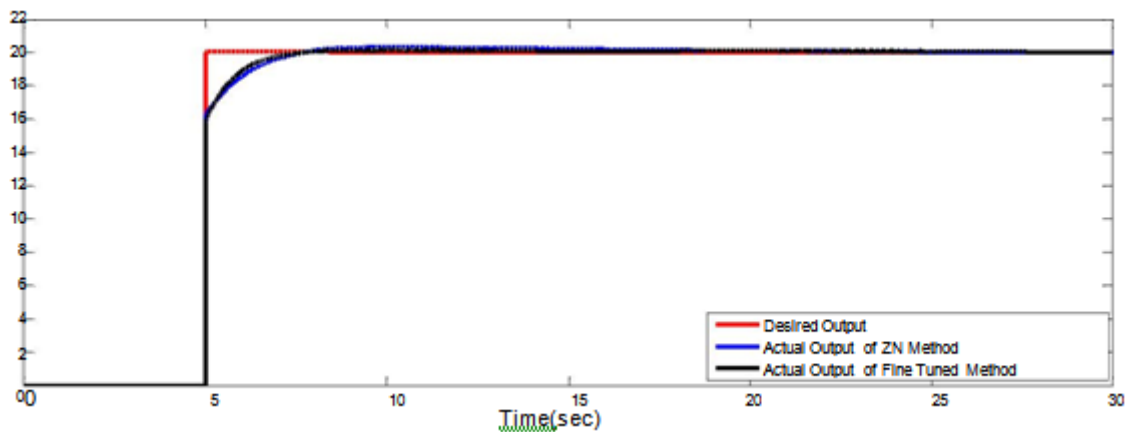
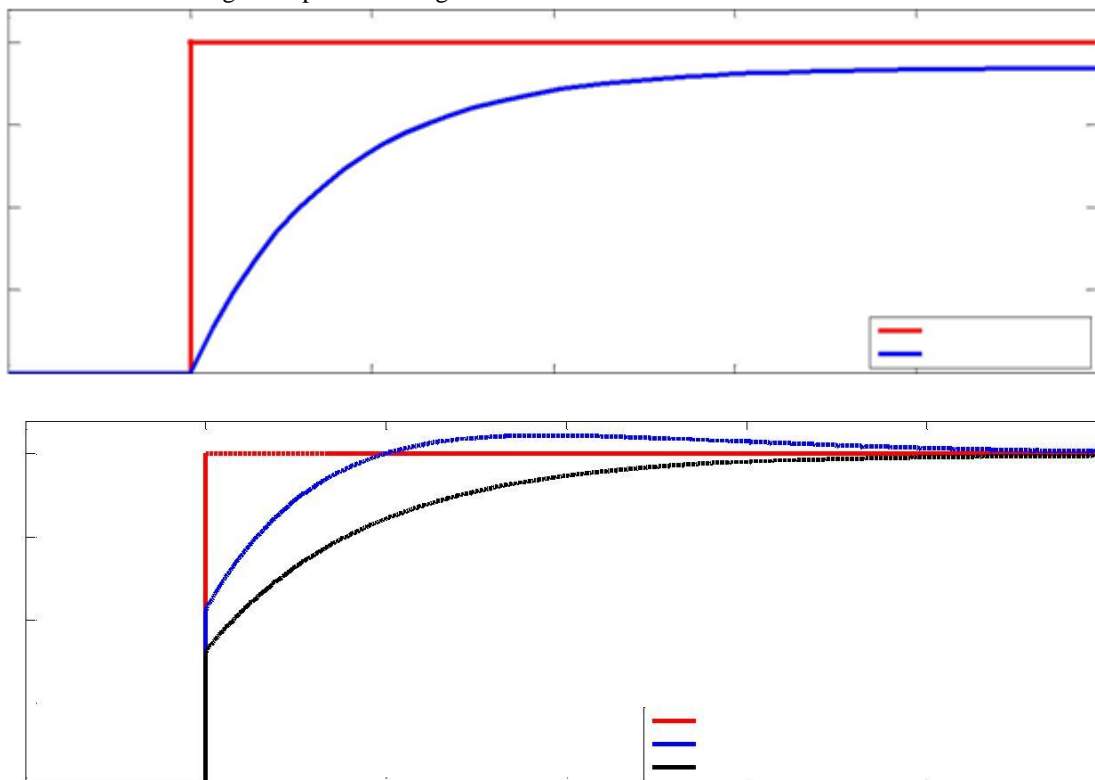


Fig.6 Response of Ziegler-Nichols and Fine Tuned PID controller



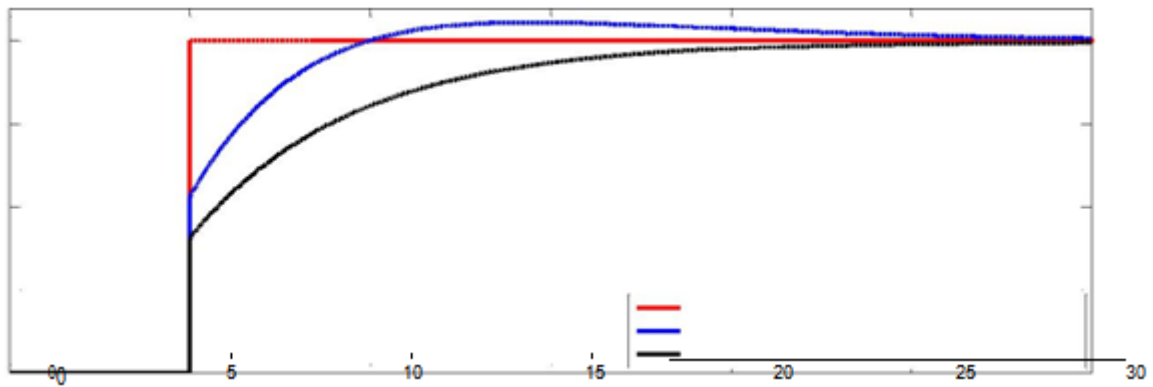


Fig.7. Response for Cohen Coon and CHR method

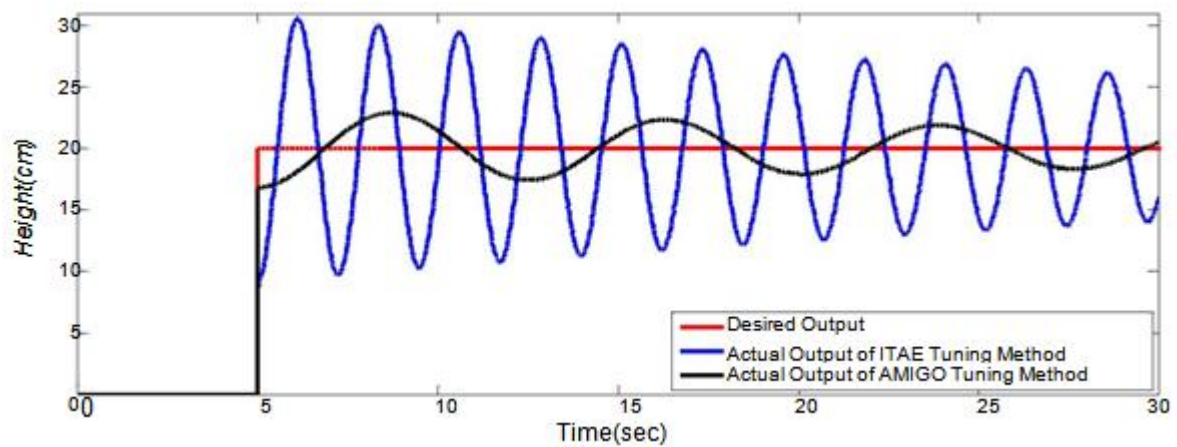


Fig.8. Response for ITAE Tuning and AMIGO Method