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A metaheuristic Globalized Bounded Nelder-Mead Algorithm applied to upstream Multiuser digital subscriber lines

Hernan X. Cordova J.*

**Applied Optimization Research Center, MGI PHIKARI GROUP, Ecuadorian Branch (Guayaquil, Ecuador), 1249 Martin Blv, Orlando, USA*

ABSTRACT

The demand for high bit-rates has forced the emerging of new technologies like 5G in wireless systems and VDSL2 and G.fast in wireline systems. In order to achieve higher bit rates in wireline system, both technologies make use of higher bandwidth thereby reaching higher bit rates. However, this incremental bandwidth causes that the FEXT (crosstalk) which is frequency and length dependent becomes a more severe problem in the upstream direction for a typical distributed topology. This effect is known as the near-far problem. In order to solve this problem whilst optimizing a bitrate, several methods have been proposed. By using anenriched global Nelder-Mead algorithm approachand by constraining the datarate at a certain distance (which we call reference length), we have improved the convergence time of the sum-rate optimization problem. In order to find the optimal UPBO parameters to shape the power spectral density such that the crosstalk does not “kill” other users, whilst maximizing the expected data rate at certain distance, we have applied this enriched “Global” bounded Nelder-Mead algorithm, optimizing its initialization and thereby leading to improvements of 1-to-2 dB’s in average (related to PSD_ref). This is translated into a better optimization of the datarate at user(s) located up to the selected distance (aka. reference length). Furthermore, we proved that the algorithm proposed in [10] corresponds to a subcase of our general proposed algorithm (Algorithm 1), whose situation is given when the reference length equals the distance corresponding to the user with the minimum datarate. Finally, our proposed enriched GBNM algorithm has higher probabilities to reach a better solution (at shorter convergence time) than [10] due to its probabilities restart procedure.

Keywords–metaheuristic algorithm, GBNM, Nelder-Mead, Upstream Power backoff, DSL

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I. INTRODUCTION

Recent years have witnessed huge advances in computer technology and communication networks. The trend continues and even new emerging technologies, like 5G are expected to further contribute to more connectivity and new requirements at physical layer (lower delay, higher bandwidth, superior spectral efficiency whilst still managing the energy-efficiency of the overall system). In systems where the number of connections are growing, they often concern the minimization (or maximization) of objective functions (maximization of user rates, minimization of power consumed, minimizing energy efficiency, maximizing spectral efficiency, etc) involved in the design of the networks or the optimization of their performance. Combinatorial optimization problems in telecommunications (and other areas too) involve finding optimal solutions from a discrete set of feasible solutions. However, even with the advent of new computer (and more efficient) technologies and parallel processing, many of these problems cannot be solved to optimality in reasonable computation times, due to their inner nature or to their size, or any combination therein.

Moreover, reaching optimal solutions could become meaningless in many practical situations, since we are often dealing with rough modelling simplifications of reality and the available data is not entirely precise. The goal of approximate algorithms (or heuristics) is to quickly produce good approximate solutions towards the global optimal (or good enough optimal), without necessarily providing any guarantee of the “best” solution quality but at a reasonable fair solution for the given (practical) constraints.

Specifically in wireline systems, the demand for high bit-rates has forced the emerging of new technologies like VDSL2 and G.fast. In order to achieve higher bit rates, both technologies make use of more bandwidth (up to 30 MHz though e restrict our study in this paper up to 12 MHz for VDSL2 and 106MHz and 212MHz under study for G.fast). This selected frequency range (up to 12 MHz) corresponds to an extra physical bandwidth of approximately 11 and 5.5 times when compared with ADSL (up to 1.1 MHz) and ADSL2+ (up to 2.2

MHz) systems, respectively and therefore, higher bit rates are achieved, [1]. However, this bandwidth incremental does not come for free. It causes that the FEXT (crosstalk) which is frequency and length dependent becomes a more severe problem in the upstream direction for a distributed topology. This effect is known as the near-far problem, [2],[3] and becomes even more troublesome in the use of even higher frequencies (e.g. G.fast).

Many upstream power back-off (UPBO) methods have been proposed for VDSL, as described in [4],[5]. Standardization bodies, however, have agreed to use the reference Power back-off method (PBO) method [6] where different reference Power Spectral Densities (PSD) have been defined for each upstream bands. The actual parameters used for the reference PBO in the current VDSL standards were established in [5] and in [7]. They both used a kind of exhaustive search to find optimized UPBO parameters, which is time consuming. To circumvent this problem, a method to calculate the UPBO parameters by using the Nelder-Mead simplex algorithm has been proposed in [8]. The concept of user unique PBO (UUPBO) was introduced in [9], where the UPBO parameters are optimized for each line, separately. In [10], the concept of cable-bundle PBO (CBPBO) was explored, where the UPBO parameters are optimized per cable. The author proposed (internal report and due to confidentiality reasons was not published) and implemented over Dutch practical lines an off-line (exhaustive search) algorithm where almost all constraints can be met focusing on the maximization of the upstream performance at some specific (reference) length, say L_R.

In this paper, we discussed that approach, explore the possibility to focus on the optimization of the upstream performance within a cable-bundle as in [10] but following the same principle as in the off-line exhaustive search algorithm, that is, keeping the idea that an operator would like to “guarantee” certain datarate of their users up to certain distance L_R. The difference relies that In order to find the optimal UPBO parameters, we will make use of an enhanced Nelder-Mead algorithm (Global Nelder-Mead) leading to fast convergence time and even the possibility to run it every certain period (when pilots are initialized, to adjust for new -crosstalk-conditions). The algorithm will be described later in section III.

Section II describes the system model and problem statement; an improved Global Nelder-Mead algorithm is explained in Section III and conclusions are drawn in section IV.

II. SYSTEM MODEL AND PROBLEM DEFINITION

2.1 Data Rate and crosstalk calculations

The data rate of a particular user m could be expressed in a channel capacity formula by introducing a $SNR_{gap}\,\Gamma$, given by:

$$R_m = f_{sym}\,.\sum_{n=1}^{N} b_i(n) \tag{1}$$

Whereas the number of bits can then be expressed by:

$$b_i(n) = \log_2(1 + \frac{P_i(n)\left|H_{ii}(n)\right|^2}{\Gamma(\sigma_i^2(n) + \sum_{\substack{j=1\\ j\neq i}}^{M} P_j(n)\left|H_{ji}(n)\right|^2)} \tag{2}$$

And the total noise is given by:

$$N_i(n) = \sigma_i^2(n) + \sum_{\substack{j=1\\ j\neq i}}^{M} P_j(n)\left|H_{ji}(n)\right|^2 \tag{3}$$

In (3), the term $\sum_{\substack{j=1\\ j\neq i}}^{M} P_j(n)\left|H_{ji}(n)\right|^2$ represents the received FEXT sensed by user i at subcarrier n, $P_{i,fext}(n)$. Therefore the aggregate datarate in a multiuser environment is given by:

$$R_{total} = \sum_{i=1}^{M} R_i = f_{sym}\,.\sum_{i=1}^{M}\sum_{n=1}^{N} b_i(n) \tag{4}$$

Where f_{sym} is the symbol rate (a typical value for DMT systems is 4000 symbols/s).

We will need to determine the power spectral density at the upstream side which is given by the following expression:

$$UPBO_{PSD}(f) = -\alpha - \beta\sqrt{f} \tag{5}$$

2.2 Deriving a normalized far-end crosstalk (FEXT) calculation

We follow a similar approach as in [10], where we normalize the FEXT as a function of the $PSD_{ref}(n)$ which equals the $UPBO_{PSD}(f)$. However, we will extend this initial analysis to an expression where our proposed algorithm can be applied to. We will refer to $PSD_{ref}(n)$ for estimations of the received power spectral density at the received side and $UPBO_{PSD}(f)$ as the actual received power spectral density, inter-exchangeable.

Having $P_{i,fext}$ at subcarrier n defined by:

$$P_{i,fext} = \sum_{\substack{i=1\\ j\neq i}}^{M} P_{ref}(n)\frac{|H_{ij}(n)|^2}{|H_{jj}(n)|^2}$$

(6)

Then we can compute the normalized FEXT by:

$$|H_{i,fext}^{norm}(n)|^2 = \sum_{\substack{i=1\\ j\neq i}}^{M} \frac{|H_{ij}(n)|^2}{|H_{jj}(n)|^2} = \frac{P_{i,fext}(n)}{P_{ref}(n)}$$

(7)

Where the “normalized” total noise is now given by:

$$N_i^{norm}(n) = \sigma_i^2(n) + P_{ref}(n)|H_{i,fext}^{norm}(n)|^2$$

(8)

And therefore, the number of bits is given by:

$$b_i(n) = log_2(1 + \frac{P_{ref}(n)}{\Gamma N_i^{norm}(n)}) \qquad (9)$$

2.3 Problem Definition

We follow the same principle of optimizing the UPBO parameters per cable (CBPBO) as in [10] and [11] but at some specific length, say L_R. The problem formulation mathematically can be expressed as:

$$\max(R_{i\_at\_L_R})$$

Subject to

$$UPBO(kl_o, f) \leq UPBO_{mask}$$

$$40 \leq \alpha \leq 80, 1 \leq \beta \leq 40$$

$$R_x UPBO(kl_R, f) = UPBO_{PSD\_Ref}(i)\ \forall i, i = 1, \ldots M$$

$$kl_o = \min\{\frac{LOSS(f)}{\sqrt{f}}\}$$

$P_i \leq P_{total}, \forall i, \quad i = 1, \ldots M$

(10)

Where $UPBO_{PSD_{Ref}}(i, L_R) = -\alpha - \beta\sqrt{f}$ is the received PSD for all i-users ($i = 1, \ldots M$). This optimization problem results in optimizing the performance of all the users located up to L_R of distance from the cabinet. Those users located after L_R will get a performance based on best effort, only, and its performance is expected to decrease rapidly.

That is, our optimization problem can be translated to: find the proper values of $\{\boldsymbol{\alpha_i}, \boldsymbol{\beta_i}\}$, for $\boldsymbol{i = 1, \ldots, SB}$ (SB=upstream subbands) such that $P_{ref}(n), \forall n$ (so, $UPBO_{PSD\_Ref}$) is optimized to achieve a specific target ($R_{i\_at\_L_R}$) at some specific (reference) length, L_R. Therefore, we derive an expression to calculate $P_{ref}(n), \forall n$.

Starting from (9), and after some manipulation, we get:

$$P_{ref}(n) = \frac{f(b_i, n).\sigma_i^2(n)}{(1 - |H_{i,fext}^{norm}(n)|^2 f(b_i, n))}$$

(11)

Where $f(b_i, n) = \Gamma.(2^{b_i(n)} - 1)$

Furthermore, our objective function translates to:

$$\psi(b_i, \alpha, \beta) = -\alpha - \beta\sqrt{f} - \frac{f(b_i, n).\sigma_i^2(n)}{(1 - |H_{i,fext}^{norm}(n)|^2 f(b_i, n))}$$

subject to

$$UPBO(kl_o, f) \leq UPBO_{mask}$$

$$40 \leq \alpha \leq 80.95, 1 \leq \beta \leq 40.95$$

$$R_x UPBO(kl_R, f) = P_{ref}(i)\ \forall i, i = 1, \ldots M$$

$R_i \geq R_{i,target}$, for a specific user i

(12)

III. THE GLOBALIZED BOUNDED NELDER-MEAD (GBNM) ALGORITHM

3.1 Initial Overview of GBNM

A good overview on current state-of-the-art search methods for global and local searches is provided in [14]. The GBNM algorithm is suitable for functions of less than 20 variables, following the local-global strategy, which basically implements a restart procedure based on adaptive probability density that keeps a memory of past local searches, [14].

The probability function, $f_p(x)$, of having sampled at point x is described by a Gaussian-Parzen-windows approach, as described in [15], given by:

$$f_p(x) = \frac{1}{\xi_{sampled}} \sum_{t=1}^{\xi_{sampled}} f_p^t(x) \qquad (13)$$

Where $\xi_{sampled}$ corresponds to the number of points that have been already sampled, and $f_p^t(x)$ is the normal multidimensional probability density function given by:

$$f_p^t(x) = \frac{1}{(2\pi)^{\frac{n}{2}}(det(\Sigma))^{\frac{1}{2}}}.exp\left(-\frac{1}{2}(x-x_i)^T\Sigma^{-1}(x-x_i)\right)$$
(14)

Where n is the dimension (number of variables, i.e. two per upstream band under study) and Σ is the covariance matrix, given by:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n^2 \end{bmatrix}$$
(15)

Where each of the variances is described by the following expression:

$$\sigma_j^2 = \alpha(x_j^{max} - x_j^{min})^2$$
(16)

Where α is a positive parameter that manages the lengths of the Gaussians; x_j^{max} and x_j^{min} are the bounds in the j^{th} direction. In our study, in order to keep the approach simple and cost-effective, we fix the variances to be constant. However, this approach might lead to an increase in the total number of analysis. Despite of this cost, the authors think this is still a good overall strategy for the problem under study, as supported by the simulation results presented hereafter.

This probabilistic restart procedure can be applied to almost any local optimizer. In this case, we follow the proposal in [14], applying the probabilistic restart leading to an improved Nelder-Mead algorithm. Clearly, the probability of having located a global optimum increases with the number of probabilistic restarts.

As the problem in (6) is non-convex, the application of GBNM suits well, supporting also discontinuities (so, no gradient information needed); hence, the improvement over a typical Nelder-Mead Algorithm [16] consists of the proper detection of simplex degenerations and handling this through proper re-initialization as explained in this section. The simplex is said to be degenerated if it has collapsed into a subspace of the search domain. This is the most common symptom of a failed Nelder–Mead search [17] because the method cannot escape the subspace. More precisely, a simplex is called degenerated in our study (as in [14]) if it is neither small, nor touches a variable bound, and one of the two following conditions is satisfied:

$$\frac{min_{k=1,\dots,n}\|e^k\|}{max_{k=1,\dots,n}\|e^k\|} < \xi_{s3} \text{ or } \frac{det\,[e]}{\prod_k\|e^k\|} < \xi_{s4}$$
(17)

Where e^k is the k^{th}edge, e is the edge matrix, $\|.\|$ represents the Euclidean form, and ξ_{s3} and ξ_{s4} are small positive constants. Further details of this algorithm are extensively described in [14].

3.2 Proposed Algorithm and Discussion

Our main focus is on the maximization of the datarate of user i at some specific lengthL_R. However, an improved algorithm (Algorithm 2) is also proposed for the maximization of the minimum datarate (as in [10]) over the lines. Algorithm 2 outperforms the algorithm as proposed in [10] because it provides a better estimation of the UPBO_(PSD_Ref) in practical DSL lines and converges faster to the optimal results.

To solve the optimization problem presented in (6) we make use of an optimized Nelder-Mead (GBNM) Algorithm. Similar concerns as described in [10] are relevant in our study. However, because we use the initial results obtained in [11], we can better estimate the minimum target rates ensuring we can indeed achieve a feasible target datarate up to certain specific (reference) length, L_R, ensuring convergence. This allows operators to easily select a type of service per cable (and even per class cabinet) so that a majority of users can be served at specific datarates. It was proved in [11] that significant improvement is achieved when distinguishing among cabinets that have different user-distributions.

The first step defines the expected power signal at the received for all users. This initial estimation is done by following a similar approach as in [11]. Step 2 calculates the normalized FEXT coupling by holding the value of $UPBO_{PSD\_Ref}$ as proposed in [10], assuming that each modem provides a "good" estimation of the background noise σ_i^2, and the total noise N_i. This way, the parameters to be found ($\{\alpha_i, \beta_i\}$) become "almost" independent of the topology of the network. However, we could use any other initialization method. Next, the main loop starts where initial values of $\{\alpha_i, \beta_i\}$ are provided to the main function (GBNM). This is repeated until the target bit rate at (reference) length L_R is reached.

Algorithm 1: Optimal GBNM, variant 1
1. Select a suitable $UPBO_{PSD\_Ref}$ using the simplified approach proposed in [11] as a best starting point.
2. Calculate normalized FEXT couplings for each line as in [10].
3. **for** i= $\{US_1, .., US_{max\_us}\}$ all upstream bands (where max_us corresponds to the maximum number of upstream bands to be considered in

```
the study)
        $\varphi_i = \{\alpha_i, \beta_i\}$
        Repeat
            $\varphi_i$ = GBNM (@RateCalc_at_$L_R$, $\varphi_i$)
        until a specific accuracy (target) is reached
    end for
```

```
function $R^{min}$ = RateCalc_at_$L_R(\varphi_i)$
    Calculate $UPBO_{PSD}$ with $\varphi_i = \{\alpha_i, \beta_i\}$, as given by (5).
    Calculate $R_i$ for all lines, that is, for $i = 1, \ldots, M$ according to (2).
    Create a new set $\emptyset_\mu(l_i, L_R)$ of length $\mu$ (where $\mu \le M$) where all lines with distances $l_i \le L_R$ are selected
    Calculate $R_i$ for $i = 1, \ldots, \mu$ according to (2).
    Calculate $R^{min} = min_{\emptyset_\mu} R_{i,\emptyset_\mu}$ in the set $\emptyset_\mu$ where $\mu \le M$ and $i = 1, \ldots, \mu$
```

Algorithm 2 addresses the same problem as described in [10]. The difference is twofold: on one hand, Algorithm 2 makes use of a better initial estimation as proposed in [11] and on the other hand, it makes use of an enhanced direct simplex algorithm, GBNM as proposed in [14]. Further analysis (considering the comparison between GBNM and NM in [14]) shows that even not using the initial estimation as proposed in [11], Algorithm 2 will still perform better than [10] because it gets not trapped into any simplex degeneration that is likely probable by only using the typical Nelder-Mead approach as in [15].

Algorithm 2: Optimal GBNM, variant 2 (the max-min)

```
Select a suitable $UPBO_{PSD\_Ref}$ using the simplified approach proposed in [11] as a best starting point.
Calculate normalized FEXT couplings for each line as in [10].
for i= $\{US_1, \ldots, US_{max\_us}\}$ all upstream bands
(where $max\_us$ corresponds to the maximum number of upstream bands to be considered in the study)
    $\varphi_i = \{\alpha_i, \beta_i\}$
    Repeat
        $\varphi_i$=GBNM(@RateCalcMin, $\varphi_i$)
    until a specific accuracy (target) is reached
end for
```

```
function $R^{min}$ = RateCalcMin ($\varphi_i$)
    Calculate $UPBO_{PSD}$ with $\varphi_i = \{\alpha_i, \beta_i\}$, as given by (5).
    Calculate $R_i$ for all lines, that is, for $i = 1, \ldots, M$ according to (2).
    Calculate $R^{min} = \min_i\{R_i\}$
```

Notice that the Kuhn and Tucker conditions of mathematical programming are not applicable to the present non-differentiable but direct search approach. The tolerances for small and degenerated simplices, [ξ_s3, ξ_s4], respectively, (this is also applicable to ξ_s1 when defining a "small" simplex as in [14]), may be difficult to adjust, so that a simplex which is becoming small may be tagged as degenerated before. Thus, if a degeneration is detected twice consecutively at the same point, the point is taken as a possible optimum, and a probabilistic restart is called. Similarly, if a degeneration is detected after a small test, this point is also saved as a possible optimum, and a large test is ordered.

IV. SIMULATION RESULTS

We have setup a VDSL2 environment using band-plan 998, mask B8-12b. The design is based on a "TP150 " cable which is a Dutch 0.5mm cable, also known as "KPN_L1". Only three users have been considered though it is straightforward to extend the analysis to any number of users. The focus is on the upstream and a maximum power of 14.5dBm is applied to the system (per user). The Γ was chosen to be 12.75dB corresponding to a BER of 10^{-7}, 3dB of coding gain and a noise margin of 6dB. No power mask constraints have been setup (though it is straightforward to incorporate them). Default value for EL-FEXT is used (-45dbm/Hz at 1km and 1MHz). The scenario under study is depicted in Fig 1. Our target reference, $\mathbf{L_R}$, is 500m; hence, we will maximize the upstream datarate at this distance, providing best effort for users behind this length (e.g. the user at 700m).

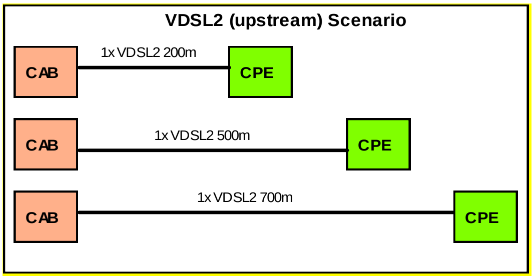

Fig 1: Simulation Scenario for VDSL2 -upstream-

First step in our proposed algorithm is to estimate the initial transmitted upstream signal according to [10],[11] (that is, calculating PSD_{ref}). Fig 2 shows this result.

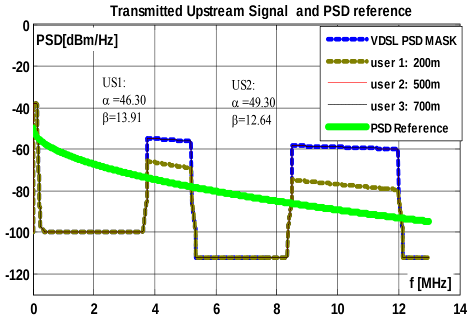

Fig 2: Transmit Upstream Signal for each user. Clearly user 3 (at 700m) will transmit at full power. User 2 does this by design. $PSD_{ref}$is shown in light green.

Fig. 3 shows the initial received FEXT at the LT side when each user is considered under study; that is, Fig. 3 shows the simulation of 3 runs: one per user when no UPBO measures have been considered. It becomes obvious the devastating effect for user 2 and 3.

Once Algorithm 1 is implemented, a better estimation of PSD_{ref} is obtained, leading to better performance results at the selected value of L_R (e.g. 500m). From extensive simulations, we have observed an average gain of 1-2 dB (per band).

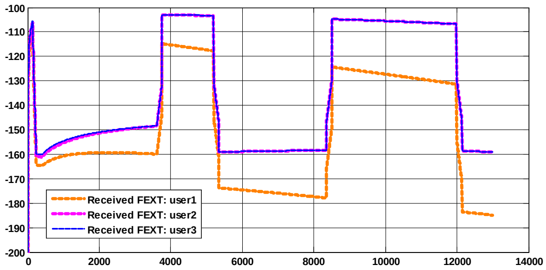

Fig 3: Comparison of the total FEXT received at the upstream side. Axis x represents the frequency in KHz and axis y the PSD in dBm/Hz

As our algorithm makes use of a "good" initial estimation of PSD_{ref}, it converges after very few iterations (in average, less than 5).

Results from the implementation of Algorithm 2 lead to similar results. This is easy to explain: variant 2 (as well as the proposal in [10]) is a subclass of our proposed algorithm where L_R equals the distance corresponding to the user with the minimum datarate.

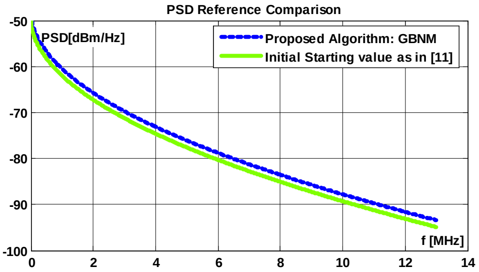

Fig 4: Comparison of PSD_{ref} calculated as proposed here via the GBNM algorithm vs the best estimation in [10]

V. CONCLUSIONS

We have studied the near-far effect in VDSL2 systems, showing the high FEXT generated from "short" loop towards "long" loops. We have focused on the optimization of the upstream performance within a cable-bundle as in [10] but following the same principle as in [11] and improving that approach by speeding up the convergence time via the usage of a global Nelder-Mead algorithm, that is, "committing" to certain datarate until some to distance L_R. As said, in order to find the optimal UPBO parameters to maximize the data rate at certain distance, we have used a "Global" bounded Nelder-Mead algorithm, which leads to improvements of 1-to-2 dB's in average (related to PSD_{ref}). This is translated into a better optimization of the datarate at user(s) located up to the selected distance, L_R. Furthermore, the algorithm proposed in [10], which is covered in our study by variant 2, proves to be a subcase of our general proposed algorithm (Algorithm 1), when L_R equals the distance corresponding to the user with the minimum datarate. However, the proposed application of the GBNM algorithm has higher probabilities to reach a better solution than [10] due to its probabilities restart procedure.

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