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The Use of Matlab/Simulink Tool in Modeling the Hysteresis Phenomenon of Piezoelectric Actuators

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ABSTRACT: The piezoelectric effect is the ability of some materials such as crystals, to generate an electric potential when mechanical stressis applied, this effect was discovered in 1880 by the brothers Pierre Curieand Jacques Curie. Nowadays, these materials were used more in applications for various fields such as lighters, sensors, ultrasonic devices, piezoelectric motors, etc. In particular, the piezoelectric materials can used to produce the piezoelectric actuators, these actuators are widely used in micropositioning, vibration control, and manufacturing applications. However, they exhibit hysteresis nonlinearities that could affect to the precise control of actuators. Thus, lot of mathematical modeling have been proposed to characterize the hysteresis phenomenon in smart actuators, such as Preisach model, Krasnoselskii–Pokrovskii model, Prandtl– Ishlinskii model, etc. This paper presents the modeling for hysteresis phenomena of piezoelectric actuators using Matlab/Simulink based on the Prandtl - Ishlinskii model. The results of the model was examined and evaluated on actual measured data and showed the accuracy of this model.
Key words:piezoelectric actuator, hysteresis, Prandtl– Ishlinskii model, modeling and simulation

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I. INTRODUCTION

Piezoelectric effect is the ability of some crystal materials such as Aquamarin, Tuamalin, quartz and PZT ceramics, v.v capable of converting mechanical energy into electrical energy and vice versa.Direct piezoelectric effect was discovered in 1880 by the brothers Pierre Curie and Jacques, The characteristic of this effect is a quartz crystal under the influence of a mechanical force (mechanical energy) will produce a voltage between the two ends of the crystal (electrical energy). Soon, M. G. Lippmann showed that piezoelectric materials are also capable of converting energy from electric form to mechanical energy, this effect is known as inverse piezoelectric effect. (Tichý, Erhart, Kittinger, & Prívratská, 2010).

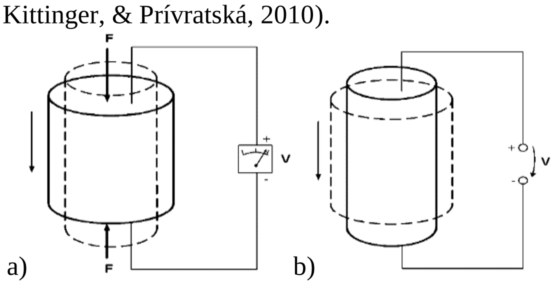

Figure 1. Piezoelectric effect: a) Direct piezoelectric effect; b) Inverse piezoelectric effect.

Because the characteristics work under both direct and inverse piezoelectric effects, piezoelectric materials are also considered intelligent materials with applications in the role of actuators, such as actuators compact, linear motor, rotary motor, and micro-pump. They are also used in the role of sensors such as load cells, pressure sensors, and acceleration sensors. Besides, when piezoelectric materials come into contact with a harmonic oscillator voltage that will produce continuous vibration so that a sound source can be created, these sound sources are used with the purpose to control noise in passenger compartments of cars, trams and airplanes (Jan Holterman and Pim Groen, 2013). Another striking application of piezoelectric materials is the ability to harvest the electrical energy received from plates made from material placed under floors, elevators or locations where there are often fluctuations, etc. (Caliò, Rongala, Camboni, Milazzo, Stefanini, de Petris, & Oddo, 2014). However, piezoelectric materials always have disadvantages in their working characteristics which are hyteresis. This phenomenon creates non-linearity in the transfer function of the material, which is considered to be the cause of oscillation in the system's response, making the accuracy of the intelligent actuators limited. in automatic control system (Tichý et al., 2010).

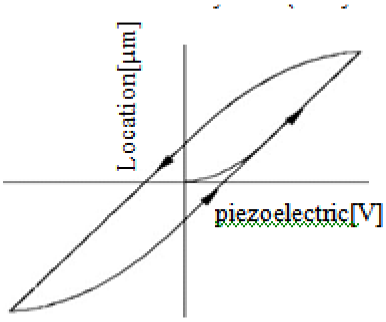

Figure 2. Hyteresis in PZT ceramic piezoelectric materials

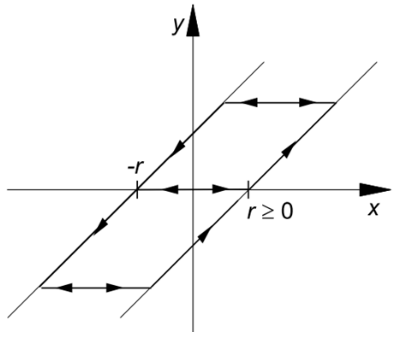

Figure 3. Representing the input-output relationship of thes play hysteresis operator.

For piezoelectric actuators, the effect of hysteresis is very large and clear. Therefore, to solve this problem, there are many proposed mathematical models for the purpose of delay compensation in typical control and control applications such as Preisach model, Krasnosel'skii-Pokrovskii model, Prandtl – Ishlinskii model, v.v. In the context of this article, the authors focus on presenting the problem of building a model describing the hysteresis of piezoelectric actuators based on thePrandtl - Ishlinskii mathematical model.

II. THE MATHEMATICAL MODEL.

2.1. 2.1. Play hysteresis operator

For any input x(t) lies in the space of continuous monotonous elements $C_m[0, t_E]$. With input values x(t) divided in time intervals $0 = t_0 < t_1 < \ldots < t_i < t < t_{i+1} < \ldots < t_N = t_E$so that the values of the first function x(t) are monotonically continuous in each segment $[t_i, t_{i+1}]$. Then the output function value y(t) of the play hysteresis operator is defined by Kuhnen (2001,2003):

$y(t) = \mathcal{F}_r[x, y_0](t)$

(1)

In which, r is the threshold variable value of the play hysteresis operator. Play hysteresis operator is expressed in recursive form:

$y(t) = \max\{x(t) - r, \min\{x(t) + r, y(t_i)\}\}$(2)

With the initial condition of the output signal at time t_0 is determined by:

$y(t_0) = \max\{x(t_0) - r, \min\{x(t_0) + r, y_0\}\}$(3)

The diagram represents the relationship between the input value x(t) and the output y(t) of a play hysteresis operator with the threshold value r> 0 as shown in Figure 3.

ay hysteresis operator.

In special cases, when the threshold variable value r = 0, from expression (2) we have:

$y(t) = \max\{x(t), \min\{x(t), y(t_i)\}\} = x(t)$(4)

Now the relationship between the input and output of the play delay operator is a linear relationship with a gain equal to 1 and completely independent of the y element $y(t_i)$. Play hysteresis operator can be reset as follows:

$\mathcal{F}_{r=0}[x, y_0](t) = I[x](t)$

(5)

I[x] describes the working characteristics of the ideal transfer function, no longer the existence of a hysteresis in the transfer function of the material. If you call the mathematical description function of a system that is G-1 [x] (t) and the system's mathematical description function is $G^{-1}[x](t)$ then I[x](t) can also be defined by the expression:

$G[G^1[x]](t)=I[x](t)$

(6)

Expression (6) demonstrates that it is possible to linearize any transfer function characteristic of the system by using the reverse model to compensate for the factors associated with nonlinearity that exist in the working characteristic of system.

2.2. Prandtl – Ishlinskii hysteresis model

The Prandtl - Ishlinskii model was built and developed based on the Prandtl model proposed by Prandtl (1928) in 1928, later proposed by Ishlinskii (1944) in 1944. This is a mathematical model that has been studied and applied a lot for hysteresiscompensation in the characteristics of actuators made from smart materials. Prandtl-Ishlinskii modelis classified into two types: play type and stop type (Visintin, 1994). In the context of this article, the authors only mention play type and briefly called Prandtl - Ishlinskii model.

The Prandtl - Ishlinskii model is determined based on a combination of n + 1 play hysteresis operators characterized by different values of threshold r_i variable corresponding to the weight values w_i of the model, with i = 0 ... n, the Prandtl - Ishlinskii model's mathematical expression is described by Kuhnen (2001,2003):

$$\mathcal{H}[x](t) = \sum_{i=0}^{n} w_i \mathcal{F}_{r_i}[x, y_{0i}](t) \quad (7)$$

The principle of building Prandtl - Ishlinskii hysteresis model from expression (7) is shown in Figure 4.

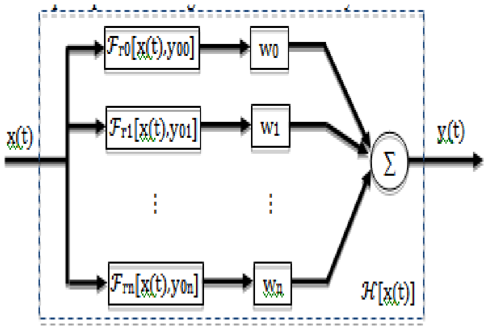

Figure 4. Illustrating the principle of modeling simulation

In which threshold variables r_i, weight values wi and initial output values y_{0i} with i = 0 ... n are represented in turn as vectors respectively

$r^T = (r_0 \quad r_1 \quad \cdots \quad r_n)$(8)

$w^T = (w_0 \quad w_1 \quad \cdots \quad w_n)$(9)

$y_0^T = (y_{00} \quad y_{01} \quad \cdots \quad y_{0n})$(10)

For threshold values r_i, selected according to the condition:

$0 = r_0 < r_1 < \ ... < r_i < \ ... < r_n < \infty$(11)

An important property of the Prandtl - Ishlinskii delay model is the ability to identify the reverse model from the mathematical model of the object, and to have the reverse model of Prandtl-Ishlinskii hysteresis model, then the weighting values must satisfy the following concurrent conditions (Kuhnen, 2001):

$$\begin{cases} w_0 > 0 \\ w_i \geq 0 \end{cases} \; i = 1 \dots n (12)$$

III. SIMULATION AND MODEL IDENTIFICATION

3.1. Simulation model using Matlab / Simulink

The simulation and identification of Prandtl – Ishlinskii hysteresis model will be calculated and performed by numerical calculation method, so expressions expressing mathematical models will also be converted to discrete form functions as follows:

For the playhysteresis operator, the form is represented with a discrete function according to the expression (13) and the recursive form as the expression (14).

$$y(k) = \mathcal{F}_r[x, y(k-1)](k) (13)$$

$$\begin{cases} y(k) = \max\{x(k) - r, \min\{x(k) + r, y(k-1)\}\}; \; k = 1 \dots N \\ y(0) = 0 \end{cases} \quad (14)$$

With N is the number of received input and output values according to the timelines divided in $[0,\ t_E]$. From expression (14), it is possible to build the model of the play hysteresis operator shown in Figure 5

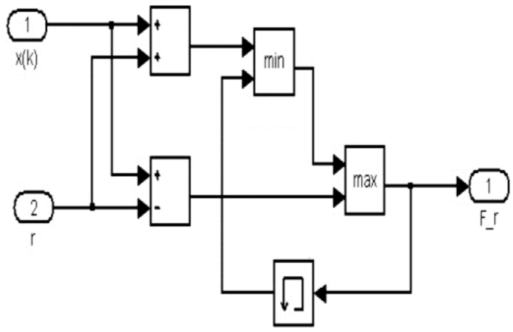

Figure 5. The play hysteresis operator is built from Matlab/Simulink

For the Prandtl – Ishlinskiihysteresismodel, the expression (7) will be represented as

$$\mathcal{H}[x](k) = \sum_{i=0}^{n} w_i \mathcal{F}_{r_i}[x, y(k-1)](k) \quad (15)$$

Prandtl – Ishlinskii hysteresismodel built from Simulink tool on Matlab software based on expression (13) is shown in Figure 6.

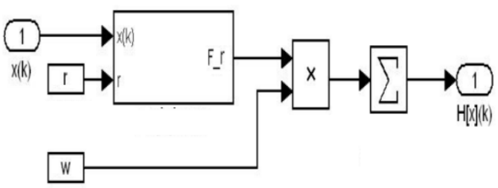

Figure 6. Simulation of Prandtl late model - Ishlinskii

According to the expression (12), the values of the elements in vector y_0 according to the expression (10) are replaced by the memory value of the output signal taken from the play delay operator with y_{00} = 0. Therefore, the mmode only works based on two parameters: threshold variable r and weight w, parameter values of this model will be provided as vectors as in expressions (8) and (9).

3.2. Method and algorithm to identify model parameters

3.2.1. Math facility

From expressions (7) and (13), it can be seen that the accuracy of the Prandtl- Ishlinskii model depends on the following factors

- The number of play hysteresis operators or values is also known as the order number of the model n
- The value of elements in the threshold variable vector r
- Value of elements in weight vector w

Set the value $r_0 = 0$ and set $v = w_0$, expression (13) can be rewritten as:

$$\mathcal{H}[x](k) = vI[x](k) + \sum_{i=1}^{n} w_i \mathcal{F}_{r_i}[x, y_{0i}](k) \qquad (16)$$

According to Kuhnen & Janocha (2002) the parameter values of Prandtl - Ishlinskii model are determined based on the dependence of the input value x(t) and the output value y(t) expressed by the wake up later:

The threshold variable values, determined by

$$r_i = \frac{i}{n+1}\max\{|x(k)|\};\ i = 1 \ldots n \qquad (17)$$

The values of weights are determined as follows:

$$v = w_0 = \frac{\max\{|y(k)|\}}{\max\{|x(k)|\}} \qquad (18)$$

and

$$w_i = \frac{\max\{|y(k)|\}}{\max\{|x(k)|\}}\frac{1}{r_i} = \frac{v}{r_i};\ i = 1 \ldots n \qquad (19)$$

However, the results of these parameters are used by Kuhnen in combination with stop type model of Prandtl - Ishlinskii hysteresis model, and some other hysteresis models to create a model called Prandtl - Ishlinskii improved model (Kuhnen and Janocha, 2002), (Kuhnen, 2001). Therefore, the parameter values calculated in the form (17), (18) and (19) will not give accurate descriptive results when only the Prandtl - Ishlinskii hysteresis model is used independent

3.2.2. Method to identify model parameters by Matlab / Simulink

In this section, the author presents method of identifying Prandtl-Ishlinskii hysteresis model based on simulation model combined with input value sets x(t) and output value y(t) achieved by the measured from experiment. The principle diagram used in identifying the parameters of the model is described by the authors as shown in Figure 7.

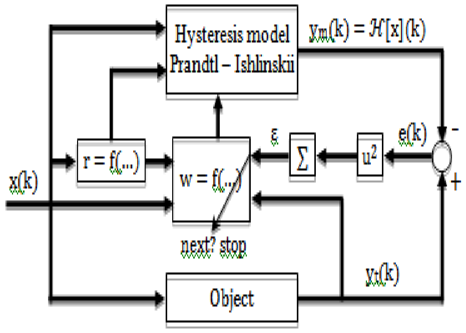

Figure 7. Principle of identifying parameters of Prandtl – Ishlinskiihysteresis model.

The parameters of the model such as threshold variable vector r, weight vector w, are calculated and selected according to the least squares method of the error of the measurement output of the real object $y_t(k)$ and the output of model $y_m(k)$. With a predetermined model error value trình, the process automatically determines the number of steps of the model n, until the error of the process is less than or equal to ε (Tran, 2010).

IV. RESULT

The results of this model are checked with the measurement data of piezoelectric actuator with the input value of triangular voltage signal U(t) and the output value is the displacement interval transfer of devices (t). Specific values are presented in Table 1.

Bảng 1.

Bảng 2. Table 1. Parameter values table with piezoelectric actuators

Measurement data	Value	Unit
Input voltage U	$-51{,}2 \le U(t) \le 51{,}2$	V
The amount of displacement s	$-19{,}2 \le U(t) \le 19{,}2$	µm

By calculating and identifying the parameters of the model as described in Section III, the authors received the model parameters at the value $\varepsilon = 5.3517.10^{-9}$, with the number of steps of the tissue figure (number of play operators) n = 32 (Tran, 2010). The results of the model's output signal comparison and time-dependent objects are shown in Figure 8.

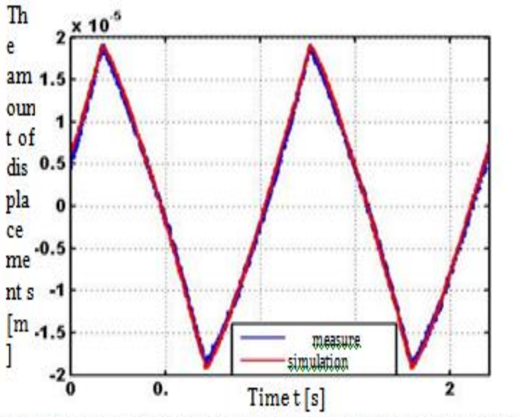

Figure 8. Compare output signals of models and objects over time

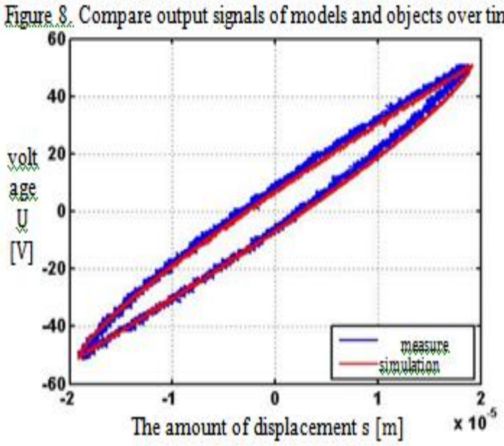

Figure 9. Results of simulation of hysteresis by Prandtl-Ishlinskiim hysteresis model

Figure 9 shows the simulation result of the object hysteresis phenomenon, which is a piezoelectric actuator whose characteristic line between input quantity is the voltage value U and the output quantity is the displacement range with very small value (about 20µm). It can be seen that the simulation results with the Prandtl-Ishlinskiihysteresis model show very close results with the actual measurement data, this shows the correctness in using the model and giving the model parameter values exactly.

V. CONCLUDE

The paper presents the use of Matlab/Simulink tool in modeling the hysteresis phenomenon of piezoelectric actuators by the late model Prandtl-Ishlinskii.At the same time, based on the simulation model combining actual measurement data from piezoelectric actuators, the authors set out the method of calculating and identifying the model parameters automatically. The results of the model have been evaluated and successfully applied in controlling the displacement position of the actuator using the inverse model (Tran, 2010)

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