Garmsiri Journal of Engineering Research and Application *www.ijera.com*
ISSN : 2248-9622 Vol. 9,Issue 9 (Series -II) September 2019, pp 20-26

RESEARCH ARTICLE OPEN ACCESS

Negative mass gravity theory(general relativity).

Garmsiri
member of the Astrophysical Society of Bushehr Province in Persian golf, dehdashti koi hosseinieh, bushehr state.

ABSTRACT: According to the theory of effective negative mass, published in the journal of physical Review letters, If force is applied to a negative mass That mass is accelerating That it accelerates unlike the direction Force applied. Therefore, based on this theory, the material can have both positive and negative masses. here we show that negative mass has negative gravity and create curvature in spacetime which basis the shape of the cosmos is after the big bang. The combination of two curves created by positive mass and negative mass basis of the orbits of the planets is around the stars…..

Date Of Submission: 30-08-2019 Date Of Acceptance: 16-09-2019

I. INTRODUCTION

According to the theory of effective negative mass, published in the journal of physical Review letters, If force is applied to a negative mass That mass is accelerating That it accelerates unlike the direction Force applied [1]. Therefore, based on this theory, the material can have both positive and negative masses. But the important thing is that the acceleration of the positive mass is opposed to the acceleration of the negative mass, Thus, according to the theory of general relativity, Albert Einstein, Given the two Hypothesis, he says:
1) Accelerated mass and gravitational mass are both one and the same.
2) The function of the accelerator and the gravity device are the same.
Therefore, we can state the theory of gravity of negative mass It is proved with these two hypotheses but for a negative mass, we must add two another hypothesis, in general, with the following three hypotheses so that:
1) we assume that the negative mass has a constant mass (that is, it has size, volume and radius).
2) The acceleration that the positive mass takes due to the force applied be equal to the acceleration that negative mass due to the same force applied.
3) Accelerated negative mass and gravitational negative mass are both one and the same.
4) The function of the accelerator and the gravity device are the same.

Therefore, expresses gravitational negative mass theory that the movement of a smaller negative mass leads to a larger negative mass, It is due to the curvature that generates a larger negative mass in spacetime And gravity is due to this curvature. (In this paper, for a smaller negative mass, the negative mass of the substance is smaller and for a larger negative mass, the negative mass of the substrnce is larger).

The Fields Equation Of Negative Mass

In order to prove that the negative mass in spacetime produces a curvature, we know energy impact tensor must be related to the spacetime curvature. We show the energy density flux with the symbol ρ and the hydrostatic pressure density with p symbol. Energy impact tensor
is as follows:
$T^{\mu\nu} = (p + \rho)u^{\mu}u^{\nu} + pg^{\mu\nu}$,
In the energy impact tensor, the energy density flux and the hydrostatic pressure are the result of a positive mass, and we call it a stress-energy tensor, or a material-energy, which is a positive mass. But in order to obtain a stress- negative Energy tensor, we must have
$T'^{\mu\nu} = (p_{nm} + \rho_{nm})u^{\mu}u^{\nu} + p_{nm} g^{\mu\nu}$,
where
$T'^{\mu\nu}$: negative energy impact Tensor,
ρ_{nm} : Energy density flux of negative mass,
p_{nm} : hydrostatic pressure,
u^{μ}: Four velocity vector,
$g^{\mu\nu}$: Metric,

Now this negative energy impact tensor should have a linear relationship with the Ricci tensor. If the Covariant derivative is zero for the negative energy impact tensor and the Ricci tensor is zero, then the problem is solved. But we know that Albert Einstein in general relativity proved that the Covariant derivative material-energy tensor is zero, but the Ricci tensor, with the help of bianchi identity, turned into the Ricci tensor and the Ricci scalar In this case, the covariant derivative for this tensor is zero. So, since Einstein proved that the new tensor has a zero covariant derivative, here is an negative energy impact tensor with the Einstein equalizer, and since Einstein's tensor has a covariant derivative of zero, here we should only find the covariant derivative for the negative energy impact tensor to gain. If $T'^{\mu\nu}$,is an negative energy impact tensor symbol, we have

$T'^{\mu\nu} = (p_{nm} + \rho_{nm})u^{\mu}u^{\nu} + p_{nm}g^{\mu\nu}$,

where

$u^{\mu}u^{\nu}g_{\mu\nu} = 1,$

$u^{\mu} = (c, 0,0,0),$

$$g^{\mu\nu} = \begin{pmatrix} -c^{-2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

We calculate the covariant derivative for this tensor is the metric and the Covariant derivative is zero for the metric tensor [2] (appendix A). so, $g_{ij,k} = 0$,

We can also the covariant derivative vector u^i is zero [3] (appendix B). so: $u^i = 0$, and

so: $T'^{\mu\nu} = 0$,

We want to say that, as if the mass too hight, it creates a curvature in the spacetime, the negative mass if too hight also creates a curvature in spacetime. Therefore, we proved that the derivative of negative Energ impact tensor is equal zero, and derivative of Ricci tensor that Einstein it to turned to Ricci tensor and scalar Ricci is zero . so, There should be a linear relationship between the tow. Therefore should a side be geometrical involving the spacetime metric, Ricci tensor and Ricci scalar, and the another side involves the negative Energy-momentum tensor of the matter with negative mass fields multiplier k , Therefore we use Einstein-Hilbert action, the difference is that we here use Eistein-Hilbert action including matter with negative mass.

We know Einstein-Hilbert action including matter is [4],

$I = I_g + I_m$,
(1)

Now if a matter be with negative mass equation (1) changes to

$I = I_g + I_{(m*)}$,

Therefor in continuum mechanics, the action is the (flat) spacetime integral of a lagrangian negative density . so,

$$I = I_g + I_{(m*)} = \frac{1}{2k}\int\sqrt{-g}\,d^4xR + \int\sqrt{-g}\,d^4x\mathcal{L}_{(m*)} ,$$

Where

$\mathcal{L}_{(m*)}$, is the lagrangian negative density for matter with negative mass,

We have proved that the negative impact energy tensor derivative is zero and We know that the Einstein tensor derivative that obtained from the Ricci and Ricci scalar tensors is zero so, We use Einstein's experience and postulate the equation for matter with negative mass field (appendix C)

As

$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -kT'^{\mu\nu}$,
(2)

where

$T'^{\mu\nu}$:negative impact energy tensor.

In the Einstein equation, the constant k is to be determined. Earlier, when we obtained the Newoton's equation in the weak and static field approximation, we had found that [5],

$g_{44} = 1 + \frac{2\phi}{c^2} = 1 - \frac{2GM}{c^2r}$,
(3)

The Newotonian theory in the presence of matter gives poisson equation [5],

$\vec{\nabla}^2\phi = 4\pi G\rho$,
(4)

We know that in Newton's law of gravity the force of gravity between two positive masses is with the product of two masses direct proportion and with square of distance two masses reverse ratio [6]. This law also applies for a negative mass, but to make proportions equal to that we must multiply the negative mass gravitational constant that's it showing with symbol G_{nm},(appendix D) so, we have

Fn1 = Fn2 = G_{nm} (m*1) (m*2) /(r²).

So, based on the first hypothesis, we can define Newoton's equation for negative mass, so that we had found that

$g_{44} = 1 + \frac{2\phi'}{c^2} = 1 - \frac{2G_{nm}(M*)}{c^2r}$,
(5)

where

G_{nm} :gravity constant of negative mass.

So, the Newotonian theory in the presence of matter with negative mass gives poisson equation

$\vec{\nabla}'^2\phi' = 4\pi G_{nm}(\rho_{nm})$,
(6)

where

G_{nm} :gravity constant of negative mass.

Now we define equation (6) gravitational potential of negative mass and show it with $\vec{\nabla}'^2\phi'$,We want to express the gravity theory with the negative mass, so that the negative mass produces a curvature in spacetime. Multiplying the equation (2) by$g_{\mu\nu}$ and denoting $T'^{\mu\nu}$ $g_{\mu\nu}$,by T', we find R = k T, Then, this equation can be as

$R_{\mu\nu} = -k(T'_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T')$,
(7)

Compare this equation with (6). We know that 4-velocity $u^{\mu} = \frac{dx^{\mu}}{ds}$ .u^{μ} (x), and also

$g_{\mu\nu}u^{\mu}$ u^{ν} =1 ,Therefore, using the result that the metric $g_{\mu\nu}$,is covariantly constant, we have

$\left(g_{\mu\nu}u^{\mu}u^{\nu}\right)_{;\lambda} = g_{\mu\nu}\left(u^{\mu}u^{\nu}_{;\lambda} + u^{\mu}_{;\lambda}u^{\nu}\right) = 2g_{\mu\nu}u^{\mu}u^{\nu}_{;\lambda} = 0.$.

Leading to

$u_v u^v_{;\lambda} = 0.$
(8)
Introduce a scalar$\rho_{nm}(x)$, so that $\rho_{nm} u^\mu$, gives that flow of matter with negative mass and negative density, as $\rho_{nm} u^i\sqrt{-g}$, and $\rho_{nm} u^4\sqrt{-g}$, respectively. Then the conservation of matter with negative mass is gowerned by
$\partial_\mu\left(\rho_{nm}(\mathcal{X})u^\mu\sqrt{-g}\right) = 0.$
(9)
Expanding
$\partial_\mu\left(\rho_{nm}(\mathcal{X})u^\mu\sqrt{-g}\right) = \partial_\mu(\rho_{nm}(\mathcal{X})u^\mu)\sqrt{-g} + (\rho_{nm}(\mathcal{X}))u^\mu\left(\partial_\mu\sqrt{-g}\right) = 0$, .
Using
$\Gamma^\sigma_{\mu\sigma} = \frac{1}{\sqrt{-g}}(\partial_\mu\sqrt{-g}),$.
The above equation becomes
$\sqrt{-g}\{\partial_\mu(\rho_{nm}(\mathcal{X})u^\mu) + \Gamma^\sigma_{\mu\sigma}(\rho_{nm}(\mathcal{X})u^\mu)\} = \sqrt{-g}\{D_\mu(\rho_{nm}(\mathcal{X})u^\mu)\} = 0$, .
That is
$(\rho_{nm}(\mathcal{X})u^\mu)_{:\mu} = 0.$
(10)
The matter with negative mass has an negative energy density $\rho_{nm}(x)u^4u^4\sqrt{-g}$,and momentum flux $\rho_{nm}(x)u^iu^i\sqrt{-g}$,and if we take
$T'^{\mu v} = \rho_{nm}(\mathcal{X})u^\mu u^v$,
(11)
Then$\sqrt{-g}T'^{\mu v}$, gives the negative energy density and the momentum flux. Thus this $T'^{\mu v}$, is the negative energy-momentum tensor for the matter with negative mass. We use this $T'^{\mu v}$, ,In equation (2). We have to verify $T'^{\mu v} = 0$
$T'^{\mu v}_{;v} = (\rho_{nm}(\mathcal{X})u^\mu u^v)_{;v} = u^\mu(\rho_{nm}(\mathcal{X})u^v)_{;v} + (\rho_{nm}(\mathcal{X}))u^v u^\mu_{;v}$, (12)
In above, the first term on the right side vanishes by (10) and the second term vanishes if the elements of the matter with negative mass move along the geodesics.(flow $u^\mu u^p_{;\mu} = 0$, and $u^\mu = \frac{dx^\mu}{ds}$.) now we use equation (11) we have $T' = g_{\mu v}\{\rho_{nm}(\mathcal{X})u^\mu u^v\} = \rho_{nm}(\mathcal{X})$ since$g_{\mu v}u^\mu u^v = 1$, Then (7) becomes
$R_{\mu v} = -k(\rho_{nm}(\mathcal{X}))\left\{u_\mu u_v - \frac{1}{2}g_{\mu v}\right\}$,
(13)
As we want to compare (7) with (6) with later in the Newtonian case, we consider weak gravitational field by expanding $g_{\mu v}$, as
$g_{\mu v} \simeq \eta_{\mu v} + h_{\mu v}$; $g^{\mu v} \simeq \eta^{\mu v} - h^{\mu v}$,
(14)
Then,
$\Gamma^\mu_{v\rho} = g^{\mu\lambda}\Gamma_{v\rho,\lambda} = \frac{1}{2}g^{\mu\lambda}\{\partial_v g_{\rho\lambda} + \partial_\rho g_{v\lambda} - \partial_\lambda g_{v\rho}\} \simeq \frac{1}{2}\eta^{\mu\lambda}\{\partial_v h_{\rho\lambda} + \partial_\rho h_{v\lambda} - \partial_\lambda h_{v\rho}\}.$ (15)
In the expression for
$R_{\mu v} = (\partial_\rho\Gamma^\rho_{\mu v} - \partial_v\Gamma^\rho_{\mu\rho} + \Gamma^\rho_{\sigma\rho}\Gamma^\sigma_{\mu v} - \Gamma^\rho_{\sigma v}\Gamma^\sigma_{\rho\mu})$,
The terms involving the derivatives of Γs are retained. They are, using (13):
$\partial_\rho\Gamma^\rho_{\mu v} \simeq \frac{1}{2}\eta^{\rho\lambda}(\partial_\rho\partial_\mu h_{v\lambda} + \partial_\rho\partial_v h_{\mu\lambda} - \partial_\rho\partial_\lambda h_{\mu v})$,
(16)
And
$\partial_v\Gamma^\rho_{\mu\rho} \simeq \frac{1}{2}\left(\partial_v\partial_\mu h\right), \quad h = \eta^{\rho\lambda}h_{\rho\lambda}$,
(17)
So,
$R_{\mu v} \simeq \frac{1}{2}\eta^{\rho\lambda}\left(\partial_\rho\partial_\mu h_{v\lambda} + \partial_\rho\partial_v h_{\mu\lambda} - \partial_\rho\partial_\lambda h_{\mu v}\right) - \frac{1}{2}\left(\partial_v\partial_\mu h\right),$
(18)
We have considered weak field approximation. For static approximation, (16) gives
$R_{44} \simeq \frac{1}{2}\eta^{\rho\lambda}\partial_\rho\partial_\lambda h_{44} = \frac{1}{2}\underset{\nabla}{\rightarrow}'^2 h_{44}.$
(19)
Now we use consider (13) in the above approximation. In this case $u^\mu = (c,0,0,0)$, and $g_{44} \simeq c^2$,Then, the (44) component of the right side of (13) is
$-k(\rho_{nm}(\mathcal{X}))\left\{c^2 - \frac{1}{2}g_{44}\right\} = -\frac{1}{2}k(\rho_{nm}(\mathcal{X}))c^2,$
Then using (19), we have
$\underset{\nabla}{\rightarrow}'^2 h_{44 = -k(\rho_{nm}(x))c^2},$
,
From (5), $h_{44} = \frac{2\Phi'}{c^2}$
And so $\underset{\nabla}{\rightarrow}'^2\Phi' = -\left(\frac{kc^4}{2}\right)(\rho_{nm}(x))$,
When compared with (6), k is determined as
$\kappa = -\frac{8\pi G_{nm}}{C^4}$,
And equation (2) becomes
$R^{\mu v} - \frac{1}{2}g^{\mu v}R = \frac{8\pi G_{nm}}{C^4}T'^{\mu v}.$
(20)

The left side (20) is geometrical involving the spacetime metric, Ricci tensor and Ricci scalar. Once the spacetime metric is given, the left side can be calculated and in this sense it is geometrical, property of spacetime. The right side involves the negative energy – momentum tensor of the matter with negative mass fields (nongravitational).

CNCLUDE

The substance has two positive and negative masses. Given Einstein's relationship, (Assuming space time if flat) If a positive mass increases, it will curvature in spacetime (figure 1). We also proved that the negative mass also increases, It causes curvature in spacetime (fig 2). Assuming space time if flat, The curvature of spacetime in Einstein's relationship is due positive mass, Different from the curvature of spacetime by negative mass according to equation (20) Because the acceleration that takes two positive and negative masses of a substance is in the opposite direction. Consequently, the curvature of spacetime by matter, consequence is curvature of spacetime a positive mass and a negative mass. This means that the combination of these two curves together, Causes cut off the space-time lines of positive mass and negative mass. The result of this intersection is a groove that Surrounded by a material that has curvature in spacetime. Here is an example of Earth and the sun. We know that the mass of the sun is greater than the mass of the earth Therefore, the curvature of the space-time created by the sun is more than Earth When the Earth is on the slope of the curvature of the Sun's spacetime The motion of the earth toward the sun is the same as the positive mass motion in the gravitational field of the sun At the same time, the negative mass of the Earth moves to the gravitational field by the negative mass of the sun, Two positive and negative masses of the earth go up to the ridge. if the mass be large, the groove getting closer.(so, the mass that going up on the slope, it causes the groove to move toward itself that this displacement depends on the size of the mass.) But in the groove, Positive mass encounters an obstacle Which is the curvature of space-time generated by a negative mass And the negative mass also hits the barrier That is, the curvature of spacetime is a positive mass Here, two positive and negative masses can only have rotational motion Which is the transition of the earth around the sun But the movement of the positive and negative masses is in the opposite direction. Finally, the groove is the planetary orbit that orbiting around the stare. (fig 3).

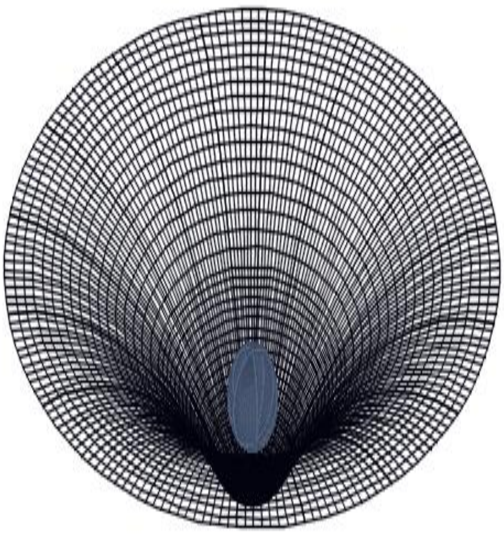

FIG 1. space-time curvature is created with positive mass and this slop same gravity of center mass.(garmsiri).

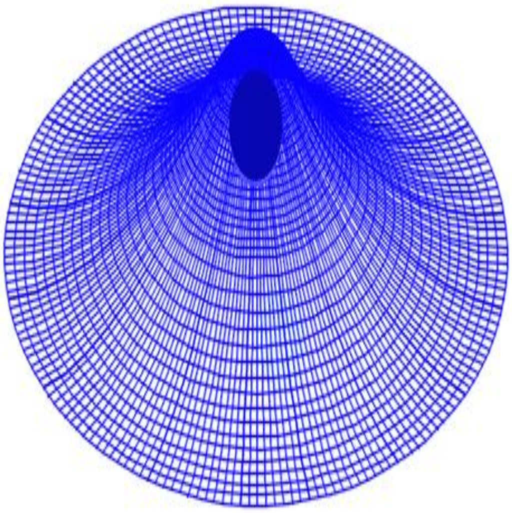

FIG 2. space-time curvature is created with negative mass and this slop same gravity of center negative mass.(garmsiri).

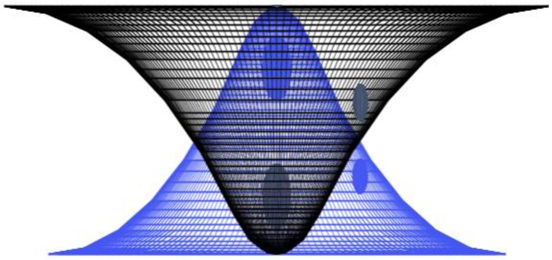

FIG 3. spacetime that is black (color online), created by positive mass and located a larger mass at the center, and a smaller mass at the slope of curvature. spacetime that is blue (color online), created by negative mass and located a larger negative mass at the center, and a smaller negative mass at the slope of curvature. A combination of two spacetimes, causes two grooves around negative and positive mass becomes central and small masses fall into these grooves, so that the small negative mass in the lower groove rotates around the central negative mass and the small positive mass in the upper groove rotates around the central pocitive mass. The direction of rotation of the two smaller masses is opposite to each other.(garmsiri).

ACKNOWLEDGMENTS

Here are not financial support.

APENDICES

Appendix A:

We calculate the covariant derivative for this tensor is the metric and the Covariant derivative is zero for the metric tensor. so:

$$\mathbf{g_{ij,k} = g_{ij,k} - g_{pj}\tau^{p}_{ik} - g_{ip}\tau^{p}_{jk}} \quad , \quad \text{(A1)}$$

$$\mathbf{g_{pj}\tau^{p}_{ik} = [ik, j], g_{ip}\tau^{p}_{jk} = [jk, i]} \quad , \quad \text{(A2)}$$

$$\mathbf{g_{ij,k} = [ik, j] + [jk, i]} \quad , \quad \text{(A3)}$$

$$\mathbf{[ik, j] + [jk, i] - [ik, j] - [jk, i] = 0} \quad , \quad \text{(A4)}$$

Appendix B:

We know that 4-velocity $u^{\mu} = \frac{dx^{\mu}}{ds}$ .u^{μ} (x), and also

$g_{\mu\nu}u^{\mu}\ u^{\nu}$ =1 , Therefore, using the result that the metric $g_{\mu\nu}$,is covariantly constant, we have

$$\mathbf{\left(g_{\mu\nu}u^{\mu}u^{\nu}\right)_{;\lambda} = g_{\mu\nu}\left(u^{\mu}u^{\nu}_{;\lambda} + u^{\mu}_{;\lambda}u^{\nu}\right) = 2g_{\mu\nu}u^{\mu}u^{\nu}_{;\lambda} = 0,} \quad \text{(B1)}$$

Leading to

$$\mathbf{u_{\nu}u^{\nu}_{;\lambda} = 0.} \quad \text{(B2)}$$

Appendix C:

We know Einstein-Hilbert action including matter is

$$\mathbf{I = I_{g} + I_{m}} \quad , \quad \text{(C1)}$$

Now if a matter be with negative mass equation (C1) changes to

$$\mathbf{I = I_{g} + I_{(m*)}}$$

Therefor in continuum mechanics, the action is the (flat) spacetime integral of a lagrangian negative density . so,

$$\mathbf{I = I_{g} + I_{(m*)}} \ ,$$
$$\mathbf{= \frac{1}{2k}\int\sqrt{-g}d^{4}xR + \int\sqrt{-g}d^{4}x\mathcal{L}_{(m*)}} \quad , \quad \text{(C2)}$$

Where

$\mathcal{L}_{(m*)}$,is the lagrangian negative density for matter with negative mass,

We vary the above action with respect to the metric$g_{\mu\nu}(x)$,and write

$$\mathbf{\delta I = \frac{1}{2k}\int d^{4}x\delta\left(\sqrt{-g}R\right) + \delta\int\sqrt{-g}d^{4}x\mathcal{L}_{(m*)},}$$
$$\mathbf{\equiv \int d^{4}x\delta\mathcal{L}_{g} + \delta\int\sqrt{-g}d^{4}x\mathcal{L}_{(m*)}} \quad , \quad \text{(C3)}$$

Whit

$$\mathbf{\delta\mathcal{L}_{g} = \frac{1}{2k}\left\{\left(\delta\sqrt{-g}\right)R + \sqrt{-g}\delta R\right\},} \quad \text{(C4)}$$

Since $R = g^{\mu\nu}R_{\mu\nu}$, we have

$$\mathbf{\delta\mathcal{L}_{g} = \frac{1}{2k}\left\{\left(\delta\sqrt{-g}\right)R + \sqrt{-g}(\delta g^{\mu\nu})R_{\mu\nu} + -g g\mu\nu\delta R\mu\nu,\right.} \quad \text{(C5)}$$

We have $\delta g = gg^{\mu\nu}\delta g_{\mu\nu}$,and so

$$\mathbf{\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g}g^{\mu\nu}\delta g_{\pi\nu},} \quad \text{(C6)}$$

And know that form $g^{\mu\nu}g_{\nu\rho} = \delta^{\mu}_{\rho,}(\delta g^{\mu\nu})g_{\nu\rho} = -g^{\mu\nu}\left(\delta g_{\nu\rho}\right)$, from which we have

$$\mathbf{\delta g^{\mu\nu} = -g^{\mu\rho}g^{\sigma\nu}\left(\delta g_{\rho\sigma}\right),} \quad \text{(C7)}$$

Substituting (C6) and (C7) in (C5), we find

$$\mathbf{\delta\mathcal{L}_{g} = \frac{1}{2k}\left\{\frac{1}{2}\sqrt{-g}Rg^{\mu\nu}\left(\delta g_{\mu\nu}\right) - \sqrt{-g}g^{\mu\rho}g^{\nu\sigma}R_{\mu\nu}\left(\delta g_{\rho\sigma}\right) + \right. -g g\mu\nu\delta R\mu\nu,} \quad \text{(C8)}$$

Now We need to examine$\delta R_{\mu\nu}$, We know that

$$\mathbf{R_{\mu\nu} = \partial_{\rho}\Gamma^{\rho}_{\mu\nu} - \partial_{\nu}\Gamma^{\rho}_{\mu\rho} + \Gamma^{\rho}_{\sigma\rho}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\rho}_{\sigma\nu}\Gamma^{\sigma}_{\rho\mu}} \quad , \quad \text{(C9)}$$

And so

$$\mathbf{\delta R_{\mu\nu} = \partial_{\rho}\left(\delta\Gamma^{\rho}_{\mu\nu}\right) - \partial_{\nu}\left(\delta\Gamma^{\rho}_{\mu\rho}\right) + \left(\delta\Gamma^{\rho}_{\sigma\rho}\right)\Gamma^{\sigma}_{\mu\nu} + \Gamma^{\rho}_{\sigma\rho}\left(\delta\Gamma^{\sigma}_{\mu\nu}\right) - \left(\delta\Gamma^{\sigma}_{\sigma\nu}\right)\Gamma^{\sigma}_{\rho\mu} - \Gamma^{\rho}_{\sigma\nu}\left(\delta\Gamma^{\sigma}_{\rho\mu}\right).} \quad \text{(C10)}$$

We help from (C7), we have

$$\mathbf{\delta\Gamma^{\rho}_{\mu\nu} = \delta\left(g^{\rho\lambda}\Gamma_{\mu\nu,\lambda}\right),}$$
$$\mathbf{= \left(\delta g^{\rho\lambda}\right)\Gamma_{\mu\nu,\lambda} + g^{\rho\lambda}\left(\delta\Gamma_{\mu\nu,\lambda}\right),}$$
$$\mathbf{= \left(\delta g^{\rho\lambda}\right)g_{\lambda\sigma}\Gamma^{\sigma}_{\mu\nu} + g^{\rho\lambda}\left(\delta\Gamma_{\mu\nu,\lambda}\right),}$$
$$\mathbf{= -g^{\rho\alpha}g^{\beta\lambda}\left(\delta g_{\alpha\beta}\right)g_{\lambda\sigma}\Gamma^{\sigma}_{\mu\nu} + g^{\rho\lambda}\left(\delta\Gamma_{\mu\nu,\lambda}\right),}$$
$$\mathbf{= -g^{\rho\alpha}(\delta g_{\alpha\sigma})\Gamma^{\sigma}_{\mu\nu} + \frac{1}{2}g^{\rho\lambda}\left\{\partial_{\mu}(\delta g_{\nu\lambda}) + \right. \partial\nu\delta g\mu\lambda - \partial\lambda\delta g\mu\nu.} \quad \text{(C11)}$$

Consider

$$\mathbf{\frac{1}{2}g^{\rho\lambda}\left\{(\delta g_{\nu\lambda})_{;\mu} + \left(\delta g_{\mu\lambda}\right)_{;\nu} - \left(\delta g_{\mu\nu}\right)_{;\lambda}\right\},} \quad \text{(C12)}$$

As$g_{\mu\nu}$ is a covariant tensor of (rank 2),$\delta g_{\mu\nu}$ is also a covariant tensor of rank 2. Then, using the covariant derivative of rank two covariant tensors. So the above expression is

$$\mathbf{\frac{1}{2}g^{\rho\lambda}\left\{\partial_{\mu}(\delta g_{\nu\lambda}) - \Gamma^{p}_{\nu\mu}\left(\delta g_{p\lambda}\right) - \Gamma^{p}_{\lambda\mu}\left(\delta g_{p\nu}\right) + \right.}$$

∂νδgμλ−ΓμνpδgpλΓλνpδgpμ−∂λδgμν+Γμλpδgpν+Γνλpδgpμ=−gρλΓμνpδgpλ+12gρλ∂μδgνλ+∂νδgμλ−∂λδgμν.

Substituting the expression in the curly bracket in the right side of the above in (C11), We see

$$\mathbf{\delta\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\lambda}\left\{(\delta g_{\nu\lambda})_{;\mu} + \left(\delta g_{\mu\lambda}\right)_{;\nu} - \left(\delta g_{\mu\nu}\right)_{;\lambda}\right\},} \quad \text{(C13)}$$

Expression $\delta\Gamma^{\rho}_{\mu\nu}$, in terms of the covariant derivatives. Thus $\delta\Gamma^{\rho}_{\mu\nu}$, is a mixed tensor of rank 3. It is to be noted that thought$\Gamma^{\rho}_{\mu\nu}$, is not a mixed tensor of rank 3,$\delta\Gamma^{\rho}_{\mu\nu}$, is. This result allows us to counsider the covariant derivative of $\delta\Gamma^{\rho}_{\mu\nu}$, as the covariant derivative of a mixed rank 3. Then it easy to see that

$(\boldsymbol{\delta\Gamma^{\sigma}_{\mu\nu}})_{;\sigma} - (\boldsymbol{\delta\Gamma^{\sigma}_{\mu\sigma}})_{;\nu} = \boldsymbol{\delta R_{\mu\nu}}$.

(C14) We have from term of 3 in the right side of equation (C8)

$\sqrt{-g}g^{\mu\nu}(\delta R_{\mu\nu}) = \sqrt{-g}g^{\mu\nu}\{((\delta\Gamma^{\sigma}_{\mu\nu})_{;\sigma} -$ **$(\delta\Gamma\mu\sigma\sigma);\nu=-g(g\mu\nu\delta\Gamma\mu\nu\sigma);\sigma-(g\mu\nu\delta\Gamma\mu\sigma\sigma);\nu$,**

(C15)

$$\sqrt{-g}g^{\mu\nu}(\delta R_{\mu\nu}) = \partial_{\sigma}\left(\sqrt{-g}g^{\mu\nu}(\delta\Gamma^{\sigma}_{\mu\nu})\right) - \partial_{\nu}\left(\sqrt{-g}g^{\mu\nu}(\delta\Gamma^{\sigma}_{\mu\sigma})\right),$$

.

Which isa total divergence (the difference of total divergences). In the variation of the action integral, such terms vanish when we integrate over all spacetime. Or, such total divergences will not contribute to the classical equation of motion. Then the remaining terms in (C5) are

$$\delta\mathcal{L}_g = \frac{1}{2k}\left\{\frac{1}{2}\sqrt{-g}Rg^{\mu\nu}(\delta g_{\mu\nu}) - \sqrt{-g}g^{\mu\rho}g^{\nu\sigma}R_{\mu\nu}(\delta g_{\rho\sigma})\right\}.$$

$$= \frac{\sqrt{-g}}{2k}\left(\frac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right)(\delta g_{\mu\nu}).$$

With this, the variation of action becomes

$$\delta I = \int\sqrt{-g}\frac{1}{2k}\left(\frac{1}{2}g^{\mu\nu}R - R^{\mu\nu}\right)(\delta g_{\mu\nu})d^4x + \delta\int\sqrt{-g}\mathcal{L}_{(m*)}d^4x.$$

We introduce the'negative energy – momentum tensor'$T'^{\mu\nu}$, for the matter with negative mass fields by

$$\delta\int\sqrt{-g}\mathcal{L}_{(m*)}d^4x \equiv -\frac{1}{2}\int T'^{\mu\nu}(\delta g_{\mu\nu})\sqrt{-g}d^4x.$$

So that for arbitrary variation the classical equations are given by

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -kT'^{\mu\nu}$$

.

Appendix D:

we can by supposing tow negative masses and it is proved that force of gravity between tow negative mass, by the multiplication of tow negative masses, the ratio is direct and with distance square of tow masses the ratio is indirect.

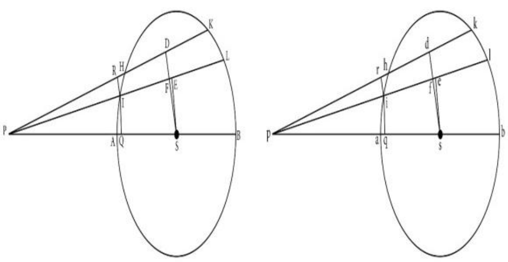

FIG 4. (garmsiri).

there are two spheres, then, according to above (fig 4), the following assumptions are available:

Two spherical surfaces = AHKB, ahkb ,

Diameter of circle = AB, ab ,

Circle center = S, s ,

Two particles located along the diameter = P, p ,

The radius of negative gravity force emitted to the particle = (PK, PL), (pk, pl),

As can be seen in the figure, when the negative gravity force radius are emitted to the particle, some arcs are obtained on the circle:

The arcs obtained from circles = HK, hk, IL, il ,

Hk = hk,

IL = il,

(SE) ⊥ (PL), (se) ⊥ (pl),

(SD) ⊥ (PK), (sd) ⊥ (pk),

(IQ) ⊥ (AB), (iq) ⊥ (ab),

(IR) ⊥ (PK), (and) ⊥ (pk),

Location of intersection = F, f ,

Now if the following lines which collide the particle at an angle, simultaneously approach zero, we will have

(PL, pl) → 0, (PK, pk) → 0,

Because,

(DS, ds) = (ES, es),

The following lines can be considered equal,

(PE, PF) = (pe, pf),

DF = df,

Because, as said above, when the following angles simultaneously approach zero, their proportions eventually equate to each other.

Angle DPE → 0,

Angle dpe → 0,

So, considering all the conditions stated above, we can write:

(PI) /(PF) =(RI) /(DF),

(Pf) /(pi) =(DF) /(ri),

By multiplying the two above terms and simplifying the product, we have

(PI.pf) / (PF.pi) =(RI) /(ri) = arc (IH) / arc (ih), (D1)

This means that the negative gravity force emitted from the surface of the sphere affects the particles outside the surface of the sphere. It is directly proportional to the surface of the sphere.

Again, we can have

(PI) /(PS) =(IQ) /(SE),

(Ps) /(pi) = (SE) /(iq),

By multiplying these two terms and simplifying its product, we have

(PI.ps) / (PS.pi) =(IQ) /(iq) , (D2)

By multiplying equations (D1), and (D2), we have

$(PI^2.pf.ps) / (pi^2.PF.PS) = (HI.IQ) / (ih.iq)$,

From the above result, it can be said that the arc of the spherical surface, where the negative gravity force emitted from the surface of the sphere to the particle, is inversely proportional to the squared distance between the sphere surface from which the negative gravity force emitted and the particle. Since the negative gravity force is directly proportional to the arc, as proved before (Eq.(D1)), the negative gravity force is inversely proportional to the distance between the sphere surface and the particle. Then the negative gravity force between the two masses can be equation as follows:

$F_{n1}=F_{n2}= G_{nm}$ (m^*_1) (m^*_2) /(r^2),

where

G_{nm} : Negative gravity constant,

F_n : negative gravity force,

M^*_1: Negative mass 1,

M^*_2: Negative mass 2,

r : The distance between the center of negative mass 1 and the center of negative mass 2.

REFERENCES

[1]. sci_news.com

[2]. Joshi A. W, 1995, Matrices and Tensors in physics, Publisher: (New york: wiley).

[3]. Parthasarthy, R. introduction to general relativity, alpha science international Ltd. Oxford, U.K. ISBN 978-1-84265-949-6.

[4]. Parthasarthy, R. introduction to general relativity, alpha science international Ltd. Oxford, U.K. ISBN 978-1-84265-949-6.

[5]. Parthasarthy, R. introduction to general relativity, alpha science international Ltd. Oxford, U.K. ISBN 978-1-84265-949-6.

[6]. Newton, I, 1803, Ed., the mathematica principhes of natural philosophy (the principia), proposition LXXI, theorem XXXI, pag 812 – 817, this edition of sir isac newton's principia, translated into English by andrew motte, in three volumes.

Garmsiri" Negative mass gravity theory (general relativity)." International Journal of Engineering Research and Applications (IJERA), Vol. 09, No.09, 2019, pp. 20-26