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A New Optimized way of designing a Sliding Mode Controller

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ABSTRACT: In this paper a new approach of designing the Sliding Mode Controller (SMC) for large scale discrete time systems is attempted. The SMC is designed via a new low order optimized model for a given dynamic system. In the proposed optimization method, the numerator coefficients are obtained using Interpolation criteria while the denominator polynomial is obtained by using, one of the stability preserving methods, the Routh method. An optimized reduced order model is derived with minimum ISE. Further, it has been observed that the sliding mode control designed for the dynamic system improves the performance of the controlled system.

Index Terms: Interpolation criterion; sliding mode control; order reduction; controller simplification

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I. INTRODUCTION

In dgital SMC, the control input is held once in every sampling interval and is kept constant during this period. Owing to the finite sampling frequency, the state trajectory of the discrete time system may not move along the sliding surface but may yield a quasi sliding mode motion. In spite of analysis and design complexity, the SMC has several advent features over other conventional controllers for dynamic non-linear systems. A single sliding surface caters for the analysis of number of uncertainties.

In the literature [1-3,7-9] useful order reduction techniques in the frequency domain for linear continuous systems are available. Some stable mixed methods [2,5,9] have been playing vital roles in model reduction that overcome the instability problems associated with the reduced order models.

SMC is one of the robust variable structure controllers with adaptive nature that finds major place in aero space technologies and unmanned aerial vehicles [4,6,10]. A reduced order based SMC attributes its results for the better performance of large scale non linear systems.

An optimum dynamic model is built in this paper by varying the interpolation points that lead to the design of SMC.

Reduction Procedure

Consider an n^{th} order linear time invariant discrete system represented by

$$G_n(z)=\frac{p_0+p_1z+...+p_{n-1}z^{n-1}}{q_0+q_1z+...+q_nz^n}=\frac{p_n(z)}{q_n(z)} \quad (1)$$

By applying transformation, the transfer function of a higher order system be represented by [6], [7]

$$G_n(s)=\frac{d_0+d_1s+...+d_{n-1}s^{n-1}}{e_0+e_1s+...+e_ns^n}=\frac{d_n(s)}{e_n(s)} \quad (2)$$

Where d_i, i=0, 1… n-1and e_i, i=0, 1… n are constants.

For the high order system a reduced k^{th} order model is proposed as given below,

$$R_k(s)=\frac{a_0+a_1s+...+a_{k-1}s^{k-1}}{b_0+b_1s+...+b_{k-1}s^{k-1}+b_ks^k}=\frac{a_k(s)}{b_k(s)} \quad (3)$$

Where the a_i, i=0, 1… k-1 and b_i, i=0, 1…. k are constants.

By applying inverse transformation, the k^{th} order reduced model as given,

$$R_k(s)=\frac{\hat{p}_0+\hat{p}_1z+...+\hat{p}_{k-1}z^{k-1}}{\hat{q}_0+\hat{q}_1z+...+\hat{q}_{k-1}z^{k-1}+\hat{q}_kz^k}=\frac{\hat{p}_k(z)}{\hat{q}_k(z)} \quad (4)$$

Reduced order denominator:

Step 1: The denominator $b_k(s)$ of reduced model can be obtained from the Routh Stability array of the denominator of the original system as given below:

The Routh table for the denominator of the system is given below:

$$
\begin{array}{lllll}
b_{11}=e_n & b_{12}=e_{n-2} & b_{13}=e_{n-4} & b_{14}=e_{n-6} & \dots \\
b_{21}=e_{n-1} & b_{22}=e_{n-3} & b_{23}=e_{n-5} & b_{24}=e_{n-7} & \dots \\
b_{31} & b_{32} & b_{33} & \dots\dots & \\
\dots\dots & & & & \\
b_{k-1,1} & b_{k-1,2} & & & \\
b_{k,1} & & & & \\
b_{k+1,1} & & & &
\end{array}
\qquad (3)
$$

$$Where \;\; b_{i,j}=b_{i-2,j+1}-\frac{b_{i-2,1}b_{i-1,j+1}}{b_{i-1,1}}, \qquad (4)$$

$$where\; i\geq 3 \,and\; 1\leq j\leq \left[\frac{(k-i+3)}{2}\right]$$

b_k (S) may be easily constructed from the (n+1-k) th and (n+2-k) th and (n+2-k) t h rows of the above to give

$$b_k(s) = b_{k+1-n.1}s^n + b_{k+2-n.1}s^{n-1} + b_{k+1-n.2}s^{n-2} + \dots \qquad (5)$$

Reduced order numerator:

Step 1: Choose 2k point's s_0, $s_1 \dots s_{2k-1}$, s_{2k} $\in_C$ (they can be multiple) from the location of the poles of the original systems and obtain g(s) as given below:

$$
\begin{aligned}
g(s) &= (s-s_0)(s-s_1)\dots(s-s_{2k-1}) \\
&= (s-s_0)^{k_0}(s-s_1)^{k_1}\dots(s-s_j)^{k_j} \\
&= s^{2k} + g_{2k-1}s^{2k-1} + \dots + g_1 s + g_0
\end{aligned}
$$

Step 2: Compute $d_n(s)b_k(s)$ and $e_n(s)a_k(s)$, respectively,

$$
\begin{aligned}
d_n(s)b_k(s) &= (d_{n-1}s^{n-1}+\dots+d_0)(b_k s^k+\dots+b_0) \\
&= c^{(0)}_{k+n-1}s^{k+n-1}+\dots\dots+c^{(0)}_1 s + c^{(0)}_0
\end{aligned}
$$

$$c^{(0)}_0 = b_0 d_0,$$

$$c^{(0)}_1 = b_0 d_1 + b_1 d_0,$$

$$\dots\dots$$

$$c^{(0)}_{k+n-2} = b_k d_{n-2} + b_{k-1}d_{n-1},$$

$$c^{(0)}_{k+n-1} = b_k d_{n-1},$$

$$and$$

$$e_n(s)a_k(s) = (e_n s^n + \dots + e_0)(a_{k-1}s^{k-1}+\dots+a_0)$$

$$
\begin{aligned}
&= d^{(0)}_{k+n-1}s^{k+n-1} + d^{(0)}_{k+n-2}s^{k+n-2} + \dots \\
&\quad + d^{(0)}_1 s + d^{(0)}_0,
\end{aligned}
$$

$$d^{\,0}_0 = e_0 a_0$$

$$d^{(0)}_1 = a_0 e_1 + a_1 e_0,$$

$$\dots\dots$$

$$d^{(0)}_{k+n-2} = a_k e_{n-1} + a_{k-2}e_n,$$

$$d^{(0)}_{k+n-1} = a_{k-1}e_n.$$

Step 3: Divide $d_n(s)b_k(s)$ by g (s) to get f(s):

$$
\begin{array}{r|l}
 & c^{(0)}_{k+n-1}s^{n-k-1} + c^{(1)}_{k+n-2}s^{n-k-2} \quad + \dots \\
\hline
s^{2k}+g_{2k-1}s^{2k-1}+\dots+g_1 s+g_0 & c^{(0)}_{k+n-1}s^{k+n-1}+c^{(0)}_{k+n-2}s^{k+n-2} \quad +\dots+c^{(0)}_{n-k-1}s^{n-k-1}+\dots+c^{(0)}_0 \\
 & \underline{c^{(0)}_{k+n-1}s^{k+n-1}+g_{2k-1}c^{(0)}_{k+n-1}s^{k+n-2}+\dots+g_0 c^{(0)}_{k+n-1}s^{n-k-1}} \\
 & c^{(1)}_{k+n-2}s^{k+n-2}+\dots+c^{(1)}_{n-k-2}s^{n-k-3}+\dots+c^{(1)}_0 \\
 & \underline{c^{(1)}_{k+n-2}s^{k+n-2}+\dots+g_0 c^{(1)}_{k+n-2}s^{n-k-2}} \\
 & c^{(2)}_{k+n-3}s^{k+n-3}+\dots+c^{(2)}_0 \\
 & \underline{\dots \quad \dots \quad \dots \quad \dots} \\
 & c^{(n-k)}_{2k-1}s^{2k-1}+\dots+c^{(n-k)}_0
\end{array}
$$

Thus get the recursive relations:

$$c^{(1)}_i = c^{(0)}_i - c^{(0)}_{k+n-1}g_{i+k-n+1},$$

$$i = 0,1,\dots,k+n-2,$$

$$c^{(2)}_i = c^{(1)}_i - c^{(1)}_{k+n-2}g_{i+k-n+2},$$

$$i = 0,1,\dots,k+n-3,$$

$$c^{(3)}_i = c^{(2)}_i - c^{(2)}_{k+n-3}g_{i+k-n+3},$$

$$i = 0,1,\dots,k+n-4,$$

$$\dots\dots\dots$$

$$c^{(l)}_i = c^{(l-1)}_i - c^{(l-1)}_{k+n-l}g_{i+k-n+l},$$

$$c^{(n-k)}_i = c^{(n-k-1)}_i - c^{(n-k-1)}_{2k}g_i,$$

$$i = 0,1,\dots,2k-1,$$

Divide $e_n(s)a_k(s)$ by g (s) to get h(s):

$$
\begin{array}{r|l}
 & d^{(0)}_{k+n-1}s^{n-k-1} + d^{(1)}_{k+n-2}s^{n-k-2} \quad + \dots \\
\hline
s^{2k}+g_{2k-1}s^{2k-1}+\dots+g_1 s+g_0 & d^{(0)}_{k+n-1}s^{k+n-1}+d^{(0)}_{k+n-2}s^{k+n-2} \quad +\dots+d^{(0)}_{n-k-1}s^{n-k-1}+\dots+d^{(0)}_0 \\
 & \underline{d^{(0)}_{k+n-1}s^{k+n-1}+g_{2k-1}d^{(0)}_{k+n-1}s^{k+n-2}+\dots+g_0 d^{(0)}_{k+n-1}s^{n-k-1}} \\
 & d^{(1)}_{k+n-2}s^{k+n-2}+\dots+d^{(1)}_{n-k-2}s^{n-k-3}+\dots+d^{(1)}_0 \\
 & \underline{d^{(1)}_{k+n-2}s^{k+n-2}+\dots+g_0 d^{(1)}_{k+n-2}s^{n-k-2}} \\
 & d^{(2)}_{k+n-3}s^{k+n-3}+\dots+d^{(2)}_0 \\
 & \underline{\dots \quad \dots \quad \dots \quad \dots} \\
 & d^{(n-k)}_{2k-1}s^{2k-1}+\dots+d^{(n-k)}_0
\end{array}
$$

Thus get the recursive relations:

$$d_i^{(1)} = d_i^{(0)} - d_{k+n-1}^{(0)} g_{i+k-n+1},$$
$$i = 0,1,\dots,k+n-2,$$
$$d_i^{(2)} = d_i^{(1)} - d_{k+n-2}^{(1)} g_{i+k-n+2},$$
$$i = 0,1,\dots,k+n-3,$$
$$d_i^{(3)} = d_i^{(2)} - d_{k+n-3}^{(2)} g_{i+k-n+3},$$
$$i = 0,1,\dots,k+n-4,$$
$$\dots\dots\dots$$
$$d_i^{(l)} = d_i^{(l-1)} - d_{k+n-l}^{(l-1)} g_{i+k-n+l},$$
$$d_i^{(n-k)} = d_i^{(n-k-1)} - d_{2k}^{(n-k-1)} g_{i,}$$
$$i = 0,1,\dots,2k-1,$$

When k <0, let g_o=0. In the above the recursive relations, the superscript n in $c_i^{(n)}$ represent the coefficients which are obtained after carrying out the algorithm n steps. And the subscript 'i' in $c_i^{(n)}$ represents the corresponding degree about the variables.

Step 4: By using the basic theorem of algebra, it is obtained that [1],

$$f(s) \equiv h(s).$$

It is found that the coefficient of each term in f(s) in is the linear combination of $a_{0,} a_1 ...., a_{2k-1}$ and the coefficient of each term in h(s) is the linear combination of $b_0, b_1 ..., b_{2K-1}$.

A linear system with 2k+1 unknowns and 2k equations is formed assuming $b_k = 1$ thus, the reduced model is obtained as given equation (2).

II. DESIGN OF SLIDING MODE CONTROLLER

The design of sliding mode controller involves designing a switching surface s(k)=0 to represent a desired system dynamics which is lower order than the given plant and then designing a suitable control, such that any state of the system outside the switching surface is driven to reach the surface in finite time.

Consider the n-th order, m-input discrete-time LTI system

$$x(k+1) = \mathrm{A}x(k) + \mathrm{B}u(k) \qquad (6)$$

$$y(k) = Cx(k)$$

Let, the system be transformed into normal form through a transformation $\hat{x}(k) = Tx(k)$, with the dynamics

$$\hat{x}(k+1) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} u$$

A sliding surface is designed; the reaching law based sliding mode control based by Gao et. al[5], aims at designing a control so as to satisfy the reaching condition given as

$$s(k+1) - s(k) = -q^T s(k) - \varepsilon \operatorname{sgn}(s(k))$$

Where r>0, is a sampling interval of the discrete time system, q>0 is a control parameter that satisfies the condition 1-qr>0. A reaching law based discrete time control law has been derived in [5] for an LTI system in equn. (6) And stable sliding surface S (k) =c^T x (k) =0, to be of the form

$$u(k) = Fx(k) + \gamma \operatorname{sgn}(s(k)) \qquad (7)$$

Where

$$F = -(c^T B_\tau)^{-1} c^T (q\tau - 1 + A_\tau)$$

$$\gamma = -(c^T \Gamma_\tau)^{-1} \varepsilon\tau$$

The inequality for (1-qr>0) must hold to guarantee the stability of reaching phase of the closed loop system. This implies that the choice of r>0 is restricted. Also the presence of the signum term guarantees that once the trajectory has crossed the switching plane the first time, it will cross the plan again in every successive sampling period resulting in a zigzag motion [9].

III. NUMERICAL EXAMPLES

Example 1: Consider the 8th order discrete time system as follows:

$$x(k+1) = \begin{bmatrix} 0.6308 & 0.4185 & -0.0788 & 0.0570 & -0.1935 & -0.0983 & 0.0165 & -0.0022 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} x(k)$$
$$+ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = [0.2102 \quad 0.1395 \quad -0.0262 \quad 0.0190 \quad -0.0645 \quad -0.0328 \quad 0.0055 \quad -0.0008] x(k)$$

An optimum second order reduced model is obtained for the above higher order system by using the proposed method as,

$$z(k+1) = \begin{bmatrix} 1.7390 & -0.8224 \\ 1 & 0 \end{bmatrix} z(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = [0.2301 \quad -0.1470] \; z(k)$$

From section-3; the values ofc^T, q and ε are of the SMC for the low order model is obtained as given below:

c^T= [0.2301 -0.1470] and controller parameters q=2 and ε=0.05. The discrete time control law is derived as given in equn., (7).

$$u(k) = [2.247 \quad 0] \; z(k) - 0.01\operatorname{sgn}([0.2301 \quad -0.1470] \; z(k))$$

The step response of the sliding mode controlled higher order system is shown in fig.1

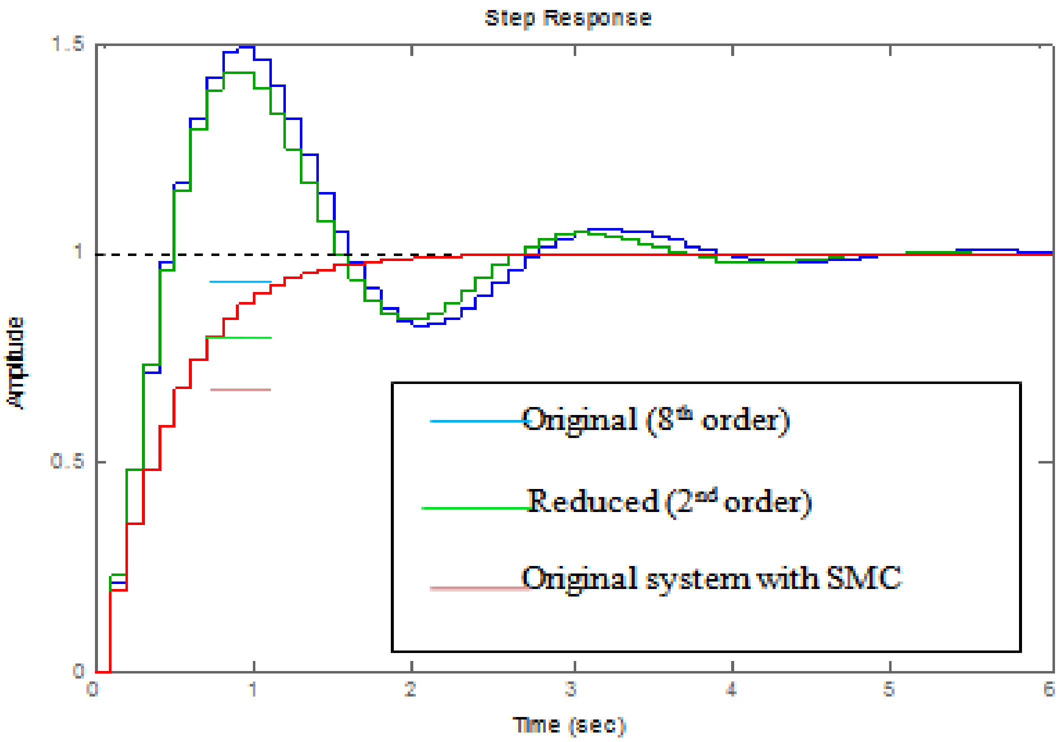

Fig 1: step response of the controlled system

Example 2: Consider the 5th order discrete time system as follows:

$$x(k+1)=\begin{bmatrix} 3.7 & -5.47 & 4.037 & -1.4856 & 0.2173 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} x(k)+\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(k)$$

$$y(k)=\begin{bmatrix} 3 & -8.8860 & 10.0221 & -5.0920 & 0.9811 \end{bmatrix} x(k)$$

An optimum second order model is,

$$z(k+1)=\begin{bmatrix} 1.863 & -0.8754 \\ 1 & 0 \end{bmatrix} z(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k)=\begin{bmatrix} 1.2190 & -0.9702 \end{bmatrix} z(k)$$

A discrete time sliding mode control designed for the reduced order system as given in equn., (7).

$$u(k)=[0.656 \quad 0]z(k)-0.01\operatorname{sgn}([1.219 \quad -0.9702]z(k))$$

In the given fig.2, steps response of the SMC higher order system.

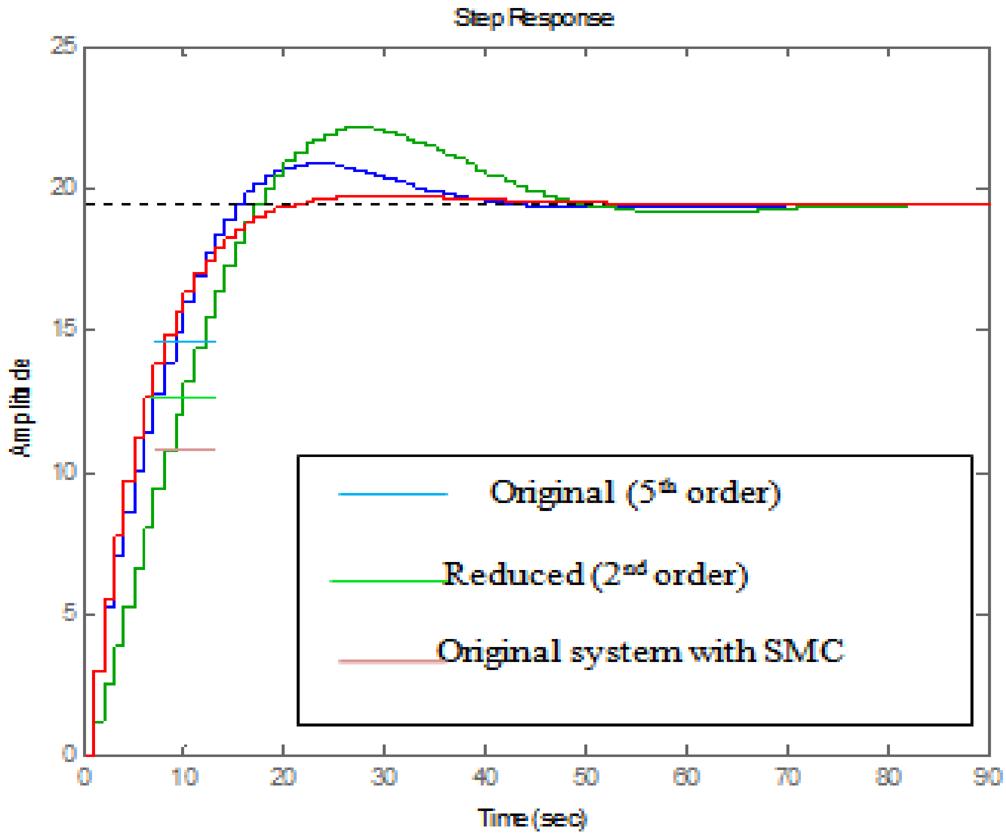

Fig 2: step response of the controlled system

IV. CONCLUSION

A new method for designing discrete-time sliding mode control for higher order system is presented via a new low order optimized model.The new proposed low order model is based on optimising the interpolation points that yields a better stable approximation. This method has been tested on two numerical examples chosen from the literature and the step responses of the original, reduced and controlled system are compared. The results were observed to be satisfactory.

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