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Analysis of a mathematical reliability model for a system with fuzzy constant failure rate using trapezoidal fuzzy number

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ABSTRACT: Mathematical Reliability Models based on Markov Chain have extensive application in reliability of electrical and electronically equipments of any complex system. In this paper, a system with three parallel and identical elements with fuzzy constant failure rate is analyzed and the results are in consideration of increasable failure rates as form of trapezoidal fuzzynumbers.
Keywords: Fuzzy, Reliability, Mean Time To Failure, Constant Failure Rate, trapezoidal fuzzy number.

I. INTRODUCTION

Nowadays, Reliability models are considered one of the most important applications of Markov Chains, and most electronic systems come across these models [2]. Earlier different researches have discussed the reliability characteristics of system using different type of fuzzy number [4 ,6 and 7] and also discussed about trapezoidal fuzzy number [5 and 8]with their applications in reliability evaluation. This is an extensive system and for every electrical system a specific model is designed and implemented. In these models failure ratesare constant. Since these failure rates are driven from gathering data and usage of probability distribution functions, or the opinions of the experts on the matter, uncertainty is also, an obvious parameter.

Hence, in this paper, one particular system with three parallel elements is reviewed and the results are in consideration of fixed and equal failure rate. To demonstrate the uncertainty in calculation of failure rate, these parameters areestimated through trapezoidal fuzzy number using α-cut because α-cut method is a standard method for performing different arithmetic operations like addition, multiplication, division, subtraction.The remaining paper is structured as follows. In Section 2, basic definitions related to trapezoidal fuzzy number are given.Section 3 shows the notifications. Section 4 introduces the discussed sample. Calculations with respect to fuzzy failure ratesare described in Section 5. A numerical example is presented to illustrate how to calculate the mean time to failure (MTTF) using α-cut of trapezoidal fuzzy number in Section 6. Section 7 includes the conclusion of the study.

II. PRELIMINARIES

Definition 2.1:A trapezoidal fuzzy number $\tilde{\lambda}$ is shown by $\tilde{\lambda}=(a,b,c,d)$, with the membership function as follows:

$$\mu_{\tilde{\lambda}}(x)=\begin{cases}0, & x\le a\\ \dfrac{x-a}{b-a}, & a\le x\le b\\ 1, & b\le x\le c\\ \dfrac{d-x}{d-c}, & c\le x\le d\\ 0, & d\le x\end{cases}$$

It is apparent that a triangular fuzzy number is a special trapezoidal fuzzy number with $b=c$.

Definitin 2.2: α-cut for trapezoidal fuzzy number:
Let $\tilde{\lambda}=(a,b,c,d)$ be a trapezoidal fuzzy number. An α-cutfor $\tilde{\lambda}$, $\tilde{\lambda}(\alpha)$ is computed as:

$$\alpha=\frac{x-a}{b-a}\Rightarrow A=x=\left[a+(b-a)\alpha\right],$$

$$\alpha=\frac{d-x}{d-c}\Rightarrow B=x=\left[d-(d-c)\alpha\right],$$

Where $\tilde{\lambda}(\alpha)=[A,B]$ is the corresponding α-cut.The α-cut for triangular fuzzy number is obtained by using the above equation considering b=c.

III. NOTIFICATIONS

The notification that used in this article is as followed:

λ: Failure rate of elements.

Pu (t) : Probability of the system at the t moment to be in condition u .

R_P (t): Probability of the functionality of the system.

MTTF : Mean time to failure.

IV. INTRODUCTION TO THE DISCUSSED SAMPLE

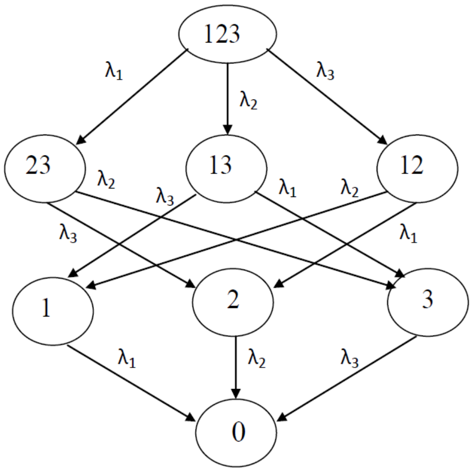
In this paper, a system that working with three parallel elements is considered. Assuming that the system will stop working when entire of elements have failed, we can consider the following eight conditions for the system:

State	Condition of first part	Condition of Second part	Condition of third part
123	working	working	Working
12	Working	working	failed
13	working	Failed	Working
23	Failed	Working	working
1	Working	failed	Failed
2	Failed	Working	failed
3	failed	Failed	Working
0	failed	failed	failed

Table 1: States of the system

And the flow diagram for this system will be as follow:

Fig. 1: States of the system



In respect to the above descriptions, the probability that the system work at themoment is calculated as followed :

$$R_p(t) = 1 - P_0(t) \quad ...(1)$$

We also know that:

$$P_{123}(t) + P_{12}(t) + ... + P_0(t) = 1 \quad ...(2)$$

In this system, the purpose is to find Pu(t). For the nods '123' through 'O' in fig. 1
we have:

$$P_{123}(t+\Delta t) = P_{123}(t) - (\lambda_1 + \lambda_2 + \lambda_3) \times \Delta t \times P_{123}(\quad 3)$$

$$P_{12}(t+\Delta t) = P_{12}(t) + \lambda_3 \times \Delta t \times P_{123}(t) - (\lambda_1 + \lambda_2) \quad (4)$$

$$P_{13}(t+\Delta t) = P_{13}(t) + \lambda_2 \times \Delta t \times P_{123}(t) - (\lambda_2 + \lambda_3) \quad (5)$$

$$P_{23}(t+\Delta t) = P_{23}(t) + \lambda_1 \times \Delta t \times P_{123}(t) - (\lambda_2 + \lambda_3 \quad (6)$$

$$P_1(t+\Delta t) = P_1(t) + \lambda_1 \times \Delta t \times P_{13}(t) + \lambda_2 \times \Delta t \times P_{12} \quad (7)$$

$$P_2(t+\Delta t) = P_2(t) + \lambda_1 \times \Delta t \times P_{12}(t) + \lambda_2 \times \Delta t \times P_2 \quad ...(8)$$

$$P_3(t+\Delta t) = P_3(t) + \lambda_1 \times \Delta t \times P_{13}(t) + \lambda_2 \times \Delta t \times P_2 \quad (9)$$

And by solving the equations (03) through (09)can calculate the values Pu(t), asfollowed:

$$P_{123}(t) = e^{-3\lambda t} \quad .(10)$$

$$P_{12}(t) = P_{13}(t) = P_{23}(t) = e^{-2\lambda t} - e^{-3\lambda t} \quad .(11)$$

$$P_1(t) = P_2(t) = P_3(t) = e^{-\lambda t} - 2e^{-2\lambda t} + e \quad ..(12)$$

And the system MTTF is also calculated as followed:

$$R_p(t) = P_{up}(t) = e^{-3\lambda t} + 3\left(e^{-2\lambda t} - e^{-3\lambda t}\right) \quad (13)$$

V. CALCULATIONS WITH RESPECT TO FUZZY FAILURE RATES

Most important consideration is that the values ofλ, are not fixed. Since they are driven from collected data or the opinions of the experts, uncertainty of the value isan undeniable fact.

Most of the times, these failure rates considered as a known value or have a knowndistributions function.Of course in this condition, failure rates are considered in the form of a trapezoidal fuzzy number as follows:

$$\bar{\lambda} = (a/b/c/d) \quad ...(14)$$

The α-cut of these failure rates can be calculated as follow:

$$\bar{\lambda}(\alpha) = \left[a + (b-a)\alpha,\ d - (d-c)\alpha\right]$$

$$\bar{\lambda}(\alpha) = [A, B] \quad ...(15)$$

where $A = \left[a + (b-a)\alpha\right]$ and $B = \left[d - \right.$

Now we can calculate the Pu(t), in Fuzzy condition by using extension principle [3].

Assume $\overline{P}_u[t,\alpha]=\left[P_{u(1)}(t,\alpha),P_{u(2)}(t,\alpha)\right]$,

therefore we will have [1]:

$$P_{123(1)}(t,\alpha)=e^{-3[d-(d-c)\alpha]} \qquad 16$$

$$P_{123(2)}(t,\alpha)=e^{-3[a+(b-a)\alpha]} \qquad 17$$

$$P_{12(1)}(t,\alpha)=P_{13(1)}(t,\alpha)=P_{23(1)}(t,\alpha) \qquad 18$$

$$P_{12(2)}(t,\alpha)=P_{13(2)}(t,\alpha)=P_{23(2)}(t,\alpha \qquad 19$$

$$P_{1(1)}(t,\alpha)=P_{2(1)}(t,\alpha)=P_{3(1)}(t,\alpha)= \qquad 20$$

$$P_{1(2)}(t,\alpha)=P_{2(2)}(t,\alpha)=P_{3(2)}(t,\alpha)= \qquad 21$$

$$R_{p(1)}(t,\alpha)=4e^{-3At}+3e^{-2At}+3e^{-At}- \qquad 22$$

$$R_{p(2)}(t,\alpha)=4e^{-3Bt}+3e^{-2Bt}+3e^{-Bt}- \qquad 23$$

$$R_p(t,\alpha)=\left[R_{p(1)}(t,\alpha),R_{p(2)}(t,\alpha)\right] \qquad 24$$

$$MTTF=\int_0^\infty R_p(t,\alpha)\,dt \qquad 25$$

Assume

$MTTF[\alpha]=\left[MTTF_1(\alpha),MTTF_2(\alpha)\right]$,

therefore we will have:

$$MTTF_1(\alpha)=\int_0^\infty R_{p(1)}(t,\alpha)\,dt$$

$$MTTF_1(\alpha)=\frac{4}{3A}+\frac{3}{2A}+\frac{3}{A}-\frac{3}{3B}-\frac{6}{3B}=\frac{35}{6A}-\frac{4}{B} \qquad \text{...(26)}$$

$$MTTF_2(\alpha)=\int_0^\infty R_{p(2)}(t,\alpha)\,dt$$

$$MTTF_2(\alpha)=\frac{35}{6B}-\frac{4}{A} \qquad \text{...(27)}$$

$$MTTF[\alpha]=\left[\frac{35}{6[d-(d-c)\alpha]}-\frac{4}{[a+(b-a)\alpha]},\frac{35}{6[a+(b-a)\alpha]}-\frac{4}{[d-(d-c)\alpha]}\right]$$

$$MTTF[\alpha]=\left[\frac{35}{6B}-\frac{4}{A},\frac{35}{6B}-\frac{4}{A}\right] \qquad \text{...(28)}$$

And to calculate the Fuzzy Number related to the mean time to failure of system(MTTF) we have:

$$\overline{MTTF}=[a/b/c/d] \qquad \text{...(29)}$$

$$MTTF[0]=\left[\frac{35}{6d}-\frac{4}{a},\frac{35}{6a}-\frac{4}{d}\right] \qquad \text{...(30)}$$

$$MTTF[1]=\left[\frac{35}{6c}-\frac{4}{b},\frac{35}{6b}-\frac{4}{d}\right] \qquad \text{...(31)}$$

$$\overline{MTTF}=\left[\frac{35}{6d}-\frac{4}{a},\frac{35}{6c}-\frac{4}{b},\frac{35}{6b}-\frac{4}{d},\frac{35}{6a}-\frac{4}{d}\right] \qquad \text{...(32)}$$

VI. NUMERICAL SAMPLE

Assume that based on an opinion of an expert the value of fuzzy trapezoidal number ofλ is as followed:

$$\overline{\lambda}=(0.05/0.07/0.09/0.11)$$

Then the fuzzy number of MTTF is:

$$\overline{MTTF}=[-26.9698,7.6718,44.9697,80.3$$

The lower limit of fuzzy number is -26.9698 and because of the bounds of the fuzzynumber is not negative, it changes to 0.

So $\overline{MTTF}=[0,7.6718,44.9697,80.3030]$

VII. CONCLUSION

Because of using fuzzy failure rates as form of trapezoidal fuzzy numbers, the system condition is more realistically thancrisp condition, and these are the advantages of this model.The system discussed in this paper is one of hundreds of the actual existing systemsthat have already been produced based on definite parameters.

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