SELF-TUNING OF PID CONTROLLER FOR DC MOTOR USING MRAC TECHNIQUE

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Abstract- The main objective of this paper is to control the speed of a separately excited DC motor using a self tuning PID controller based on Model Reference Adaptive control (MRAC) approach. This auto tuning controller aims to Tune the parameters of the PID controller in response to changes in plant and disturbance real time by referring to the reference model that specifies the properties of the desired control system.

The simulation results show the effectiveness of the self tuning controller both in transient and steady state responses when compared to the fixed parameter PID controller.

Keywords - PID controller, MRAC Technique, Tuning and DC motor

1. Introduction

The speed of DC motors can be adjusted within wide boundaries so as to have easy controllability and high performance. The DC motors used in many applications such as steel rolling mills, electric trains, electric vehicles, electric cranes and robotic manipulators require speed controllers to perform their tasks. Generally DC motor systems have uncertain and non linear characteristics which decade performance of controllers. It is often used to handle any worst-case control environment such as parametric perturbations, external disturbances, stick-slip friction and etc.

Many control methods such as 2-DOF PID [1], Auto tuning PID [2], and CDM [3] have been applied to DC motor for solving their problems.

In the speed control applications 90% of the control loops are of the PID Type. The performance of a PI controller is mainly determined by the choice of its parameters. Tuning of PID controller implies choosing suitable values for its adjustable parameters in order to obtain the desired performance. Significant efforts have been developed to develop tuning methods for PID controllers [4], Automatic or self tuning is an on demand tuning procedure, as opposed to continuous adaptation.

This paper presents design methodology of self tuning PID controller using MRAC technique for solving the problem of DC motor. The proposed method can adjust the controller parameters in response to changes in plant and disturbance real time by referring to the reference model that specifies properties of the desired control system.

2. MATHEMATICAL MODELLING OF DC MOTOR

DC motors are widely used for industrial and domestic applications. Examples are as robotic and actuator for automation process, mechanical motion, and others. Accurate speed control of the motor is the basic requirement in such applications. The electric circuit of the DC motor is shown in Fig. 1. Objective is to control the speed of the motor by armature voltage control. The reference signal determines the desired speed. For simplicity, a constant value as a reference

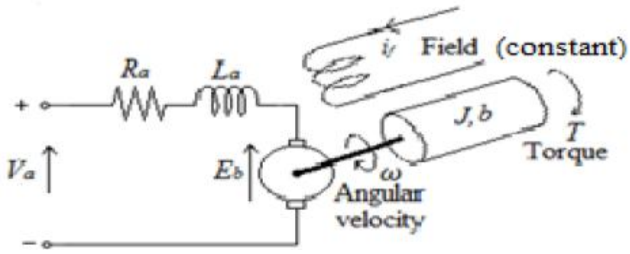

Fig: 1 The structure of DC Motor

The differential equations governing the dynamics of the system is given by

$$T(t) = J\frac{d\omega}{dt} + B\omega(t) \quad (1)$$

Where $\omega(t)$ represents angular velocity in radian/second, J represents the moment of inertia in kgm^2/s^2 and B is the coefficient of viscous friction which opposes the direction of motion in Nms. The Torque T generated by the armature current in Nm is given by

$$T(t) = K_t i_a(t) \quad (2)$$

Where i_a (t) is the armature current in Amp and K_t is Torque constant in Nm/Amp. This in turn is assumed to satisfy Kirchhoff's voltage

$$V_a(t) - E_b(t) = R_a i_a(t) + L_a \frac{di_a}{dt} \quad (3)$$

Where Ra and La are the armature inductance in H ad resistance in ohm respectively and E_b represents electromotive force in Volts given by

$$E_b(t) = K_b(t)\omega(t) \quad (4)$$

Where Kb is the back Emf constant in Vs/rad. The input terminal voltage Va is taken to be the controlling variable.

Assuming the Armature inductance of the DC motor is negligibly small. Then the transfer function of the DC motor becomes

$$\frac{\omega(s)}{V_a(s)} = \frac{K_T/(B+K_bK_t)}{S\left(\frac{J}{B+K_bK_t}\right)+1} \quad (5)$$

Table 1 Parameters of DC motor

Parameter	Symbol	Value
Armature Resistance	Ra	0.6 Ω
Armature Inductance	La	0.012H
Back Emf Constant	Kb	0.8 Vs/rad
Torque constant	Kt	0.8 Vs/rad
Moment of Inertia	J	0.0167 Kgm^2/S^2
Damping friction	B	0.0167 Kgm^2/S

Using the parameters given in Table 1, transfer function of the DC motor with angular velocity as controlled variable and input terminal voltage as manipulating variable is determined as given below

$$\frac{\omega(s)}{V_a(s)} = \frac{Km}{S(Tm)+1} \quad (6)$$

$$\frac{\omega(s)}{V_a(s)} = \frac{1.1218}{S(0.025430)+1} \quad (7)$$

2.1 PID CONTROLLER

Today most control systems in industries are based on PID technology; PID stands for proportional-integral-derivative controller. This algorithm is most popular feedback controller used within the process industries. It has been successfully used for over 50 years. It is a robust easily understood algorithm that can provide excellent control performance despite the varied dynamic characteristics of process plant. The PID algorithm consists of three basic modes, the Proportional mode, the Integral and the Derivative modes. The mathematical representation is

$$mv(s) = k_c e(s) + \frac{1}{T_i s} e(s) + T_D s e(s) \quad (8)$$

A PID controller uses a three-term transfer function to calculate a signal (u) from an error signal (e). The signal (u) is equal to the proportional gain (K_p) times the magnitude of the error plus the integral gain (K_i) times the integral of the error plus the derivative gain (Kd) times the derivative of the error. This signal (u) will be sent to the plant, and the new output (Y) will be obtained. Each element of the PID controller refers to a particular action taken on the error [6].

The performance of a PID controller is mainly determined by the choice of its parameters. It is commonly recognized that industrial controllers of the PID type often operate with poor tuning. Tuning a PID controller implies choosing suitable values for its adjustable parameters in order to obtain the desired control performance.

2.2 MODEL REFERENCE ADAPTIVE CONTROLLER (MRAC)

The model reference adaptive control (MRAC) was originally proposed to solve a problem in which the specifications are given in terms of a reference model that tells how the process output should responds to the command signal. The MRAC which was proposed by Whitaker in 1958 is an important adaptive controller [5], [13]. A block diagram of MRAC is illustrated in Fig. 1.

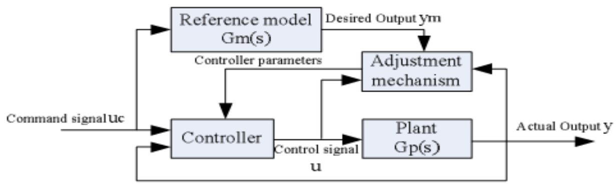

Fig 2: Block diagram of MRAC controller

The controller presented above can be thought of as consisting of two loops. The ordinary feedback loop which is called the inner loop composed of the process and controller. The parameters of the controller are adjusted by the adaptation loop on the basis of feedback from the difference between the process output y and the model output y_m. The adaptation loop which is also called the outer loop adjusted the parameter in such a way that makes the difference becomes small. An important problem associated with the MRAC system is to determine the adjustment mechanism so that a stable system that brings the error to zero is obtained. The following parameter adjustment mechanism, called the MIT rule, was originally used in MRAC:

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta} \quad (9)$$

In equation, e ($e = y - y_m$) denotes the model error. The components of $\frac{\partial e}{\partial \theta}$ are the sensitivity derivatives of the error with respect to the adjustable parameter vector θ. The parameter γ is known as the adaptation gain. The MIT rule is a gradient scheme that aims to minimize the squared model error e^2.

2.3 TUNING OF PID CONTROLLER USING MRAC TECHNIQUE

The goal of this section is to develop parameter adaptation laws for a PID control algorithm using the MIT rule.

Consider a system described by 1st order model of a DC motor mentioned above as **Gp(s) = b / (S+a)**

Where b=km/Tm and a=1/Tm

$$G_p(s) = \frac{y(s)}{u(s)} = \frac{b}{s+a} \qquad (10)$$

Where a and b are positive constants

Consider also the following PID control Law, where the Laplace Transform of the control signal is given by:

$$U(s) = K_p\{U_c(s)-Y(s)\} + \frac{Ki}{s}\{Uc(s) - Y(s)\} + K_d SY(s) \quad (11)$$

The main difference between equation 11 and the equation 8 is that the derivative action acts on the measured output, rather than on the control error. In the time domain, using the differential operator $p=\frac{d}{dt}$, Equation 8 can be written as follows:

$$U(t) = K_p\{U_c(t)-Y(t)\} + \frac{Ki}{p}\{Uc(t) - Y(s)\} + K_d PY(t) \quad (12)$$

The process transfer function can be written in the time domain as follows:

$$G_p(s) = \frac{y(t)}{u(t)} = \frac{b}{p+a} \qquad (13)$$

It is possible to show that applying control law 13 to system 13 gives the following closed loop transfer function:

$$y(t) = \frac{\dfrac{bK_p p}{(1+bK_d)} + \dfrac{bK_i}{(1+bK_d)}}{p^2 + \dfrac{(a+bK_p)}{(1+bK_d)}p + \dfrac{bK_i}{(1+bK_d)}} u_c(t) \qquad (14)$$

Consider also the following second order reference model:

$$G_m(s) = \frac{Y_m(s)}{U_c(s)} = \frac{\alpha s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \qquad (15)$$

Which has until steady state gain, natural frequency ω_n, and damping ratio ς, Notice the presence of a real zero at $-\frac{\omega n}{\alpha}$ in the reference model. This zero is introduces to match the structure of equation (14).

Recall that the model error is defined as the difference between the process output y and the reference model output y_m (e). It is then possible to derive adaptation rules for the controller parameters Kp, ki and Kd of control law 12 using the MIT rule (9) with parameters θ =Kp,Ki,Kd. Given that the process parameters a and b are not Known, the exact formulas that are derived using the MIT rule (9) cannot be used. Instead some approximations are required. An approximation made, which is valid when the parameters are close to their ideal values, is as follows:

$$p^2 + \frac{(a+bK_p)}{(1+bK_d)}p + \frac{bK_i}{(1+bK_d)} \cong p^2 + 2\zeta\omega_n p + \omega_n^2 \qquad (16)$$

Then, the approximate parameter adaptation laws are as follows:

$$K_i = \left[\frac{-\gamma}{p}\right] e \frac{1}{p^2 + 2\zeta\omega_n p + \omega_n^2}(u_c - y) \qquad (17)$$

$$K_p = \left[\frac{-\gamma}{p}\right] e \frac{p}{p^2 + 2\zeta\omega_n p + \omega_n^2}(u_c - y) \qquad (18)$$

$$K_d = \left[\frac{\gamma}{p}\right] e \frac{p^2}{p^2 + 2\zeta\omega_n p + \omega_n^2}(y) \qquad (19)$$

A diagrammatic Representation ofadaptation scheme is depicted in the figure as:

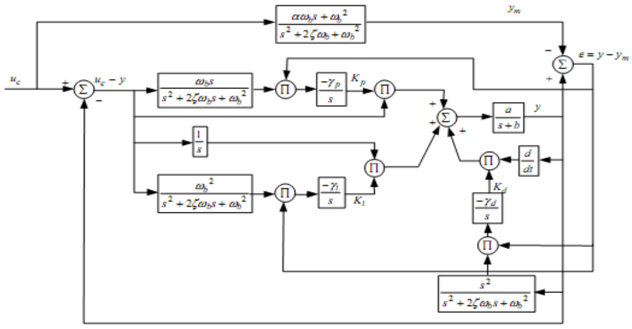

Fig 3: Block Diagram of a PID controller using MRAC

3. Simulation Results:

Automatic controller tuning using MRAC can be carried out in two steps. First, the adaptation is switched on and the process is excited adequately in order to enable the parameter adaptation to take place. When the parameters have adapted after a period of time, then the adaptation is switched off and the PID controller operates with fixed parameters.

Simulation Data:

a= 39.9

b=44.4

geta = 0.7

ω_n=1

γ =0.5

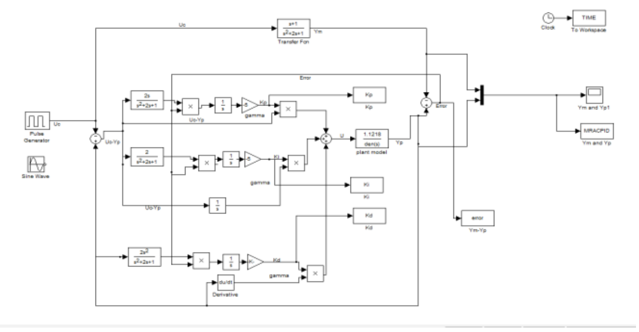

Fig 4: Simulink Block diagram of MRAC PID controller

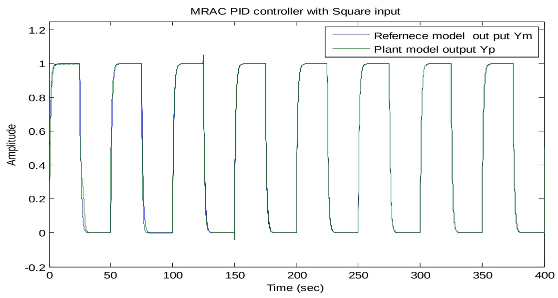

Fig 5: MRAC PID Controller output for Square Input

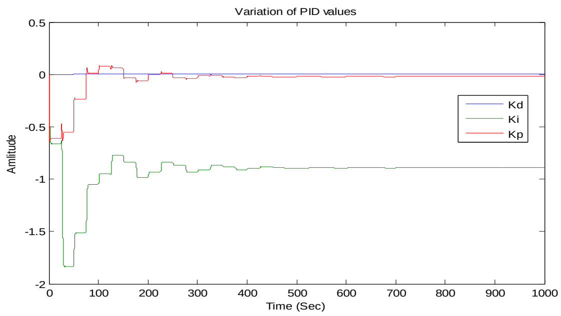

Fig 6: MRAC PID controller parameter variations of Square wave input

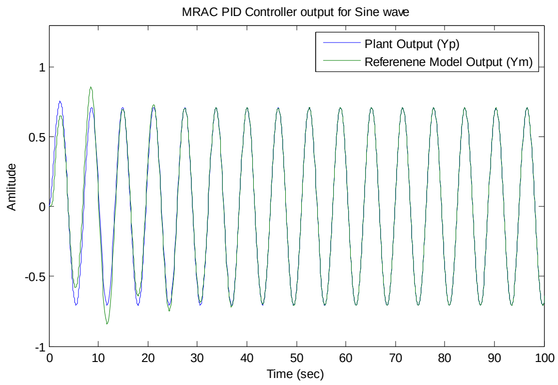

Fig 7: MRAC PID Controller output for Sine wave Input

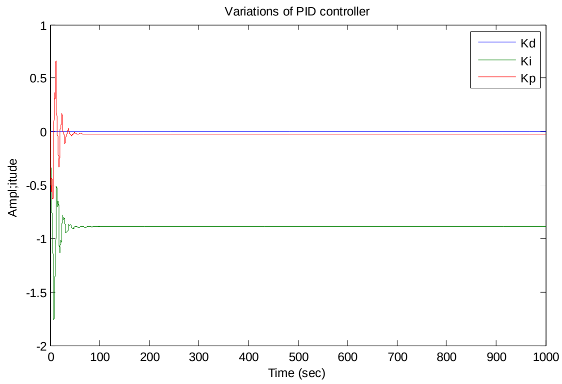

Fig 7: MRAC PID controller parameter variations of Sine wave input

4. Conclusion:

The automatic tuning of PID controllers has been investigated using Model Reference Adaptive control concepts and the MIT rule for the speed control of DC motor. Simple adaptation laws for the controller parameters have been presented assuming that the process under control can be approximated as first order transfer function. It will be extended to higher order systems with Non linear Dynamics also.

5. References

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